Weak Open Sets in Ideal Bitopological Spaces

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Abstract

The paper introduces (1,2)-semi-I-open sets, (1,2)-pre-I-open sets,(1,2)-α-I-open sets and (1,2)-β-I-open sets in ideal bitopological spaces. The relationship between them are established. Some of their basic properties are discussed. As an application some new types of sets are introduced and the relationship between them is derived.

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1 Introduction

kelly[8] has introduced the concept of bitopological spaces by defining two topologies on a set. Lellis Thivagar et.al.[6] have defined(1,2)semi-open,(1,2)pre-open,(1,2)α-open and (1,2)β-open sets in bitopological spaces.Ideals play an important role in topology. Jankovic and Hamlet[7] have introduced the notion of I-open sets in topological spaces. Kuratowski[10] has introduced local function of a set with respect to a topology τ and an ideal and its properties are investigated. Hatir and Noiri [5] introduced α-I-open sets, semi-I-open sets and β-I-open sets and derived a decomposition of continuity.In this paper a new closure operator using $\tau_1\tau_2$-open set is defined and some of its basic properties are discussed.(1,2)-semi-I-open sets, (1,2)-pre-I-open sets,(1,2)-α-I-open sets and (1,2)-β-I-open sets are defined in ideal bitopological spaces. The relationship between them and other existing sets are derived. Some properties of the sets are discussed.

2 Preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset A of (X, τ) are denoted by $Int(A)$ and $Cl(A)$, respectively. Throughout the present paper (X, τ) and (Y, σ)(or X and Y) represent non-empty topological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition 2.1 *A subset A of a space X is called*

1. *a semi-open set [3] if* $A \subseteq Cl(Int(A))$
2. *a pre-open set [4] if* $A \subseteq Int(Cl(A))$
3. *an* α*-open set [11] if* $A \subseteq Int(Cl(Int(A)))$
4. *a* β*-open set[1] if* $A \subseteq Cl(Int(Cl(A)))$
5. *a* α^**-set[5] if* $Int(A) = Int(Cl(A))$
6. *a C-set[5] if* $A = U \cap V$ *where U is open and V is an* α^**-set*

7. *a t-set[12] if* $Int(A) = Int(Cl(A))$

8. *a B-set[12] if* $A = U \cap T$ *where* U *is an open set and* T *is a t-set*

The complement of a semi-open (resp. pre open, α*- open and* β*) set is called a semi-closed (resp. pre closed,* α*-closed and* β*-closed) set.*

Definition 2.2 (5) *An ideal* I *on a topological space* (X, τ) *is a non empty collection of subsets of* X *which satisfies the following conditions. i)*$A \in I$ *and* $B \subseteq A$ *implies* $B \in I$*.*
ii)$A \in I$ *and* $B \in I$ *implies* $A \cup B \in I$*.*
An ideal topological space (X, τ) *with an ideal* I *on* X *is denoted by* (X, τ, I)*.*

Definition 2.3 (5) *Let* (X, τ, I) *be an ideal topological space and* $A \subseteq X$*.* $A^*(I, \tau) = \{x \in X : A \cap U \notin I \ forevery \ U \in \tau(X, x)\}$ *is called the local function of* A *with respect to* I *and* τ*. For every ideal topological space* (X, τ, I) *there exists a topology* $\tau^*(I)$ *finer than* τ *defined as* $\tau^*(I) = \{U \subseteq X : Cl^*(X - U) = X - U\}$*generated by the base* $\beta(I, J) = \{U - J : U \in \tau \text{ and } J \in I\}$ *and* $Cl^*(A) = A \cup A^*$*.*

Definition 2.4 (5) *A subset* A *of an ideal topological space* (X, τ, I) *is called*

1. *semi-I-open if* $A \subseteq Cl^*(Int(A))$
2. *pre-I-open if* $A \subseteq Int(Cl^*(A))$
3. α*-I-open if* $A \subseteq Int(Cl^*(Int(A)))$
4. β*-I-open if* $A \subseteq Cl(Int(Cl^*(A)))$

Definition 2.5 *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is called*

(i) $\tau_1\tau_2$*-open[6] if* $A \in \tau_1 \cup \tau_2$

(ii) $\tau_1\tau_2$*-closed[6] if* $A^c \in \tau_1 \cup \tau_2$*.*

Definition 2.6 (6) *Let* A *be a subset of* (X, τ_1, τ_2)*. Then* $\tau_1\tau_2$*-Cl(*A*) denotes the* $\tau_1\tau_2$*-closure of* A *and is defined as the intersection of all* $\tau_1\tau_2$*-closed sets containing* A*. Also* $\tau_1\tau_2$*-*$Int(A)$ *denotes the* $\tau_1\tau_2$*-interior of* A *and is defined as the union of all* $\tau_1\tau_2$*-open sets contained in* A*.*

Definition 2.7 *A subset* A *of* (X, τ_1, τ_2) *is said to be*

(i)$(1,2)\alpha$*-open[6] if* $A \subseteq \tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(A)))$*.*

(ii)(1,2)semi-open[6] if $A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(A))$

(iii)(1,2)pre-open[6] if $A \subseteq \tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A))$ *and*

(iv)(1,2)semi-pre-open[6](briefly (1,2)sp-open) if
$A \subseteq \tau_1\tau_2\text{-}Cl(\tau_1\text{-}Int(\tau_1\tau_2\text{-}Cl(A)))$

3 New closure operator

We define a new closure operator in terms of $\tau_1\tau_2$-open sets.

Definition 3.1 *Let* (X, τ_1, τ_2) *be a bitopological space. A bitopological space together with an ideal is defined to be an ideal bitopological space and is denoted as* (X, τ_1, τ_2, I)*. Let* $A \subseteq X$ *the local function with respect to the* $\tau_1\tau_2$*-open sets is defined as* $A^*(X, I, \tau_1, \tau_2) = \{x \in X : A \cap U \notin I \ forevery \ U \in \tau_1\tau_2O(X, x)\}$ *is called the local function of* A *with respect to* I *and the two topologies* τ_1 *and* τ_2*.In short it will be denoted as* $A^*_{\tau_1\tau_2}$*.* $\tau_1\tau_2O(X, x)$ *denotes the collection of all* $\tau_1\tau_2$*-open sets containing the point* x*.*

Proposition 3.2 *Let (X, τ_1, τ_2, I) be an ideal bitopological space and A be a subset of X, then $A^*_{\tau_1\tau_2} \subseteq A^*(I, \tau_1)$ for every subset A of X.*

Proof. Let $x \notin A^*(I, \tau_1)$. Then there is a τ_1-open set U containing x such that $A \cap U \in I$. Since every τ_1-open set is $\tau_1\tau_2$-open then $x \notin A^*(I, \tau_1\tau_2)$.

