Heat Transfer of Two Immiscible Fluids Flow Through A Porous Channel

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***Abstract*— An analysis is performed to study the flow and heat transfer of two immiscible viscous fluids in a horizontal composite porous channel. The flow is modeled by Darcy-Brinkman equation with suitable boundary and interface conditions. The partial differential equations governing the flow and heat transfer have been transformed into a system of ordinary differential equations. The obtained equations are solved analytically taking the viscous and Darcy dissipation into consideration. The effect of the variation in the porous parameter, Prandtl number, Eckert number, ratios of viscosities and thermal conductivities on the velocity and temperature fields for both the fluids is discussed.**

Keywords—Heat transfer, immiscible fluids, porous medium, horizontal channel

NOMENCLATURE

C_P	specific heat at constant pressure
Ec	Eckert number
h	channel half width
K	thermal conductivity
P	pressure
$\Pr$	Prandtl number
s	permeability of porous matrix
T	temperature
T_w	wall temperature
t	time
U_0	average velocity
u	velosity component along the plate

GREEK LETTERS

α	ratio of viscosities
β	ratio of thermal conductivities
σ	porous parameter
ρ	fluid density
μ	viscosity of fluid
θ	non-dimensional temperature

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1,2 quantities for region-I and region-II respectively.

I. INTRODUCTION

Investigation of the flow through porous geometries has many scientific and engineering applications, such as in the utilization of geothermal energy, underground disposal of nuclear waste material, high-performance building insulation, crude oil extraction, petroleum industries, chemical catalytic reactor, solid matrix exchangers and many others. Berman [1] investigated the flow of two dimensional steady incompressible laminar viscous fluids through a porous channel where both channel plates have equal permeability and the flow at the center line of the channel attains maximum. Later Yuan [2] extended the problem for different values of suction and injection Reynolds numbers. The combined natural and force convective flows through a horizontal porous channel connecting two reservoirs have been investigated by Haajizzadeh and Tien [3]. Steady and transient Magnetohydrodynamic flow and heat transfer in a porous medium channel has been analyzed by Chamkha [4]. Mankinde and Mohane [5] investigated the combined effect of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with the saturated porous medium with no-slip boundary condition, further the work is extended by Mehmood and Ali [6] by considering the fluid slip at the lower wall. Nield and Bejan [7], Vafai [8,9], Pop and Ingham [10], Ingham et al. [11] and Bejan et al. [12] have made comprehensive reviews of the studies of heat transfer in relation to the above applications.

In all the above-mentioned research pertain to single fluid model. Most of the problems relating to the petroleum industry, geophysics, plasma physics, and so forth involve multiphase flow situations. The multiphase flow in porous media has attracted considerable attention for many researchers. This is due to the fact that problem involving the multiphase flow, heat transfer and multi-component mass transfer in porous media arises in engineering disciplines such as geothermal energy production, multiphase trickle bed reactors, high level radioactive waste repositories and paper machines. Multiphase flow in porous media has been approached by so called Multiphase Flow Model (MFM) (Abriola [13], Bear [14]) in which various phase are considered as distinct fluids with individual thermodynamic and transport properties and with different flow velocities. The transport phenomena are mathematically described by the basic principles of convection for each phase separately and by appropriate interfacial condition between various phases. Srinivasan and

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Vafai [15] have reported a theoretical study on immiscible fluid systems in a porous medium, taking into account the non-Darcian boundary and inertia effects. Chamkha [16] analyzed the flow of two immiscible fluids in porous and non-porous channels. Two-fluid flow and heat transfer in an inclined channel containing porous and fluid layers was studied analytically by Malashetty et al. [17]. Umavathi et al. [18, 19] analyzed unsteady flow and heat transfer of immiscible fluids in a horizontal channel. Recently, Prathap Kumar [20] has studied the effect of homogeneous and heterogeneous reactions on the dispersion of a solute in a composite porous medium between two parallel plates. The unsteady magnetohydrodynamic flow of two immiscible fluids in a horizontal channel bounded by two parallel porous isothermal plates in the presence of an applied magnetic and electric field is investigated by Linga Raju and Nagavalli [21].

The object of the present work is to study flow and heat transfer of immiscible fluids through the porous media in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices.

