# $\overline{X}$ Charts with Variable Sampling Interval, Control limits, and Warning Limits

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**Abstract - The idea of *variable sampling interval, control limits, and warning limits* (VSICWL) is proposed for $\overline{X}$ charts. Expressions for performance measures for VSICWL $\overline{X}$ charts are developed using a Markov chain approach. The performances of these charts are compared numerically with that of *variable sampling interval and warning limits* (VSIWL) and *variable control and warning limits* (VCWL) $\overline{X}$ charts. It is observed that the statistical performance of a VSICWL $\overline{X}$ char is better than that of VSIWL and VCWL $\overline{X}$ charts while its administrative performance is better than that of VSIWL $\overline{X}$ chart.**

**Key words: Adaptive control chart, average number of samples to signal, average number of switches to signal, steady-state average time to signal.**



## I. INTRODUCTION

Shewhart control chart is an effective on-line process control technique for detecting the occurrence of an assignable cause variability in manufacturing and other processes. It has three design parameters, viz, sampling interval length, sample size, and warning limit(s). The original control chart is static in the sense that its design parameters are kept fixed throughout the period of its implementation. A control chart is termed to be adaptive if at least one of its design parameter is a variable and takes a value for the next sample according to the status of the process indicated by the current sample.

It has been proved in the literature that the adaptive control charts monitor processes more efficiently than the static ones. Reynolds et al. (1988) proposed the first adaptive control chart. It is the $\overline{X}$ chart with variable sampling interval. Then, Prabhu, et al. (1993) and Costa (1994) independently proposed variable sample size $\overline{X}$ charts. Prabhu, et al. (1994) proposed variable sample size and sampling interval $\overline{X}$ charts. Costa (1999) proposed the adaptive $\overline{X}$ charts in which all the three design parameters are variable. Mahadik and Shirke (2009) proposed a special variable sample size and sampling interval $\overline{X}$ chart.

The weakness of an adaptive control chart is the inconvenience in its administration due to frequent switches between the values of its adaptive design parameters. Some modifications have been suggested in the literature in order to lessen this inconvenience. See, for example, Amin and Letsinger (1991), Amin and Hemasinha (1993), and Mahadik (2012a, b).

Some recent references on adaptive control charts include Chen et al. (2011), Dai et al. (2011), Faraz and Saniga (2011), Nenes (2011), Kooli and Limam (2011), and Lee (2011), Zhang, et al. (2011), Lee and Lin (2012), Huang (2013), Mahadik, S. B. (2013a, b), Kuo and Lee (2013), Seif, et al. (2014), and Faraz, et al. (2014).

In the present paper, the idea of *variable sampling interval, control limits, and warning limits* (VSICWL) is proposed for $\overline{X}$ charts. The performances of these charts are compared numerically with that of *variable sampling interval and warning limits* (VSIWL) and *variable control and warning limits* (VCWL) $\overline{X}$ charts. It is observed that the statistical performance of the proposed chart is better than that of VSIWL and VCWL $\overline{X}$ charts while its administrative performance is better than that of VSIWL $\overline{X}$ chart.

The remainder of the paper is organized as follows. The subsequent sections describe the design principle of a VSICWL $\overline{X}$ chart. Expressions for performance measures for this chart are derived. Its statistical and administrative performances are compared numerically with that of VSIWL and VCWL $\overline{X}$ charts. This is followed by the Conclusions.

## II. A VSICWL $\overline{X}$ CHART

Let the quality characteristic $X$ to be monitored follows a normal distribution with mean $\mu$, and a known and constant standard deviation $\sigma$. Suppose $\mu_0$ is the target value of $\mu$. An occurrence of an assignable cause results in a shift of size $\delta$ in $\mu$, where $\delta$ is expressed in □□units. It is assumed that $\delta$ remains constant following the occurrence of a shift until it is detected. A VSICWL $\overline{X}$ chart to monitor $\mu$ is as described below.