Remark 3.3 *The converse of the proposition 3.2 need not be true.*

Example 3.4 *Let $X = \{a, b, c, d\}, \tau_1 = \{\phi, \{a\}, X\}, \tau_2 = \{\phi, X\{b\}\}, I = \{\phi, \{c\}\}$*
$\tau_1\tau_2O(X) = \{\phi, X, \{a\}, \{b\}\}.A = \{\{c\}\}, A^* = X, A^*_{\tau_1\tau_2} = \{a, c, d\}$

Remark 3.5 *Neither $A^*_{\tau_1\tau_2} \subseteq A$ nor $A \subseteq A^*_{\tau_1\tau_2}$.*

Example 3.6 *Let $X = \{a, b, c, d\}, \tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}, \tau_2 = \{\phi, X, \{c\}\}, I = \{\phi, \{b\}\}$. If $A = \{a, b, c\}, A^*_{\tau_1\tau_2} = \{a, c, d\}$, if $A = \{b, d\}, A^*_{\tau_1\tau_2} = \{d\}$*

Theorem 3.7 *For subsets A and B of an ideal bitopological space (X, τ_1, τ_2) the following statements are true.*

(i)If $A \subseteq B \Rightarrow A^*_{\tau_1\tau_2} \subseteq B^*_{\tau_1\tau_2}$.

(ii) $(A \cup B)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2}$.

(iii)$(A \cap B)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} \cap B^*_{\tau_1\tau_2}$.

(iv) $A^*_{\tau_1\tau_2} - B^*_{\tau_1\tau_2} \subseteq (A - B)^*_{\tau_1\tau_2}$.

(v) If $A \in I \Rightarrow A^*_{\tau_1\tau_2} = \phi$.

(vi) $B \in I \Rightarrow (A \cup B)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} = (A - B)^*_{\tau_1\tau_2}$.

(vii) If $(A - B), (B - A) \in I$ then $A^*_{\tau_1\tau_2} = B^*_{\tau_1\tau_2}$.

(viii) $A^*_{\tau_1\tau_2} - (A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} \subseteq (A - A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2}$.

(ix) $(A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2}$

(x)$A^*_{\tau_1\tau_2} = \tau_1\tau_2\text{-}Cl(A^*_{\tau_1\tau_2})$.

(xi) If $U \in \tau_1$ then $U \cap A^*_{\tau_1\tau_2} = U \cap (U \cap A)^*_{\tau_1\tau_2} \subseteq (U \cap A)^*_{\tau_1\tau_2}$.

(xii)If $I_1 \in I$then $(A - I_1)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} = (A \cup I_1)^*_{\tau_1\tau_2}$.

Proof.

(i)Let $A \subseteq B$ and $x \notin B^*_{\tau_1\tau_2}$.There exists $U \in \tau_1\tau_2O(X, x)$ such that $U \cap B \in I$. Since $A \subseteq B$, $U \cap A \in I$ and hence $x \notin A^*_{\tau_1\tau_2}$.Hence $A^*_{\tau_1\tau_2} \subseteq B^*_{\tau_1\tau_2}$.

(ii) $A \subseteq A \cup B, B \subseteq A \cup B, A^*_{\tau_1\tau_2} \subseteq (A \cup B)^*_{\tau_1\tau_2}, B^*_{\tau_1\tau_2} \subseteq (A \cup B)^*_{\tau_1\tau_2}$. Thus $A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2} \subseteq (A \cup B)^*_{\tau_1\tau_2}$. Claim $(A \cup B)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2}$. Let $x \in (A \cup B)^*_{\tau_1\tau_2}$. Then for every $\tau_1\tau_2$-open set U of x such that $(U \cap A) \cup (U \cap B) = U \cap (A \cup B) \notin I$. Therefore $U \cap A \notin I$ or $U \cap B \notin I$. This implies that $x \in A^*_{\tau_1\tau_2}$ or $x \in B^*_{\tau_1\tau_2}$. That is $x \in A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2}$. Therefore we have $(A \cup B)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2}$.Thus we get $(A \cup B)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2}$.

(iii)$A \cap B \subseteq A, A \cap B \subseteq B$,by (i)$(A \cap B)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} \cap B^*_{\tau_1\tau_2}$.

(iv) For every subset A and B of X, $A = (A - B) \cup (A \cap B)$. By(ii) $A^*_{\tau_1\tau_2} = (A - B)^*_{\tau_1\tau_2} \cup (A \cap B)^*_{\tau_1\tau_2}$ and hence $A^*_{\tau_1\tau_2} - B^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} \cap (X - B^*_{\tau_1\tau_2}) = [(A - B)^*_{\tau_1\tau_2} \cup (B \cap A)^*_{\tau_1\tau_2} \cap (X - B^*_{\tau_1\tau_2})] = [(A - B)^*_{\tau_1\tau_2} \cap (X - B^*_{\tau_1\tau_2})] \cup [(A \cap B)^*_{\tau_1\tau_2} \cap (X - B^*_{\tau_1\tau_2})] \subseteq (A - B)^*_{\tau_1\tau_2} - B^*_{\tau_1\tau_2}+] \cup \phi \subseteq (A - B)^*_{\tau_1\tau_2}$.

(v) By definition if $A \in I$ then $A^*_{\tau_1\tau_2} = \phi$.

(vi)Let $B \in I$ and by (ii),(iv) and (v) $(A \cup B)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} \cup B^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2} = (A - B)^*_{\tau_1\tau_2}$.

(vii) Let $E = (A-B)\cup(B-A) \in I$ then $A = (A-B)\cup(A\cap B), B = (A\cap B)\cup(B-A)$,By (ii) and (vi)$A^*_{\tau_1\tau_2} = (A\cap B)^*_{\tau_1\tau_2} = B^*_{\tau_1\tau_2}$.

(viii) By(iv) $A^*_{\tau_1\tau_2} - (A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} \subseteq (A - A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2}$.

(ix) Let $x \in (A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2}$. Then for every $U \in \tau_1\tau_2 O(X,x), U\cap A^*_{\tau_1\tau_2} \notin I$ and hence $U\cap A^*_{\tau_1\tau_2} \neq \phi$. Let $y \in U\cap A^*_{\tau_1\tau_2}$. Then $U \in \tau_1\tau_2 O(X,y)$ and $y \in A^*_{\tau_1\tau_2}$. Hence $U\cap A \notin I$ and $x \in A^*_{\tau_1\tau_2}$. This implies that $(A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2}$.

