Wavelets with Application on Signal Processing Applicable To Non Linear Signals

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***Abstract*— Seismic damage is highly dependent on the rate of energy input from the earthquake waves. Ground motions with the same amplitude spectrum can cause substantially different levels of seismic damage on the same structure, depending on the time-frequency localization of the energy imparted to the structure. In this study, a wavelet-based approach to analyze the earthquake signal detects different frequency components at different time intervals are discussed. Wavelet analysis is a new mathematical technique and in the recent years enormous interest in application of engineering has been observed. This new technique is particularly suitable for non-stationary processes as in contrast to the Fourier transform. The wavelet transform allows exceptional localization, both in time and frequency domains. The distance between earth quake station and epicenter is also calculated.**

***Keywords*— Ground motions, Wavelet Transform, Earth Quake, Fourier Transform.**

I. Introduction

The earthquake records are the non linear signals. However, in many cases the most distinguished information is hidden in the frequency domain, which provides the energy, associated with a given frequency. The frequency spectrum of the signal can be obtained by the Fourier Transform (FT). The FT yields information on how much but not when (in time) the particular frequency components exist. Such information is sufficient in a case of the stationary signals as the frequency content of such signals does not change in time and all frequency components exist all the time. However, in all cases the earthquake waves change in a relatively short period of time [1].

When waves start to break, the frequency content of signal changes rapidly in time due to nonlinear interaction between elementary wave components and resulting energy transfer, and energy dissipation .In such cases, the FT provides information on the frequency content; however, the information on the frequency localization in time is essentially lost in the process. When the time localization of the spectral components is required, the transform of time series, which provides the time–frequency representation of the signal, should be developed. Transform of such type is the wavelet transform, which gives full time– frequency representation of the time series. In contrast to the FT, the wavelet transform allows exceptional localization in the time domain via translations of the so called mother wavelet, and in the scale (frequency) domain via dilations [2-3].

Wavelet transform is a relatively modern technique and in recent years enormous interest in the application of wavelets has been observed. On the other hand, the application of the wavelet transform to earthquake engineering is not frequent. Discrete and fast wavelet transforms are used for dynamic analysis of structures induced earthquake load. Then the discrete and fast wavelet transform are used for optimization of structures with earthquake loading [4-5].

In this paper the application of wavelet transform for the processing of earthquake record is discussed and an attempt to produce some useful quantitative results is made. The fundamentals of the WT are given and the difference between FT and WT is demonstrated and the application of the WT for processing of earthquake record is shown.

The paper is organized as follows. Section II gives the brief introduction of earth quake signals. Section III present the over view of Fourier Transform. Section IV gives information about Wavelet Transform. Section V describes the proposed Wavelet Transform system. Finally some conclusions are reported in Section VI.

II. EARTH QUAKE

The structure of Earth's deep interior cannot be studied directly. But geologists use seismic (earthquake) waves to determine the depths of layers of the molten and semi-molten material within Earth. Because different types of earthquake waves behave differently when they encounter material in different states (for example, molten, semi-molten, solid), seismic stations established around Earth detect and record the strengths of the different types of waves and the directions from which they came. Geologists use these records to establish the structure of Earth's interior[6].

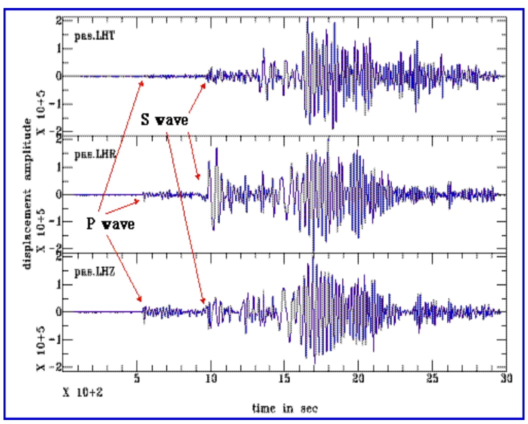

Fig.1. Earth Quake Signals.

The two principal types of seismic waves are P-waves (pressure; goes through liquid and solid) and S-waves (shear or secondary; goes only through solid - not through liquid). The travel velocity of these two wave types is not the same (P-waves are faster than S-waves). Thus, if there is an earthquake somewhere, the first waves that arrive are P-waves. In essence, the gap in P-wave and S-wave arrival gives a first estimate of the distance to the earthquake. Fig. 1 shows some typical seismograms with arrival of P- and S-waves marked [7].