The chart statistic is the standardized sample mean $Z_i = \sqrt{n}\left(\overline{X}_i - \mu_0\right)/\sigma$, where $\overline{X}_i$, $i$ = 1, 2, …, is the mean of $i^{th}$ sample of size is $n$ drawn on $X$. Note that when $\mu = \mu_0$, $Z_i \sim N$ (0, 1), and when $\mu = \mu_0 + \delta\sigma$, $Z_i \sim N$ ($\sqrt{n}\delta$, 1). Let $t(i)$ be the length of sampling interval between the $(i-1)^{st}$ and $i^{th}$ trials, $i$ = 1, 2, …. Let $L(i)$ be the distance of each control limit and $w(i)$ be the distance of each warning limit of the chart from its centerline for the $i^{th}$ trial. The values of $(t(i), L(i), w(i))$ can be either ($t_1$, $L_1$, $w_1$) or ($t_2$, $L_2$, $w_2$), where $t_1$, $t_2$, $L_1$, $L_2$, $w_1$, and $w_2$ are such that $t_{max} \geq t_1 \geq t_2 \geq t_{min}$, $t_{max}$ and $t_{min}$ being the longest and shortest possible sampling intervals, respectively, $\infty > L_1 \geq L_2 > 0$, $0 < w_1 < L_1$, $0 < w_2 < L_2$, and $w_1 \geq w_2$. When $Z_{i-1}$ falls within $(-L(i-1), L(i-1))$, the triplet of values of $(t(i), L(i), w(i))$, $i = 2, 3, \ldots$, between ($t_1$, $L_1$, $w_1$) and ($t_2$, $L_2$, $w_2$) is chosen according to the following rule

$$(t(i), L(i), w(i)) = \begin{cases} (t_1, L_1, w_1), & \text{if } Z_{i-1} \in I_1 \\ (t_2, L_2, w_2), & \text{if } Z_{i-1} \in I_2, \end{cases}$$

where $I_1 = [-w(i-1), w(i-1)]$ and $I_2 = (-L(i-1), -w(i-1)) \cup (w(i-1), L(i-1))$.

The chart signals an out-of-control state at the $i^{th}$ trial, $i = 1, 2, \ldots$, if $Z_i$ falls beyond $(-L(i), L(i))$. Figure 1 shows a typical VSICWL $\bar{X}$ chart. We note that its appearance is same as the VCWL $\bar{X}$ chart proposed by Mahadik (2013c).

At start-up the values of $(t(1), L(1), w(1))$ can be chosen using an arbitrary probability distribution. In practice, it is recommended to use the triplet ($t_2$, $L_2$, $w_2$) for the first trial to provide additional protection against the problems that may exist initially. The trial following an out-of-control signal is again treated to be the first trial and the mechanism of choosing $(t(i), L(i), w(i))$ is restarted from that.

Note that when $L_1 = L_2$, a VSICWL $\bar{X}$ chart is a VSIWL $\bar{X}$ chart proposed by Mahadik (2013d) and when $t_1 = t_2$, it is a VCWL $\bar{X}$ chart proposed by Mahadik (2013c). In the next section, expressions for performance measures for a VSICWL $\bar{X}$ chart are derived.