(x)In general $A^*_{\tau_1\tau_2} \subseteq \tau_1\tau_2$-$Cl(A^*_{\tau_1\tau_2})$.Let $x \in \tau_1\tau_2 - Cl(A^*_{\tau_1\tau_2}))$. Then $A^*_{\tau_1\tau_2}\cap U \neq \phi$ for every $U \in \tau_1\tau_2 O(X,x)$. Therefore there exist some $y \in A^*_{\tau_1\tau_2}\cap U$ and $U \in \tau_1\tau_2 O(X,x)$. Since $y \in A^*_{\tau_1\tau_2}, A\cap U \notin I$ and hence $x \in A^*_{\tau_1\tau_2}$. Hence $\tau_1\tau_2$-$Cl(A^*_{\tau_1\tau_2}) \subseteq (A^*_{\tau_1\tau_2})$. Hence $A^*_{\tau_1\tau_2} = \tau_1\tau_2$-$Cl(A^*_{\tau_1\tau_2})$.

(xi) If $U \in \tau_1 O(X)$ and $x \in U\cap A^*_{\tau_1\tau_2}$. Then $x \in U$ and $x \in A^*_{\tau_1\tau_2}$. Let V be any $\tau_1\tau_2$-open set containing x. Then $V\cap U \in \tau_1\tau_2 O(X,x)$ and $V\cap(U\cap A) = (V\cap U)\cap A \notin I$. This shows that $x \in (U\cap A)^*_{\tau_1\tau_2}$ and hence $U\cap A^*_{\tau_1\tau_2} \subseteq (U\cap A)^*_{\tau_1\tau_2}$. Hence $U\cap A^*_{\tau_1\tau_2} \subseteq U\cap(U\cap A)^*_{\tau_1\tau_2}$. By(i) $(U\cap A)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2}$ and $U\cap A^*_{\tau_1\tau_2} \supseteq U\cap(U\cap A)A^*_{\tau_1\tau_2}$. Therefore $U\cap A^*_{\tau_1\tau_2} = U\cap(U\cap A)^*_{\tau_1\tau_2} \subseteq (U\cap A)^*_{\tau_1\tau_2}$.

(xii) Since $A - I_1 \subseteq A$.By (i) $(A-I_1)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2}$.Also $(A\cup I_1)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2}\cup(I_1)^*_{\tau_1\tau_2} = A^*_{\tau_1\tau_2}\cup\phi = A^*_{\tau_1\tau_2}$. Hence $(A-I_1)^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2} = (A\cup I_1)^*_{\tau_1\tau_2}$. Since $(I_1)^*_{\tau_1\tau_2} = \phi$.

Definition 3.8 *For a subset A of an ideal topological space (X,τ,I),we define $Cl^*_{\tau_1\tau_2}(A) = A\cup A^*_{\tau_1\tau_2}$.*

Theorem 3.9 *$Cl^*_{\tau_1\tau_2}$ satisfies Kuratowski's closure axioms.*

Proof.

(i) $Cl^*_{\tau_1\tau_2}(\phi) = \phi, A \subseteq Cl^*_{\tau_1\tau_2}(A)\forall A \subseteq X$

(ii)$Cl^*_{\tau_1\tau_2}(A\cup B) = A\cup B\cup(A\cup B)^*_{\tau_1\tau_2} = A\cup B\cup A^*_{\tau_1\tau_2 *}\cup B^*_{\tau_1\tau_2} = Cl^*_{\tau_1\tau_2}(A)\cup Cl^*_{\tau_1\tau_2}(B)$.

(iii) For any $A \subseteq X, Cl^*_{\tau_1\tau_2}(Cl^*_{\tau_1\tau_2}(A)) = Cl^*_{\tau_1\tau_2}(A\cup A^*_{\tau_1\tau_2}) = A\cup A^*_{\tau_1\tau_2}\cup(A\cup A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} = A\cup A^*_{\tau_1\tau_2}\cup(A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} = A\cup A^*_{\tau_1\tau_2} = Cl^*_{\tau_1\tau_2}(A)($ since $(A^*_{\tau_1\tau_2})^*_{\tau_1\tau_2} \subseteq A^*_{\tau_1\tau_2})$.

Definition 3.10 *The topology generated by $Cl^*_{\tau_1\tau_2}$ is denoted by $\tau^*_{\tau_1\tau_2}(I)$ and is defined as $\tau^*_{\tau_1\tau_2}(I) = \{U \subseteq X : Cl^*_{\tau_1\tau_2}(X-U) = X-U\}$. Without ambiguity it will be denoted as $\tau^*_{\tau_1\tau_2}$.*

(i)$\phi \subseteq X, Cl^*_{\tau_1\tau_2}(X-\phi) = Cl^*_{\tau_1\tau_2}(X) = X$ and $Cl^*_{\tau_1\tau_2}(X-X) = Cl^*_{\tau_1\tau_2}(\phi) = \phi$. Hence $\phi, X \in \tau^*_{\tau_1\tau_2}(I)$.

(ii)Let $\{U_i\}_{i\in I} \in \tau^*_{\tau_1\tau_2}(I)$ then $Cl^*_{\tau_1\tau_2}(X-U_i) = X-U_i\forall i$. i.e $(X-U_i)\cup(X-U_i)^*_{\tau_1\tau_2} = X-U_i\forall i$. Therefore $(X-U_i)^*_{\tau_1\tau_2} \subseteq X-U_i\forall i$. Claim $Cl^*_{\tau_1\tau_2}(X-\bigcup_i U_i) = Cl^*_{\tau_1\tau_2}(\bigcap_i(X-U_i)) = X-\bigcup_i U_i = \bigcap_i(X-U_i)$. By definition $Cl^*_{\tau_1\tau_2}(\bigcap_i(X-U_i)) = \bigcap_i(X-U_i)\cup(\bigcap_i(X-U_i))^*_{\tau_1\tau_2} \Rightarrow Cl^*_{\tau_1\tau_2}(\bigcap_i(X-U_i)) \supseteq \bigcap_i(X-U_i)$.Also by hypothesis $\bigcap_i(X-U_i)\cup(\bigcap_i(X-U_i))^*_{\tau_1\tau_2} \subseteq \bigcap_i(X-U_i)$.Hence $Cl^*_{\tau_1\tau_2}(\bigcap(X-U_i)) \subseteq \bigcap_i(X-U_i)$.Thus $Cl^*_{\tau_1\tau_2}(X-\bigcup_i U_i) = X-\bigcup U_i$.