III. FOURIER TRANSFORM

The FT is probably the most popular transform being used, but for better understanding the difference between the WT and the FT, a short overview of the FT is provided. We start with a case of continuous deterministic signal x (t). If the total signal energy, Er, is finite or if:

$$Er = \int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty \quad (1)$$

then x(t) is absolute-integral over the entire domain and the FT of the x(t) exists as follows:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi fti}\, dt \quad (2)$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi fti}\, dt \quad (3)$$

Using the square of the modules of the FT the energy spectral density of the signal is obtained as:

$$S(f) = X(f) * X^*(f) = |X(f)|^2 \quad (4)$$

where the asterisk denotes the complex conjugate. The integration of the spectral density S(f) over the entire spectral domain provides the total energy, Er, of the signal:

$$E_r = \int_{-\infty}^{+\infty} |X(f)|^2\, df = \int_{-\infty}^{+\infty} |X(t)|^2\, dt \quad (5)$$

In the FT there is no resolution problem in the frequency domain, as we know exactly what frequencies exist. This perfect frequency resolution in the FT is due to the fact that the window, e2 π fti used in this transformation, lasts all the time, from minus infinity to plus infinity. Similarly, there is no time resolution problem in the time domain, since we know the value of the signal at every instant of time. In contrary, we can say that both time resolution in the FT and the frequency resolution in the time domain are zero, since we have no information about them [7].

IV. WAVELET TRANSFORM

The WT is similar to the FT as it breaks a signal down into its constituents. Whereas the FT breaks the signal into a series of sine waves of different frequencies, the WT breaks the signal into its wavelets, which are scaled and shifted versions of the so called mother wavelet. The WT allows exceptional localization both in the time domain via translations of the wavelet, and in the frequency (scale) domain via dilations. The wavelets are complex or real functions concentrated in time and frequency and having the same shape. In the WT, the signal is multiplied with the wavelet, and the transform is separately computed for different segments of the time domain signal. In general, the WT of the signal, x (t), is defined as a following inner product:

$$WT(\tau, b) = \int_{-\infty}^{+\infty} x(t)\, g^{*}(\tau, b)\, dt \qquad (6)$$

The family of continuously translated and dilated wavelets is generated from mother wavelet g(t):

$$g(\tau, b) = \frac{1}{\sqrt{b}}\, g\left(\frac{t-\tau}{b}\right) \qquad (7)$$

Where t is the translation parameter, corresponding to the position of the wavelet as it is shifted through the signal, b is the scale dilation parameter determining the width of the wavelet. The scale b>1 dilates (or stretches out) the signals, whereas scale b<1 compresses the signal. The wavelet coefficients, WT (t, b), represent the correlation (in terms of the time-scale functions) between the wavelet and a localized section of the signal. If the signal has a major component of the frequency corresponding to the given scale, then the wavelet at this scale is close to the signal at the particular location and the corresponding wavelet transform coefficient, determined at this point, has a relatively large value. Therefore, the wavelet transform is a sort of microscope with magnification b-1 and location given by parameter t, while the optics of the microscope is characterized by the function g (τ , b). For the wavelet which has the mother wavelet status, the function g (t) must satisfy several properties, such as:

1. The amplitude |g (t)| must decay rapidly to zero in the limit |t| →∞. This feature ensures the localization aspect of wavelet analysis. It means that the wavelet g [(t −τ) / b] has insignificant effect at time |t| >τcrit, where τcrit is a critical time lag.
2. The wavelet g(t) must have zero mean. This condition, known as the admissibility condition, ensures the invertibility of the wavelet transform.
3. The wavelets are regular functions such that G (w < 0) = 0. It means that wavelets need to be described in terms of positive frequencies only[1].

V. PROPOSED WAVELET TRANSFORM SYSTEM

It is aimed at improving the older techniques and to get a more reliable patter matching. In this system we are taking input data from sensors. Our aim is to find out distance of earth quake centre to the earth quake station and analyze the different frequency components in signal. Fig. 2 shows the block diagram of proposed wavelet transform system.

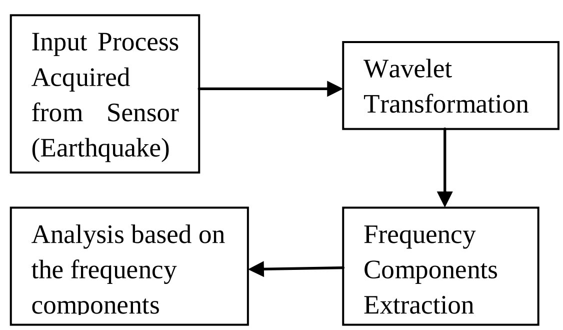

Fig.2. Block diagram of the proposed Wavelet Transform system.