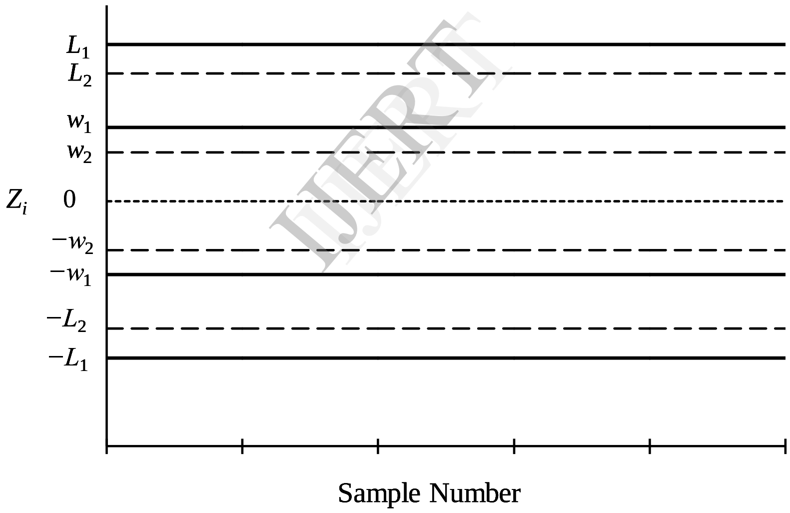


Figure 1: A VSICWL $\bar{X}$ chart

## III. PERFORMANCE MEASURES

The appropriate measures of statistical performance of a VSICWL $\bar{X}$ chart are the *steady-state average time to signal* (SSATS) and the *average number of samples to signal* (ANSS). SSATS is the expected value of the time between a shift that occurs at some random time after the process starts and the time the chart signals while ANSS is the expected value of the number of samples taken from a shift to the time the chart signals. The administrative performance can be measured through *average number of switches to signal* (ANSW). ANSW is the expected value of the number of switches between two sampling interval lengths from a shift to the signal.

Let $SSATS_\delta$, $ANSS_\delta$, and $ANSW_\delta$ be the SSATS, ANSS, and ANSW, respectively of a control chart when the process mean has shifted from $\mu_0$ to $\mu_1 = \mu_0 + \delta\sigma$. The expressions for $SSATS_\delta$ and $ANSS_\delta$ are derived below using a Markov chain approach.

Henceforth, the $i^{th}$ trial refers to the $i^{th}$ trial after a shift when $i > 0$ and the last trial before the $(i+1)^{st}$ trial when $i \leq 0$. Also, $Z_i$ refers to the sample point corresponding to the $i^{th}$ trial.

Define the three states 1, 2, and 3 of the Markov Chain corresponding to whether a sample point for the $i^{th}$ trial is plotted in $I_1$, $I_2$, and $I_3 = (-\infty, -L(i)] \cup [L(i), \infty)$, respectively $i = 1, 2, \ldots$. State 3 is the absorbing state, as the process of taking samples is restarted when a sample point falls in region $I_3$. The transition probability matrix is given by

$$\mathbf{P}^{\boldsymbol{\delta}} = \begin{bmatrix} p_{11}^{\delta} & p_{12}^{\delta} & p_{13}^{\delta} \\ p_{21}^{\delta} & p_{22}^{\delta} & p_{23}^{\delta} \\ 0 & 0 & 1 \end{bmatrix},$$

where $p_{jk}^{\delta}$ is the transition probability that $j$ is the prior state and $k$ is the current state, when the process mean has shifted by $\delta\sigma$. For example,

$$\begin{aligned} p_{12}^{\delta} &= \Pr_{\delta}[Z_i \in I_2 \mid Z_{i-1} \in I_1] \\ &= \Pr_{\delta}[Z_i \in I_2 \mid L(i) = L_1, w(i) = w_1] \\ &= \Pr_{\delta}[-L_1 < Z_i < -w_1] \\ &\quad + \mathrm{P}[w_1 < Z_i < L_1] \\ &= \Phi(-w_1 - \sqrt{n}\delta) - \Phi(-L_1 - \sqrt{n}\delta) + \Phi(L_1 \\ &\quad - \sqrt{n}\delta) - \Phi(w_1 - \sqrt{n}\delta), \end{aligned}$$

where $\Phi(\cdot)$ is the cumulative distribution function of standard normal variate.

Then, $\mathrm{SSATS}_{\delta}$ and $\mathrm{ANSS}_{\delta}$ are given by

$$\mathrm{SSATS}_{\delta} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P}_1^{\boldsymbol{\delta}})^{-1}\boldsymbol{t} - \mathrm{E}(U) \qquad (1)$$

and

$$\mathrm{ANSS}_{\delta} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P}_1^{\boldsymbol{\delta}})^{-1}\mathbf{1},$$