(iii) Let $U_1, U_2 \in \tau^*_{\tau_1\tau_2}(I)$ then $Cl^*_{\tau_1\tau_2}(X-U_i) = X-U_i, i = 1,2$. $Cl^*_{\tau_1\tau_2}(X-(U_1\cap U_2)) = Cl^*_{\tau_1\tau_2}(\bigcup_i(X-U_i)) = Cl^*_{\tau_1\tau_2}(X-U_1)\cup Cl^*_{\tau_1\tau_2}(X-U_2) = (X-U_1)\cup(X-U_2)$. Hence $U_1\cap U_2 \in \tau^*_{\tau_1\tau_2}(I)$. Hence $\tau^*_{\tau_1\tau_2}(I)$ is a topology.

Proposition 3.11 *Every $\tau^*_{\tau_1\tau_2}$ is finer than τ^*.*

Proof. Since every τ_1-open set is $\tau_1\tau_2$-open. Therefore $\tau \subseteq \tau^* \subseteq \tau^*_{\tau_1\tau_2}$.

Example 3.12 *Let $X = \{a,b,c\}, \tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}, \tau_2 = \{\phi,X,\{b\}\}, I = \{\phi,\{c\}\}$.*
$\tau^* = \{\phi,\{a\},\{a,b,d\},X\}$.(with respect to τ_1)
$\tau^*_{\tau_1\tau_2} = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,d\},X\}$. Hence $\tau \subseteq \tau^* \subseteq \tau^*_{\tau_1\tau_2}$.

Remark 3.13 *Using the new closure operator some open sets are defined in an ideal bitopological space here $Cl^*(A)$ represent the closure with respect to the topology τ_1.*

Definition 3.14 *A subset A of an ideal topological space (X, τ, I) is said to be*

(i) (1,2)semi-I-open if $A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$

(ii) (1,2)pre-I-open if $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$

(iii) (1,2)α-I-open if$A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)))$

(iv)(1,2)β-I-open if $A \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)))$

Remark 3.15 *(2,1)semi-I-open,(2,1)pre-I-open,(2,1)α-I-open and β-I-open sets are defined by replacing τ_1 by τ_2.*

Proposition 3.16 *(i)Every (1,2)semi-I-open set is semi-I-open and hence semi-open.*

(ii)Every (1,2)pre-I-open set is pre-I-open and hence pre-open.

(iii) Every (1,2)α-I-open set is α-I-open and hence α-open.

(iv)(1,2)β-I-open set is β-I-open and hence β-open.

Proof.

(i) Let A be (1,2)semi-I-open. i.e $A \subseteq Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(A) \subseteq Cl^*(\tau_1\text{-}Int(A)) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(A))$. Since $Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*(A) \subseteq \tau_1\text{-}Cl(A)$.

(ii)$A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)) \subseteq \tau_1\text{-}Int(Cl^*(A)) \subseteq \tau_1\text{-}Int(\tau_1\text{-}Cl(A))$.

(iii) $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))) \subseteq \tau_1\text{-}Int(Cl^*(\tau_1\text{-}Int(A))) \subseteq \tau_1\text{-}Int(\tau_1\text{-}Cl(\tau_1\text{-}Int(A)))$.

(iv)$A \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*(A))) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(\tau_1\text{-}Cl(A)))$

Remark 3.17 *The converse of the proposition 3.16 is not true.*

Remark 3.18 *The collection of all semi-I-open, pre-I-open, α-I-open and β-I-open sets are denoted as $SIO(X), PIO(X), \alpha IO(X)$ and $\beta IO(X)$ respectively. The collection of all (1,2)semi-I-open,(1,2)pre-I-open,(1,2)α-I-open and (1,2)β-I-open sets are denoted as (1,2)SIO(X),(1,2)PIO(X),(1,2)$\alpha IO(X)$ and (1,2)$\beta IO(X)$ respectively.*

Example 3.19 *Let $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}, \{a, b, c\}\}, \tau_2 = \{\phi, X, \{b, c\}\}$,*
$I = \{\phi, \{b\}\}.SIO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$
(1,2)SIO(X)=$\{\phi, X, \{a\}, \{a, d\}, \{a, b, c\}\}$.
$\alpha O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}\{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.
$\alpha IO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}\{a, b, c\}, \{a, b, d\}\}$.
(1,2)$\alpha IO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.
$PO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.
$PIO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}\}$.
$(1, 2)PIO(X) = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$.
$\beta O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.
$\beta IO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.
$(1, 2)\beta IO(X) = \{\phi, X, \{a\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.

Proposition 3.20 *Every τ_1-open set is*

(i) (1,2)semi-I-open.

(ii) (1,2)pre-I-open.

(iii)(1,2)α-I-open.

(iv) (1,2)β-I-open.

Proof. *Let A be an* τ_1*-open set.*

(i)$A = \tau_1\text{-}Int(A) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$

(ii)$A = \tau_1\text{-}Int(A) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$

(iii) $A = \tau_1\text{-}Int(A) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$

(iv) $A = \tau_1\text{-}Int(A) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(A)) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)))$

Remark 3.21 *The converse of the proposition 3.20 is not true. It follows from the example 3.19*

Example 3.22 *Let* $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{a, b\}\}, I = \{\phi, \{c\}\}$ *(1,2)SIO(X)=(1,2)*$\alpha IO(X) = (1, 2)PIO(X) = (1, 2)\beta O(X) =$ $\{\phi, X, \{a\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$.