II. MATHEMATICALFORMULATION

Consider the flow of an incompressible viscous fluid in a horizontal composite channel as shown in Fig. 1

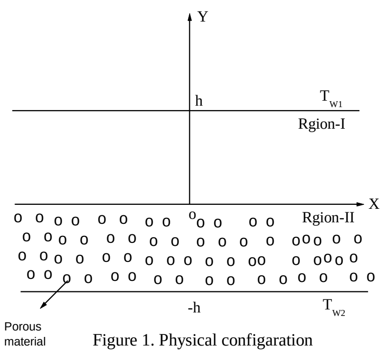

Figure 1. Physical configaration

In order to derive basic equations for the problem under consideration following assumptions are made:

i. The flow is steady, laminar and fully developed with constant physical properties.

ii. The region-I ($0 \le y \le h$) is occupied by a clear viscous fluid and the region-II ($-h \le y \le 0$) is filled with a porous matrix.

iii. All the physical dependent variables except pressure will only dependent of y.

iv. The plates of the channel are assumed to be finite and maintained at constant and different temperatures T_{w1} and T_{w2}.

v. Viscous and Darcy dissipation terms are included in this study.

vi. The flow considered in both the regions is driven by a common pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$ and temperature gradients $T_{w1} - T_{w2}$.

Under these assumptions and taking, $C_{P1} = C_{P2} = C_P$ and $\rho_1 = \rho_2 = \rho_0$ the governing equations of motion and energy are:

Region-I

$$\mu_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} = 0 \tag{1}$$

$$K_1 \frac{\partial^2 T_1}{\partial y^2} + \mu_1 \left(\frac{\partial u_1}{\partial y}\right)^2 = 0 \tag{2}$$

Region-II

$$\mu_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p}{\partial x} - \frac{\mu_2}{s} u_2 = 0 \tag{3}$$

$$K_2 \frac{\partial^2 T_2}{\partial y^2} + \mu_2 \left(\frac{\partial u_2}{\partial y}\right)^2 + \frac{\mu_2}{s} u_2^2 = 0 \tag{4}$$

where u is the x-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which required that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y=0.

The boundary and interference condition for this problem is written as

$$\begin{aligned}
&u_1 = 0, \quad T_1 = T_{w1} \quad at && y = h \\
&u_2 = 0, \quad T_2 = T_{w2} \quad at && y = -h \\
&u_1 = u_2, T_1 = T_2 \quad at && y = 0 \qquad (5) \\
&\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \quad at && y = 0 \\
&K_1 \frac{\partial T_1}{\partial y} = K_2 \frac{\partial T_2}{\partial y} \quad at && y = 0
\end{aligned}$$

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By use of the following non-dimensional quantities:

$$u_i = U_0 u_i^*,\quad y = hy^*,$$

$$\theta_i = \frac{T_{wi} - T_{w2}}{T_{w1} - T_{w2}},\quad \sigma^2 = \frac{h^2}{s},$$

$$P = \frac{h^2}{\mu_1 U_0}\left(-\frac{\partial p}{\partial x}\right), \tag{6}$$

$$\Pr = \frac{\mu_1 C_p}{K_1},\quad Ec = \frac{U_0^2}{C_p (T_{w1} - T_{w2})}$$

and for simplicity dropping the asterisks, equations (1) to (5) becomes

Region-I

$$\frac{d^2 u_1}{dy^2} + P = 0 \tag{7}$$

$$\frac{d^2 \theta_1}{dy^2} + Ec\Pr\left(\frac{du_1}{dy}\right)^2 = 0 \tag{8}$$

Region-II

$$\frac{d^2 u_2}{dy^2} - \sigma^2 u_2 + \alpha P = 0 \tag{9}$$

$$\frac{d^2 \theta_2}{dy^2} + \frac{\beta Ec\Pr}{\alpha}\left(\sigma^2 u_2^2 + \left(\frac{du_2}{dy}\right)^2\right) = 0 \tag{10}$$

where $\alpha = \frac{\mu_1}{\mu_2}$ is the ratio of viscosities and $\beta = \frac{K_1}{K_2}$ is the ratio of thermal conductivities.