where $\mathbf{I}$ is the identity matrix of order 2, $\mathbf{P}_1^{\boldsymbol{\delta}}$ is the sub matrix of $\mathbf{P}^{\boldsymbol{\delta}}$ that contains the probabilities associated with the transient states only, $\boldsymbol{t}' = (t_1, t_2)$, $\mathbf{1}' = (1, 1)$, and $\boldsymbol{b}' = (b_1, b_2)$, $b_j$ being the conditional probability that $Z_0$ falls in $I_j$ given that it falls within the control limits, $j = 1, 2$. We note that $b_2 = 1 - b_1$. The Expression for $b_1$ is derived by Mahadik (2013d) and is

$$b_1 = \frac{\dfrac{2\Phi(w_2) - 1}{2\Phi(L_2) - 1}}{1 - \left[\dfrac{2\Phi(w_1) - 1}{2\Phi(L_1) - 1} - \dfrac{2\Phi(w_2) - 1}{2\Phi(L_2) - 1}\right]}.$$

E($U$) in equation (1) is the expected value of the time $U$ between the $0^{th}$ trial and the shift. Assuming that an assignable cause of a process shift occurs according to a Poisson process, it can be shown that $\mathrm{E}(U) = \mathrm{E}[t(1)]/2$. Hence,

$$\mathrm{SSATS}_{\delta} = \boldsymbol{b}'(\mathbf{I} - \mathbf{P}_1^{\boldsymbol{\delta}})^{-1}\boldsymbol{t} - \mathrm{E}[t(1)]/2.$$

The expression for $\mathrm{ANSW}_{\delta}$ is also derived using a Markov Chain approach. For, let

$$Y_i = \begin{cases} 1, \text{ if } (Z_{i-1} \in I_1, Z_i \in I_2) \\ 2, \text{ if } (Z_{i-1} \in I_2, Z_i \in I_1) \\ 3, \text{ if } (Z_{i-1} \in I_1, Z_i \in I_1) \\ 4, \text{ if } (Z_{i-1} \in I_2, Z_i \in I_2) \\ 5, \text{ if } |Z_i| > L(i) \end{cases}, \; i = 1, 2, \ldots$$

It is easy to see that $\{Y_i, i = 1, 2, \ldots\}$ is a Markov Chain with transition probability matrix

$$\mathbf{Q}^{\boldsymbol{\delta}} = \begin{bmatrix} 0 & p_{21}^{\delta} & 0 & p_{22}^{\delta} & p_{23}^{\delta} \\ p_{12}^{\delta} & 0 & p_{11}^{\delta} & 0 & p_{13}^{\delta} \\ p_{12}^{\delta} & 0 & p_{11}^{\delta} & 0 & p_{13}^{\delta} \\ 0 & p_{21}^{\delta} & 0 & p_{22}^{\delta} & p_{23}^{\delta} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, the expression for ANSW is given by

$$\mathrm{ANSW}_{\delta} = \boldsymbol{a}'(\mathbf{I}_1 - \mathbf{Q}_1^{\boldsymbol{\delta}})^{-1}\boldsymbol{e},$$

where, $\mathbf{I}_1$ is the identity matrix of order 4, $\mathbf{Q}_1^{\boldsymbol{\delta}}$ is the sub matrix of $\mathbf{Q}^{\boldsymbol{\delta}}$ that contains the probabilities associated with the transient states only, $\boldsymbol{e} = (1, 1, 0, 0)'$, and $\boldsymbol{a} = (a_1, a_2, a_3, a_4)'$, $a_j$ being the initial probability of state $j$, $j = 1, 2, 3, 4$, given by

$$a_j = \Pr_{\delta}[Y_1 = j] = \begin{cases} b_1 p_{12}^{\delta}, & j = 1 \\ b_2 p_{21}^{\delta}, & j = 2 \\ b_1 p_{11}^{\delta}, & j = 3 \\ b_2 p_{22}^{\delta}, & j = 4 \end{cases}.$$

The following section evaluates the performances of VSICWL $\bar{X}$ charts in comparison with the VSIWL and VCWL $\bar{X}$ charts.