Proposition 3.23 *A subset A of an ideal bitopological space* (X, τ_1, τ_2, I) *is* $(1, 2)\alpha$*-I-open if and only if A is (1,2)semi-I-open and (1,2)pre-I-open.*

Proof. *Let A be* $(1, 2)\alpha$*-I-open.* $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$*. Thus A is (1,2)semi-I-open.*$A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}\tau_1\text{-}(Int(A))) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$*. Thus A is (1,2)pre-I-open. Let A be (1,2)semi-I-open and (1,2)pre-I-open.i.e.*$A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$ *and* $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$*.* $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$*. Hence A is (1,2)*$\alpha$*-I-open.*

Proposition 3.24 *For any ideal topological space* (X, τ, I)*,* $(1, 2)SIO(X) \cup (1, 2)PIO(X) \subseteq (1, 2)\beta IO(X)$

Proof. *Let A be (1,2)semi-I-open then* $A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(A) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$*. Hence A is* $(1, 2)\beta$*-I-open. Or if A be (1,2)pre-I-open. Then* $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}((A)) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$*.Hence A is (1,2)*$\beta$*-I-open.*

Remark 3.25 *The converse of the proposition 3.24 is not true.*

Example 3.26 *Let* $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}, \tau_2 = \{\phi, X, \{c\}\}$. $I = \{\phi, \{b\}\}, (1, 2)SIO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}, (1, 2)PIO(X) = \{\phi, X, \{a\}, \{b\},$ $\{a, b\}, \{a, b, c\}, \{a, b, d\}\}, (1, 2)\beta O(X) = \{\phi, X\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\},$ $\{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

Proposition 3.27 *Let* (X, τ_1, τ_2, I) *be an ideal topological space and* $\{A_\alpha : \alpha \in J\}$ *be a family of subsets of X where J is an arbitrary index set.*

(i)If $\{A_\alpha : \alpha \in J\} \in (1, 2)SIO(X)$ *then* $\bigcup\{A_\alpha : \alpha \in J\} \in (1, 2)SIO(X)$.

(ii)If $A \in (1, 2)SIO(X)$ *and* $B \in \tau_1$ *then* $A \cap U \in (1, 2)SIO(X)$

(iii) If $\{A_\alpha : \alpha \in J\} \in (1, 2)PIO(X)$ *then* $\bigcup\{A_\alpha : \alpha \in J\} \in (1, 2)PIO(X)$.

(iv) If $A \in (1, 2)PIO(X)$ *and* $B \in \tau_1$ *then* $A \cap U \in (1, 2)PIO(X)$

(v) If $\{A_\alpha : \alpha \in J\} \in (1, 2)\alpha IO(X)$ *then* $\bigcup\{A_\alpha : \alpha \in J\} \in (1, 2)\alpha IO(X)$.

(vi)If $A \in (1, 2)\alpha IO(X)$ *and* $B \in \tau_1$ *then* $A \cap U \in (1, 2)\alpha IO(X)$.

(vii)If $\{A_\alpha : \alpha \in J\} \in (1, 2)\beta IO(X)$ *then* $\bigcup\{A_\alpha : \alpha \in J\} \in (1, 2)\beta IO(X)$.

(viii)If $A \in (1, 2)\beta IO(X)$ *and* $B \in \tau_1$ *then* $A \cap U \in (1, 2)\beta IO(X)$.

Proof.

(i) Since $U_\alpha \in (1,2)SIO(X)$ for each $\alpha \in J.U_\alpha \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(U_\alpha).\bigcup U_\alpha \subseteq \bigcup_{\alpha\in J} Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(U_\alpha)) \subseteq \bigcup_{\alpha\in J}(\tau_1\text{-}Int(U_\alpha))^*_{\tau_1\tau_2} \cup (\tau_1\text{-}Int(U_\alpha) \subseteq \bigcup_{\alpha\in J}(\tau_1\text{-}Int(U_\alpha))^*_{\tau_1\tau_2} \cup \tau_1\text{-}Int(\bigcup_{\alpha\in J} U_\alpha) \subseteq (\tau_1\text{-}Int\bigcup_{\alpha\in J} U_\alpha)^*_{\tau_1\tau_2} \cup (\tau_1\text{-}Int(\bigcup_{\alpha\in J} U_\alpha)) = Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(\bigcup_{\alpha\in J} U_\alpha)$. Thus $\bigcup_{\alpha\in J} U_\alpha \in (1,2)SIO(X)$.

(ii) Let $A \in (1,2)SIO(X), B \in \tau_1$. $A \subseteq Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(A), B = \tau_1\text{-}Int(B)$. $A\cap B \subseteq Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(B) \subseteq (\tau_1\text{-}Int(A))^*_{\tau_1\tau_2} \cup \tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(B) \subseteq (\tau_1\text{-}Int(A))^*_{\tau_1\tau_2} \cap \tau_1\text{-}Int(B) \cup \tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(B) \subseteq (\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(B))^*_{\tau_1\tau_2} \cup \tau_1\text{-}Int(A\cap B) \subseteq (\tau_1\text{-}Int(A\cap B))^*_{\tau_1\tau_2} \cup \tau_1\text{-}Int(A\cap B) = Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(A\cap B))$. Hence $A\cap B \in (1,2)SIO(X)$.

(iii) Since $U_\alpha \in (1,2)PIO(X)$ for each $\alpha \in J.U_\alpha \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(U_\alpha)$. $U_\alpha \subseteq \tau_1\text{-}IntCl^*_{\tau_1\tau_2}(\bigcup U_\alpha)$. $\bigcup U_\alpha \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\bigcup_{\alpha\in J} U_\alpha))$. Thus $\bigcup_{\alpha\in J} U_\alpha \in (1,2)PIO(X)$.

(iv) Let $A \in (1,2)PIO(X), B \in \tau_1$. $A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)), B = \tau_1\text{-}Int(B)$. $A\cap B \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)) \cap \tau_1\text{-}Int(B) = \tau_1\text{-}Int[(Cl^*_{\tau_1\tau_2}(A) \cap B)] \subseteq \tau_1\text{-}Int[(A \cup A^*_{\tau_1\tau_2}) \cap B)] \subseteq \tau_1\text{-}Int[(A\cap B) \cup (A\cap B)^*_{\tau_1\tau_2}] \subseteq \tau_1\text{-}IntCl^*_{\tau_1\tau_2}(A\cap B))$. Hence$A\cap B \in (1,2)PIO(X)$.

(v) Since $U_\alpha \in (1,2)\alpha IO(X)$ for each $\alpha \in J.U_\alpha \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(U_\alpha)) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(\bigcup U_\alpha)).\bigcup U_\alpha \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(\bigcup U_\alpha))$.
Thus $\bigcup_{\alpha\in J} U_\alpha \in (1,2)\alpha IO(X)$.

*(vi)*Let A be $(1,2)\alpha I$-open. Then A is (1,2)semi-I-open and (1,2)pre-I-open. By (ii) and (iv) $A\cap B \in (1,2)SIO(X)$ and $A\cap B \in (1,2)PIO(X)$. Hence $A\cap B \in (1,2)\alpha IO(X)$.