The boundary and interference condition in the non-dimensional form becomes

$$\begin{aligned}
&u_1 = 0,\quad \theta_1 = 1 && at && y = h\\
&u_2 = 0,\quad \theta_2 = 0 && at && y = -h\\
&u_1 = u_2, \theta_1 = \theta_2 && at && y = 0\\
&\alpha\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial y}, && at && y = 0\\
&\beta\frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y} && at && y = 0
\end{aligned} \tag{11}$$

Solution: The governing equations (7) to (10) are solved subject to the boundary and interface conditions (11) for the velocity and temperature distributions in both regions.
The results are depicted graphically and are discussed in the next section.

III. RESULTS AND DISCUSSION

In this section, a detailed parametric study has been performed for the effects of the porous parameter, Prandtl number, Eckert number, ratios of viscosities and thermal conductivities on the velocity and temperature profiles which is presented graphically in figures 2 to 8. The following parameters are fixed values $P = 2.0$, $\sigma = 2.0$, $\Pr = 0.71$, $Ec = 0.2$, $\alpha = 1.0$ and $\beta = 1.0$ except one the varying.

Figs. 2 and 3 display the effects of the porous medium parameter σ on the velocity and temperature profiles respectively. As the porous medium parameter σ increases, the velocity and temperature decrease in both regions of the channel. This is expected since the porous matrix represents an obstacle to flow and therefore, reduces its velocity and temperature. This result is also similar to the case of fully developed flow through a porous medium as predicted by Rudraiah and Nagraj [22].

Figs. 4 and 5 respectively shows that an increase in Prandtl number Pr and Eckert number Ec clearly boost temperature in both the regions. Eckert number signifies the quantity of mechanical energy converted via internal friction to thermal energy. Increasing Ec values will therefore cause an increase in thermal energy contribution to the temperature profiles.

Fig. 6 depicts the effect of ratio of thermal conductivities β on the temperature profiles. From this figure it is observed that the temperature profile increases as β increases in both the regions. Also, the maximum temperature in the channel tends to move above the interface towards region-I.

The effect of the viscosities ratio on the velocity and temperature distributions is shown in Figs 7 and 8, respectively. As the viscosity ratio increases, both the velocity and temperature profile decreases. This is due to the fact that as the viscous effects increase, the fluids in both regions become thicker and hence, the flow velocity in the channel is reduced causing the temperature distribution in the channel to reduce as well.

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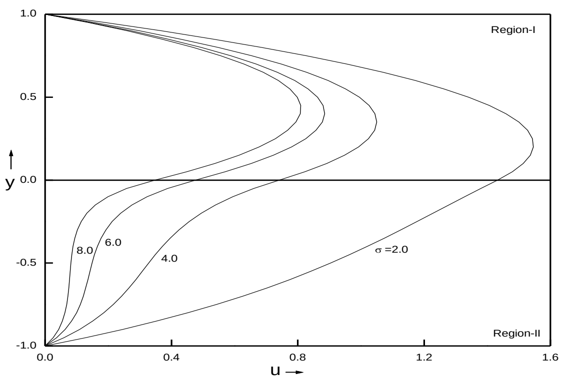