## IV. PERFORMANCE EVALUATION OF VSICWL $\bar{X}$ CHARTS

In this section, the performances of VSICWL $\bar{X}$ charts are evaluated by comparing that with that of VSIWL and VCWL $\bar{X}$ charts. The three charts are designed such that their in-control statistical performances are matched. Such charts are called matched charts. The matching of the charts is achieved by choosing the values of design parameters of the charts such that E[$t$(1)] as well as P[$|Z_1| > L(i)$] are the same for all the charts.

Obviously, the ANSS values of the matched VSIWL and VSICWL charts are the same. Further, we note that the VCWL charts are free from the problem of switches between the sampling interval lengths and thus have better administrative performance than that of VSICWL charts.

Table 1 shows the design parameters of two sets of the matched VCWL, VSIWL, and VSICWL $\bar{X}$ charts while tables 2, 3 and 4, respectively, show the ANSS, SSATS, and ANSW values of these charts for the shifts in mean of various sizes. These tables clearly indicate that the SSATS values of VSICWL charts are uniformly smaller than that of VCWL and VSIWL charts for a wide range of shift size. The ANSS values of VSICWL charts are smaller than that of VSIWL charts for small to moderate shifts and are similar to that of VSIWL charts for large shifts. Also, the out-of-control ANSW values of VSICWL charts are smaller than that of VSIWL charts although the in-control ANSW values of the two charts are almost the same. Thus, the statistical performance of a VSICWL chart is superior to that of VSIWL and VCWL charts while its administrative performance is superior to that of a VSIWL chart.

Table 1: Design parameters of the matched charts

| Chart | $n$ | $t_1$ | $t_2$ | $L_1$ | $L_2$ | $w_1$ | $w_2$ |
|---|---|---|---|---|---|---|---|
| Set 1 | | | | | | | |
| VCWL | 4 | 1.00 | 1.00 | 3.20 | 2.26 | 2.00 | 1.00 |
| VSIWL | 4 | 1.05 | 0.20 | 3.00 | 3.00 | 2.00 | 1.00 |
| VSICWL | 4 | 1.05 | 0.20 | 3.20 | 2.26 | 2.00 | 1.00 |
| Set 2 | | | | | | | |
| VCWL | 3 | 1.00 | 1.00 | 3.20 | 2.15 | 2.00 | 1.75 |
| VSIWL | 3 | 1.04 | 0.10 | 3.00 | 3.00 | 2.00 | 1.75 |
| VSICWL | 3 | 1.04 | 0.10 | 3.20 | 2.15 | 2.00 | 1.75 |



Table 2: ANSS values of the matched charts

| Chart | ANSS values for the shift in mean of size | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $0\sigma$ | $0.25\ \sigma$ | $0.5\sigma$ | $0.75\sigma$ | $1\sigma$ | $1.5\sigma$ | $2\sigma$ | $2.5\sigma$ | $3\sigma$ |
| Set 1 | | | | | | | | | |
| VCWL/ VSICWL | 370.40 | 138.25 | 30.93 | 9.44 | 4.26 | 1.81 | 1.21 | 1.03 | 1.00 |
| VSIWL | 370.40 | 155.22 | 43.89 | 14.97 | 6.30 | 2.00 | 1.19 | 1.02 | 1.00 |
| Set 2 | | | | | | | | | |
| VCWL/ VSICWL | 370.43 | 173.11 | 48.81 | 16.09 | 6.85 | 2.41 | 1.45 | 1.13 | 1.02 |
| VSIWL | 370.40 | 184.24 | 60.69 | 22.48 | 9.76 | 2.91 | 1.47 | 1.10 | 1.01 |