*(vii)*Since $U_\alpha \in (1,2)\beta IO(X)$ for each $\alpha \in J.U_\alpha \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(U_\alpha).\bigcup U_\alpha \subseteq \bigcup_{\alpha\in J} \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(U_\alpha))) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(\bigcup_{\alpha\in J} Cl^*_{\tau_1\tau_2}((U_\alpha)) \subseteq$
$\tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2} \bigcup_{\alpha\in J}(U_\alpha))$. Thus $\bigcup_{\alpha\in J} U_\alpha \in (1,2)\beta IO(X)$.

(viii) Let $A \in (1,2)\beta IO(X), B \in \tau_1$. $A \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)), B = \tau_1\text{-}Int(B)$.
$A\cap B \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))) \cap \tau_1\text{-}Int(B) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(A \cup A^*_{\tau_1\tau_2}) \cap B)) =$
$\tau_1\text{-}Cl(\tau_1\text{-}Int((A^*_{\tau_1\tau_2} \cap B) \cup (A\cap B)) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int((A\cap B)^*_{\tau_1\tau_2} \cup (A\cap B)) =$
$\tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A\cap B)))$. Hence$A\cap B \in (1,2)\beta IO(X)$.

Definition 3.28 *The largest (1,2)semi-I-open, (1,2)pre-I-open, (1,2)α-I-open and (1,2)β-I-open sets contained in A are defined as (1,2)semi-I-int(A),(1,2) pre-I-int(A),(1,2)αI-int(A) and (1,2)βI-int(A) respectively. They are denoted as (1,2)sI-Int(A),(1,2)pI-Int(A),(1,2)αI-Int(A),βI-Int(A) respectively.*

Proposition 3.29 *For any subset A of an ideal topological space the following holds.*
(i)$Cl^*_{\tau_1\tau_2}(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}[A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))]$
(ii) $Cl^*_{\tau_1\tau_2}\tau_1\text{-}(Int(A)) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq$
$Cl^*_{\tau_1\tau_2}[\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))]$.

Proof. *(i)* Let $x \in Cl^*_{\tau_1\tau_2}(A)\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$ then $x \in Cl^*_{\tau_1\tau_2}(A)$ and $x \in \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$. $x \in Cl^*_{\tau_1\tau_2}(A) \Rightarrow x \in A$ or $x \in A^*_{\tau_1\tau_2}$. If $x \in A$ then $x \in A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)) \Rightarrow x \in Cl^*_{\tau_1\tau_2}(A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. Hence $Cl^*_{\tau_1\tau_2}(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. On the other hand if $x \in A^*_{\tau_1\tau_2}$ then $x \in A^*_{\tau_1\tau_2} \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq (\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)\cap A)^*_{\tau_1\tau_2} \subseteq Cl^*_{\tau_1\tau_2}(A\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. Thus $Cl^*_{\tau_1\tau_2}(A)\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$.
(ii) Let $x \in Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$ and $x \in \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. If $x \in Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \Rightarrow x \in \tau_1\text{-}Int(A) \cup (\tau_1\text{-}Int(A))^*_{\tau_1\tau_2}$. If $x \in \tau_1\text{-}Int(A)$ then $x \in \tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. On the other hand if $x \in (\tau_1\text{-}Int(A))^*_{\tau_1\tau_2}$ then $x \in Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))\cap(\tau_1\text{-}Int(A))^*_{\tau_1\tau_2} \subseteq [\tau_1\text{-}Int(A)\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)))]^*_{\tau_1\tau_2}$
$\subseteq Cl^*_{\tau_1\tau_2}[\tau_1\text{-}Int(A)\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)))]$. Hence $Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))\cap\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$.

Lemma 3.30 *Let A be a subset of a space* (X,τ_1,τ_2,I). *If O is an* τ_1*-open subset of X then* $O \cap Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(O \cap A)$.

Proof. *By Theorem 3.7* $O \cap A^*_{\tau_1\tau_2} \subseteq (O \cap A)^*_{\tau_1\tau_2}$. $O \cap Cl^*_{\tau_1\tau_2}(A) = O \cap (A \cup A^*_{\tau_1\tau_2}) = (O \cap A) \cup (O \cap A^*_{\tau_1\tau_2}) \subseteq (O \cap A) \cup (O \cap A)^*_{\tau_1\tau_2} \subseteq Cl^*_{\tau_1\tau_2}(O \cap A)$.

Proposition 3.31 *If* A *is any subset of an ideal topological space* (X, τ_1, τ_2, I) *then the following holds.*

(i) $(1,2)sI\text{-}Int(A) = A \cap (Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int((A)))$

(ii) $(1,2)pI\text{-}Int(A) = A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$.

(iii) $(1,2)\alpha I\text{-}Int(A) = A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$

(iv) $(1,2)\beta I\text{-}Int(A) = A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$

Proof.

(i) $(1,2)sI\text{-}Int(A) \subseteq A$. *If S be any (1,2)semi-I-open set contained in A then* $S \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(S)) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Therefore it is true for all (1,2)semi-I-open set S. Therefore* $(1,2)sI\text{-}Int(A) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$,
$(1,2)sI\text{-}Int(A) \subseteq A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$*—-(1). claim:* $A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$ *is (1,2)semi-I-open.* $Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \supseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A \cap \tau_1\text{-}Int(A))) \supseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(\tau_1\text{-}Int(A))) \supseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \supseteq A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Thus* $A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$ *is (1,2)semi-I-open set contained in A.* $A \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq (1,2)sI\text{-}Int(A)$*.Hence (1,2)sI-Int(A)=A* $\cap(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$.

(ii) $(1,2)pI\text{-}Int(A) \subseteq A$. *If S be any (1,2)pre-I-open set contained in A then* $S \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(S)) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$. *This is true for all (1,2)pre-I-open set S. Therefore* $(1,2)pI\text{-}Int(A) \subseteq A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$. *claim:*$A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$ *is (1,2)preI-open.*$A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)) \subseteq \tau_1\text{-}Int(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$
$= \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. *Therefore* $A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$ *is (1,2)pre-I-open.* $A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$ *is a (1,2)pre-I-open set contained in A.* $A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq (1,2)pI\text{-}Int(A)$. *Thus* $(1,2)pI\text{-}Int(A) = A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$.

(iii) If S is any $(1,2)\alpha$*-I-open set contained in A then* $S \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(S)) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Therefore* $(1,2)\alpha I\text{-}Int(A) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \cap A$*.—-(1).* $A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) =$
$\tau_1\text{-}Int(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)))) = \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1 Int(A)) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$
$\subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)))) =$
$\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(A)))$.
Therefore $A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$ *is (1,2)*α*-I-open set contained in A.*
$A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq (1,2)\alpha I\text{-}Int(A)$*.—-(2). Therefore from (1) and (2)*
$A \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) = (1,2)\alpha I\text{-}Int(A)$.