A. *Processed Data of earth quake*

In this section, FT and WT of the EI Centro earthquake will be computed. The earthquake record and FT of the Taft are shown in Fig. 3 and 4, respectively. WT of the earthquake record in two dimensions is shown in Fig. 5. By comparing Figures 4 and 5, it is observed that the highest frequency of the earthquake record is 1.7 (Hz), which is corresponding to scale 0.55. Besides, the frequency range of 0.8-2.5 (Hz) has the most important of FT, is the same as the highest frequency range. On the other hand, we can say that the frequency range of 0.8-2.5 can be the same as the scale rang of 1.25- 0.4. Referring to Figures 5, the output programming for this purpose, we can distinguish the time of each frequency. For example, the time of the frequency 1.7 which is as correspond to scale 0.55, is 1.85 seconds. The time of the frequency range 0.8-2.5, is 1.4-2 seconds. In other words, this range of time is the most dangerous range for all constructions which have the same frequency as the dominant frequency of earthquake. The important frequency of the EI Centro earthquake is 1.7 Hertz. This frequency equals to resonance frequency of steel frame with the heights of 17m. For this particular construction, the resonance takes place due to the closeness of the frequency of the construction with the frequency of the earthquake at 1.85 seconds after the beginning of the earthquake and in theoretically the construction is destroyed. In the same way, we can compute the time of other frequencies of the earthquake record. For EI Centro Earthquake the distance is 15.2Km from earth station.

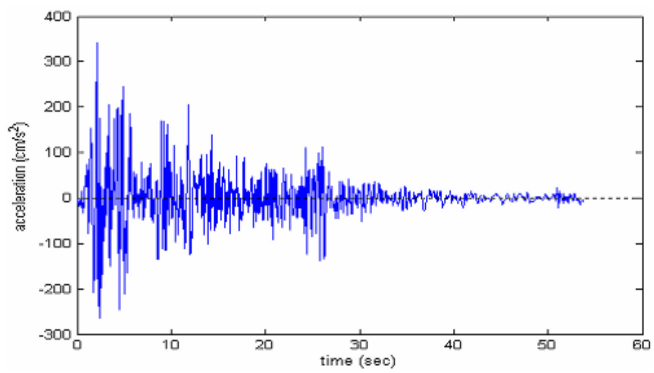

Fig.3 EI Centro Earth Quake Records

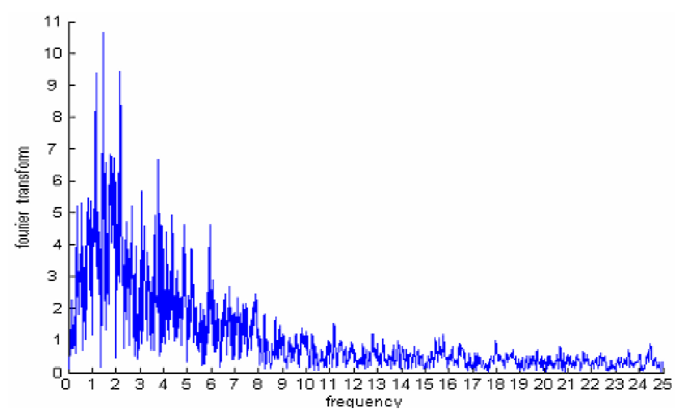

Fig.4 FT of EI Centro Earth Quake Records

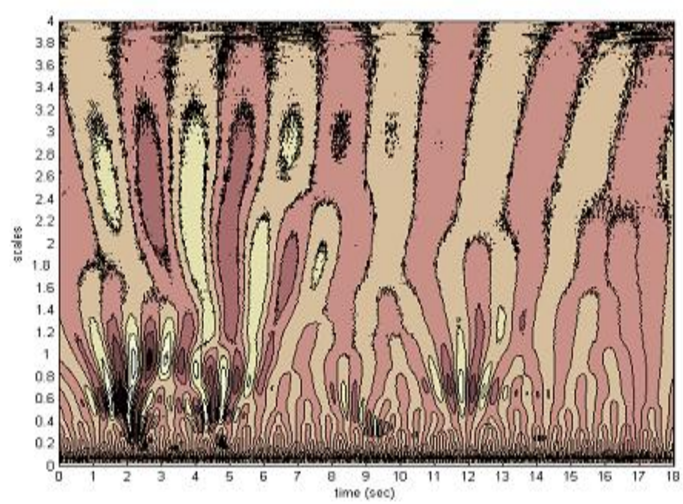

Fig.5 WT of Taft Earth Quake Records

VI. CONCLUSION

The wavelet transform technique is particularly suitable for non-stationary signals like earth quake. In contrast to the Fourier transform, the wavelet transform allows exceptional localization, both in the time domain via translation *t* of the wavelet, and in the frequency domain via dilations scales *b*, which can be changed from minimum to maximum, chosen by the user. The variations of the frequencies corresponding to the maximum of the wavelet transform at particular time are very similar to the recorded frequencies. We can also calculate distance of earth quake from earthquake station. It is interesting to note that wavelet transform shows the influence of the some low frequency components which are not clearly seen in the classical energy spectrum.

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