Table 3: SSATS values of the matched charts

| Chart | SSATS values for the shift in mean of size | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $0\sigma$ | $0.25\ \sigma$ | $0.5\sigma$ | $0.75\sigma$ | $1\sigma$ | $1.5\sigma$ | $2\sigma$ | $2.5\sigma$ | $3\sigma$ |
| Set 1 | | | | | | | | | |
| VCWL | 369.90 | 137.75 | 30.43 | 8.94 | 3.76 | 1.31 | 0.71 | 0.53 | 0.50 |
| VSIWL | 369.90 | 151.62 | 39.90 | 11.94 | 4.19 | 0.97 | 0.56 | 0.51 | 0.50 |
| VSICWL | 370.03 | 133.57 | 26.65 | 6.67 | 2.43 | 0.83 | 0.56 | 0.51 | 0.50 |
| Set 2 | | | | | | | | | |
| VCWL | 369.93 | 172.61 | 48.31 | 15.59 | 6.35 | 1.91 | 0.95 | 0.63 | 0.52 |
| VSIWL | 369.90 | 180.42 | 55.74 | 18.30 | 6.65 | 1.35 | 0.64 | 0.52 | 0.50 |
| VSICWL | 369.93 | 169.56 | 44.91 | 13.24 | 4.81 | 1.20 | 0.63 | 0.52 | 0.50 |

Table 4: ANSW values of the matched charts

| Chart | ANSW values for the shift in mean of size | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $0\sigma$ | $0.25\ \sigma$ | $0.5\sigma$ | $0.75\sigma$ | $1\sigma$ | $1.5\sigma$ | $2\sigma$ | $2.5\sigma$ | $3\sigma$ |
| Set 1 | | | | | | | | | |
| VSIWL | 29.84 | 19.26 | 10.29 | 5.27 | 2.50 | 0.55 | 0.14 | 0.02 | 0.00 |
| VSICWL | 30.30 | 16.88 | 6.60 | 2.62 | 1.23 | 0.49 | 0.18 | 0.03 | 0.00 |
| Set 2 | | | | | | | | | |
| VSIWL | 30.30 | 20.90 | 12.07 | 6.77 | 3.54 | 0.88 | 0.28 | 0.08 | 0.01 |
| VSICWL | 30.77 | 19.98 | 9.85 | 4.91 | 2.57 | 0.87 | 0.36 | 0.12 | 0.02 |
| | | | | | | | | | |

## V. CONCLUSIONS

The proposed chart is the fusion of VSIWL and VCWL $\bar{X}$ charts. The expressions for performance measures, viz, SSATS, ANSS, and ANSW for this chart are developed using a Markov chain approach. This chart exhibit better statistical performance than that of VSIWL and VCWL $\bar{X}$ charts. Also, its administrative performance is better than that of VSIWL $\bar{X}$ chart.

## APPENDIX: NOTATION

$X$ : quality characteristic to be monitored
$\mu$ : mean of $X$
$\sigma$ : standard deviation of $X$
$\mu_0$ : target value of $\mu$.
$\delta$ : size of shift in $\mu$ in $\sigma$ units
$Z_i$ : standardized sample mean
$n$ : sample size
$t(i)$ : length of the sampling interval between the $(i-1)^{st}$ and $i^{th}$ trials
$w(i)$ : distance of each warning limit from the centerline for the $i^{th}$ trial
$L(i)$ : distance of each control limit from the centerline for the $i^{th}$ trial
$I_1$ : $[-w(i), w(i)]$ for $i^{th}$ trial
$I_2$ : $(-L(i), -w(i)) \cup (w(i), L(i))$ for $i^{th}$ trial
$I_3$ : $(-\infty, -L(i)] \cup [L(i), \infty)$
$t_1$ : long sampling interval
$t_2$ : short sampling interval
$w_1$ : distance of each warning limit from the centerline for the $i^{th}$ trial when $Z_{i-1} \in I_1$
$w_2$ : distance of each warning limit from the centerline for the $i^{th}$ trial when $Z_{i-1} \in I_2$
$L_1$ : distance of each control limit from the centerline for the $i^{th}$ trial when $Z_{i-1} \in I_1$
$L_2$ : distance of each control limit from the centerline for the $i^{th}$ trial when $Z_{i-1} \in I_2$
$p_{jk}^{\delta}$ : $\Pr_{\delta}[Z_i \in I_k \mid Z_{i-1} \in I_j]$
$\Phi(\cdot)$ : cumulative distribution function of standard normal variate
$b_j$ : conditional probability that $Z_0$ falls in $I_j$ given that it falls within its control limits
$U$ : time between the $0^{th}$ trial and the shift



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