(iv) If S is a $(1,2)\beta I$*-open set contained in A then* $S \subseteq \tau\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(S))) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)))$. *Also* $(1,2)\beta I\text{-}Int(A) \subseteq A$*.Therefore* $(1,2)\beta I\text{-}Int(A) \subseteq A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)))$. *—-(1) Also* $A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))) = \tau_1\text{-}Cl(\tau_1\text{-}Int(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)))) =$
$\tau_1\text{-}Cl(\tau_1\text{-}Int[\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) \cap Cl^*_{\tau_1\tau_2}(A)] \subseteq$
$\tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}[A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))]))$. *Hence* $A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}A)))$ *is a* $(1,2)\beta I$*-open set contained in A. Therefore* $A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))) \subseteq (1,2)\beta I\text{-}Int(A)$. *Thus* $(1,2)\beta I\text{-}Int(A) = A \cap \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$

Theorem 3.32 *A subset A of a space* (X, τ_1, τ_2, I) *is (1,2)semi-I-open if and only if* $Cl^*_{\tau_1\tau_2}(A) = Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$.

Proof. *Let A be (1,2)semi-I-open then* $A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \Rightarrow Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$, $\tau_1\text{-}Int(A) \subseteq A \Rightarrow Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}(A)$. *Therefore* $Cl^*_{\tau_1\tau_2}(A) = Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$.
Conversely $Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \Rightarrow A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Hence A is (1,2)semi-I-open.*

Proposition 3.33 *A subset A of a space* (X, τ_1, τ_2, I) *is (1,2)semi-I-open if and only if there exists an* τ_1*-open set U such that* $U \subseteq A \subseteq Cl^*_{\tau_1\tau_2}(U)$.

Proof. *Let A be (1,2)semi-I-open. Then* $A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Let* $\tau_1\text{-}Int(A) = U$. *Therefore* $U \subseteq A \subseteq Cl^*_{\tau_1\tau_2}(U)$. *Conversely let* $U \subseteq A \subseteq Cl^*_{\tau_1\tau_2}(U)$ *for some* τ_1*-open set U. Since* $U \subseteq A \Rightarrow U \subseteq \tau_1\text{-}Int(A)$ *and hence* $Cl^*_{\tau_1\tau_2}(U) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$. *Therefore* $A \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A))$.

Proposition 3.34 *If a subset A of a space* $(X, \tau_1, \tau_2, , I)$ *is (1,2)semi-I-open and* $A \subseteq B \subseteq Cl^*_{\tau_1\tau_2}(A)$ *then B is (1,2)semi-I-open in* (X, τ_1, τ_2, I).

Proof. *Since A is (1,2)semi-I-open there exists an* τ_1*-open set U such that* $U \subseteq A \subseteq Cl^*_{\tau_1\tau_2}(U)$. *Therefore* $U \subseteq A \subseteq B \subseteq Cl^*_{\tau_1\tau_2}(A) \subseteq Cl^*_{\tau_1\tau_2}(U)$*.Hence by the Proposition 3.33 B is (1,2)semi-I-open.*

Proposition 3.35 *A subset A of an ideal topological space* (X, τ_1, τ_2, I) *is* $(1,2)\beta$*-I-open if and only if* $\tau_1\text{-}Cl(A) = \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$.

Proof. *Let A be* $(1,2)\beta$*-I-open. Then* $A \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. $\tau_1\text{-}Cl(A) \subseteq \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))) \subseteq \tau_1\text{-}Cl(A)$. *Hence* $\tau_1\text{-}Cl(A) = \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A)$. *Conversely* $\tau_1\text{-}Cl(A) = \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$. *Then* $A \subseteq \tau_1\text{-}Cl(A) = \tau_1\text{-}Cl(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A))$.

Proposition 3.36 *(i)If* $V \in (1,2)SIO(X)$ *and* $A \in (1,2)\alpha IO(X)$ *then*
$V \cap A \in (1,2)SIO(X)$.
(ii) If $V \in (1,2)PIO(X)$ *and* $A \in (1,2)\alpha IO(X)$ *then*
$V \cap A \in (1,2)PIO(X)$.

Proof. *(i)Let* $V \in (1,2)SIO(X)$ *and* $A \in (1,2)\alpha IO(X)$. $V \cap A \subseteq$
$Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(V) \cap \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(V) \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq$
$Cl^*_{\tau_1\tau_2}(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(V) \cap \tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(V) \cap \tau_1\text{-}Int(A)) \subseteq Cl^*_{\tau_1\tau_2}\tau_1\text{-}Int(V \cap A)$.
Hence $V \cap A \in (1,2)SIO(X)$.
(ii) $V \in (1,2)PIO(X)$ *and* $A \in (1,2)\alpha IO(X)$. $V \cap A \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(V)) \cap$
$\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) = \tau_1\text{-}Int(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(V)) \cap Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) \subseteq$
$\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(V) \cap \tau_1\text{-}Int(A)) \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(Cl^*_{\tau_1\tau_2}(V) \cap \tau_1\text{-}Int(A)))$
$\subseteq \tau_1\text{-}Int[(Cl^*_{\tau_1\tau_2}(Cl^*_{\tau_1\tau_2}(V \cap Int(A))] \subseteq \tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(V \cap A)$. *Hence* $V \cap A \in (1,2)PIO(X)$.

4 Applications

As an application new types of sets are defined and their some of their properties are derived.

Definition 4.1 *A subset A of an ideal bitopological space* X, τ_1, τ_2, I *is called a*

(i) (1,2)t-I-set if $\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(A) = \tau_1\text{-}Int(A)$.

(ii) (1,2)α^**-I-set if* $\tau_1\text{-}Int(Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A)) = \tau_1\text{-}Int(A)$.

(iii) (1,2)S-I-set if $Cl^*_{\tau_1\tau_2}(\tau_1\text{-}Int(A) = \tau_1\text{-}Int(A)$.