Fig. 2 Velocity profile for different values of σ

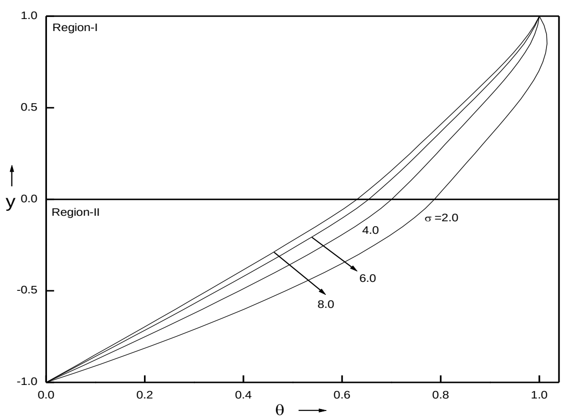

Fig. 3 Temperature profile for different values of σ

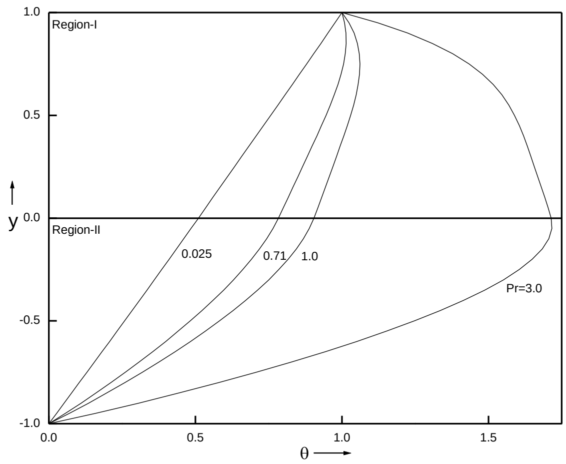

Fig. 4 Temperature profile for different values of Pr

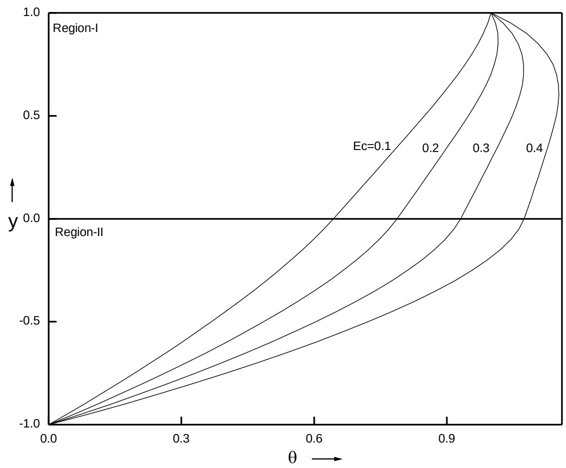

Fig. 5 Temperature profile for different values of Ec

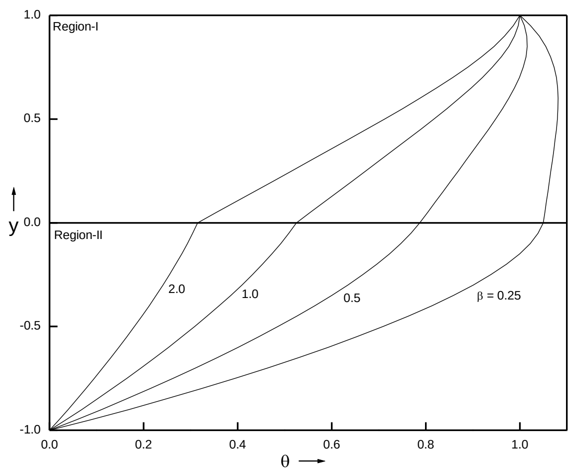

Fig. 6 Temperature profile for different values of β

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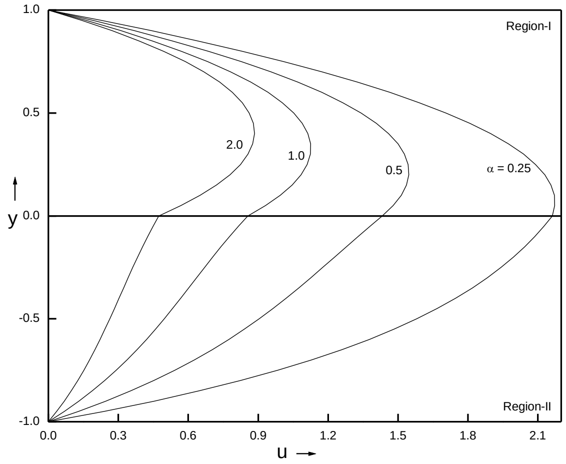

Fig. 7 Velocity profile for different values of α

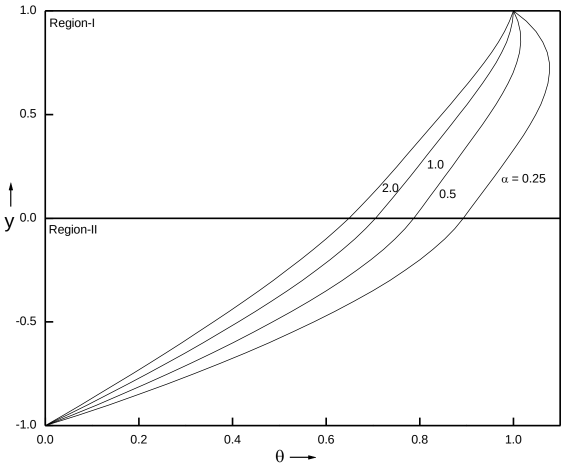

Fig. 8 Temperature profile for different values of α

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