Definition 4.2 *A subset A of an ideal bitopological space* (X, τ_1, τ_2, I) *is called a*

(i) (1,2)B_I*-set if* $A = U \cap V$ *where U is* τ_1*-open and V is a (1,2)t-I-set.*

(ii) (1,2)C_I*-set if* $A = U \cap V$*, where U is is an* τ_1*-open and V is a (1,2)*α^**-I-set.*

(iii) (1,2)S_I*-set if* $A = U \cap V$ *where U is* τ_1*-open and V is a (1,2)S-I-set.*

Example 4.3 *Let* $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}, \{a, b, c\}\}, \tau_2 = \{\phi, X, \{b, c\}\}, I = \{\phi, \{a\}\}$
$\tau_1\tau_2 O(X) = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$. *The (1,2)t-I- sets are* ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{$
(1,2)α^**-I-sets are* ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
(1,2)S-I-sets are ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
(1,2)B_I*- sets are* ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\}$
(1,2)C_I*-sets are* ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$
(1,2)S_I*-sets are* ϕ,X,$\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

Proposition 4.4 *For a subset A of an ideal bitopological space (X, τ_1, τ_2, I) the following hold.*

(i) If A is a t set with respect to τ_1, then A is a (1,2)t-I-set.

(ii) If A is a (1,2)t-I-set then A is a (1,2)B_I-set.

Proof. *(i) Let A be a t-set. τ_1-Int(A)=τ_1-Int(τ_1-Cl(A)). τ_1-Int($Cl^*_{\tau_1\tau_2}(A) = \tau_1$-Int($A^*_{\tau_1\tau_2} \cup A$) $\subseteq \tau_1$-Int(τ_1-Cl(A)∪A)) = τ_1-Int(τ_1-Cl(A)) = τ_1-Int(A)——(1).$Cl^*_{\tau_1\tau_2}(A) \supseteq A$ and τ_1-Int($Cl^*_{\tau_1\tau_2}(A)) \supseteq \tau_1$-Int(A).*
(ii)Let A be a (1,2)t-I-set . If $U = X \in \tau_1$ then $A = U \cap A$ then A is a (1,2)B_I-set.

Remark 4.5 *The converse of the proposition 4.4 is not true.*

Example 4.6 *(i)In Example 4.3 the set {a} is a (1,2)t-I-set but not a t-set.*
(ii)The set {a,b,c,} is a (1,2)B_I-set but not a (1,2)t-I-set.

Proposition 4.7 *Let (X, τ_1, τ_2, I) be an ideal bitopological space and A a subset of X. Then the following hold.*
(i) If A is an α^*-set with respect to τ_1 then A is an $(1,2)\alpha^*$-I-set.
(ii) If A is a (1,2)t-I-set then A is a (1,2)α^*-I-set.
(iii) If A is a (1,2)α^*-set then A is a (1,2)C_I-set.

Proof. *(i)Let A be an α^*-set with respect to τ_1. Then τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) = τ_1-Int(τ_1-Int(A))$^*_{\tau_1\tau_2} \cup \tau_1$-Int(A)) $\subseteq \tau_1$-Int(τ_1-Cl(τ_1-Int(A) $\cup$ τ_1-Int(A)) = τ_1-Int(τ_1-Cl(τ_1-Int(A)) = τ_1-Int(A)—-(1). Also $Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\supseteq \tau_1$-Int(A).*
Hence τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\supseteq \tau_1$-Int(A).Therefore τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) = τ_1-Int(A).
(ii)Let A be a (1,2)t-I-set then τ_1-Int(A) = τ_1-Int($Cl^*_{\tau_1\tau_2}$(A)) $\supseteq \tau_1$-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\supseteq \tau_1$-Int(A). Also τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\supseteq \tau_1$-Int(A). Hence τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) = τ_1-Int(A)
(iii) Let A be an (1,2)α^*-I-set and U=X. Then $A = U \cap A$. Hence A is a (1,2)C_I-set.

Remark 4.8 *The converse of the proposition 4.7 is not true.*

Example 4.9 *In Example 4.3 the set {a,b} is a (1,2)α^*-I-set but not a α^* and a(1,2)t-I-set. The set {a,b,c} is a (1,2)C_I-set but not a (1,2)α^*-I-set.*

Proposition 4.10 *Let(X, τ_1, τ_2, I) be an ideal bitopological space.A be a subset of X. Then the following hold.*

(i) If A is (1,2)C_I-set then $(1,2)\alpha I$-Int(A)= τ_1-Int(A).

(ii) If A is A (1,2)B_I-set then (1,2)pI-Int(A)= τ_1-Int(A).

(iii)If A is a (1,2)S_I-set then (1,2)pI-Int(A)= τ_1-Int(A).

Proof.

(i) $(1,2)\alpha I$-Int(A)$\supseteq \tau_1$-Int(A). Since A is a (1,2)C_I-set, A=U∩V, where U is an τ_1-open and V is a (1,2)α^*-I-set.A $\subseteq V \Rightarrow \tau_1$-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\subseteq \tau_1$-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(V)) = τ_1-Int(V). By the Proposition 3.31(iii)$(1,2)\alpha I$-Int(A)=A∩τ_1-Int($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A))) $\subseteq A\cap$ τ_1-Int(V) = U∩τ_1-Int(V) = τ_1-Int(U∩V) = τ_1-Int(A). Hence $(1,2)\alpha I$-Int(A)= τ_1-Int(A).

(ii)(1,2)pI-Int(A)$\supseteq \tau_1$-Int(A). Since A is a (1,2)B_I-set, A=U∩V, where U is an τ_1-open and V is a (1,2)t-I-set. $A \subseteq V \Rightarrow \tau_1$-Int($Cl^*_{\tau_1\tau_2}$(A)) $\subseteq \tau_1$-Int($Cl^*_{\tau_1\tau_2}$(V)) = τ_1-Int(V). By the Proposition 3.31(ii)$(1,2)pI$-Int(A)=A∩τ_1-Int($Cl^*_{\tau_1\tau_2}$(A))) $\subseteq A \cap \tau_1$-Int(V) = $U \cap \tau_1$-Int(V) = τ_1-Int($U \cap V$) = τ_1-Int(A). Hence (1,2)pI-Int(A)= τ_1-Int(A).

(iii)(1,2)sI-Int(A)$\supseteq \tau_1$-Int(A). Since A is a (1,2)S_I-set, A=U∩V, where U is an τ_1-open and V is a (1,2)S-I-set. $A \subseteq V \Rightarrow$ ($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A)) $\subseteq$ ($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(V)) = τ_1-Int(V). By the Proposition 3.31(i)$(1,2)pI$-Int(A)=A∩($Cl^*_{\tau_1\tau_2}(\tau_1$-Int(A))) $\subseteq A\cap\tau_1$-Int(V) = $U\cap\tau_1$-Int(V) = τ_1-Int($U \cap V$) = τ_1-Int(A). Hence $(1,2)pI$-Int(A)= τ_1-Int(A).

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