Visualization of the Key Air Properties Influencing Derived Functions

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Abstract— The main objective of this paper is to suggest the best correlation between the temperature and temperature dependent air properties. The main air properties presented in this paper are specific heat, density, dynamic viscosity, kinematic viscosity, thermal conductivity and thermal diffusivity. The reliance of these air properties with temperature is studied at atmospheric pressure. At the end, details of a MATLAB computer program are presented. This computer code would be useful for the students in further detailed studies and hand calculations.

Keywords—Temperature, air properties, specific heat, density, viscosity, thermal conductivity, diffusivity

NOMENCLATURE

Cp= specific heat in kJ/kg-K
T = temperature in K
ρ = density in kg/m^3
μ = dynamic viscosity in N-s/m^2
k = thermal conductivity in W/m-K
ν = kinematic viscosity in m^2/s
α = thermal diffusivity in m^2/s

I. INTRODUCTION

This section provides a brief survey about the relevant literature and preliminary considerations of various thermodynamic properties of air. The first author Donald W. Mueller, Jr., Hosni I. Abu-Mulaweh presented a study on isentropic compression of a gas with temperature-dependent specific heat capacities [1]. The values of coefficients to solve thermodynamic properties are taken from this paper. The work by Sanford Gordan, Bonnie J. McBride [3] has also given the polynomial equation to solve specific heat, enthalpy and entropy which are dependent on temperature values. Density relation with respect to temperature is given by F J McQuillan, J R Culham, MMYovanovichin their paper [11]. The most satisfactory law of variation of viscosity of a gas with temperature was proposed by W. Sutherland [7]. R. Byron Bird, Warren E. Stewart, Edwin N. Lightfoot, in their paper [12] have considered an example of molecular momentum transport theory and introduced "Newton's law of viscosity" along with the definition of kinematic viscosity. Cannon John Rozier in his paper [13] had introduced the term heat diffusion which describes the heat distribution in a given region which may be further used with Fourier's law to determine the heat flux. A reliable experimental equation for thermal conductivity covering a wide range of temperature is presented by K Kodaya, N Matsunaga and A Nagashima [6].

II. GOVERNING EQUATIONS (THEORY, EXPERIMENT, EMPERICAL)

A. *Specific heat*

The polynomial governing equation for specific heat as a function of temperature [1] is presented as,

$$Cp = R(a_1T^{-2} + a_2T^{-1} + a_3 + a_4T + a_5T^2 + a_6T^3 + a_7T^4)$$

The coefficients' value are given with the attached MATLAB program.

B. *Density*

Density of air can be found using the relation, ρ = P/RT which requires the values of pressure, gas constant along with the temperature. In the relation given below, at atmospheric pressure, density can be found with the help of only temperature. This inverse relationship is given in the paper [11].

$$\rho = 351.99/T + 344.84/T^2$$

C. *Dynamic Viscosity*

The simple expression for a theoretical model is given below for the dynamic viscosity of a gas.

$$\mu/\mu_0 = (T/T_0)^{0.5}((1 + C/T_0)/(1 + C/T))$$

Where, μ_0, T_0 and C denote the reference values of the dynamic viscosity, absolute temperature, and the Sutherland's constant respectively. Using Holman's values of μ/μ_0 based on experimental measurements [8], the value of C was calculated to be 113. Based on experimental data from Barus [9], the above equation holds good for a temperature range of 273K to 1200K. Since experimental data is required to specify μ_0, the Sutherland's theoretical expression takes a semi- empirical form as shown below:

$$\mu = a (T^{1.5}/(C + T))$$

Where, a = 1.47 * 10^{-6} kg/msK$^{0.5}$, calculated using μ_0 at T_0. The above equation is further modified by using a multiplication factor, as suggested by James J. Gottlieb, David V. Ritzel [10] such that dynamic viscosity could be calculated for the temperature range of 78K to 2500K.

$$\mu = a (T^{1.5}/(C + T))(1+ 1.53*10^{-4}(T/C -1)^2)$$

D. *Kinematic Viscosity*

Referring to Bird et al [12], based on the analogy of Newton's viscosity equation with heat and mass transport, kinematic viscosity, often called as momentum diffusivity, is given by:

$$\nu = \mu/\rho$$

E. *Thermal Conductivity*

The keystone of conduction heat transfer is Fourier's law which defines a crucial material/transport property viz.

thermal conductivity. This property depends on the atomic & molecular structure of matter.

The trend in thermal conductivity among the different types of matter is $k_{solid}>k_{liquid}>k_{gas}$

In gases, the molecular collisions increase with rise in temperature which indicates ‘k’ increasing with temperature.

For gases, variation of ‘k’ with temperature can be presented both theoretically and experimentally.

Theoretical: Thermal conductivity of gases depicting the effect of temperature, pressure and chemical species may be explained in terms of kinetic theory of gases [5],

$$k = ⅓ (c_v \rho v \lambda_{mfp})$$

where, v is mean molecular speed &λ_{mfp}, the mean free path which denotes the average distance travelled by a molecule prior to a collision given by,

$$\lambda_{mfp} = k_b\, T/(1.414\pi d^2\, p)$$

where, kb is Boltzmann’s constant, d is the diameter of the gas molecule & ’p’ is the pressure.

Experimental: The uncertainty over thermal conductivity of air is large compared to other thermal properties. This is quite evident from the trace of extensive amounts of experiments carried out during the mid-19thcentury [4]. In fact, the precariousness in experimental determination is larger than the theoretical way of arriving at a value of ‘k’ as a function of temperature due to the difficulty exhibited in measuring it. In order to cover a wider temperature range & reliability of results, the below equation derived from the extensive study done by K Kodaya, N Matsunaga and A Nagashima [6] is used.

$$k\,(T_r,\rho_r) = \Lambda\,[k_0\,(T_r) + \Delta k\,(\rho_r)]$$

Where,

$$k_0\,(T_r) = C_1 T_r + C_{0.5}\, T_r^{0.5} + \sum_{i=0}^{-4} C_i T_r^i$$

$$\Delta k\,(\rho_r) = \sum_{i=1}^{5} D_i \rho_r^i$$

$$T_r = T//T^*$$

and $\rho_r = \rho/\rho^*$

The list of constants to be used to arrive at final ‘k’ values is as given in the following table 1.

TABLE II. CONSTANTS CATERING EXPERIMENTAL EQUATION

Constants catering experimental equation	
T* 132.5 K	D_1 0.4022
ρ* 314.3 kg/m³	D_2 0.3566
Λ 25.9778x10^{-3} W/m.K	D_3 -0.1631
C_1 0.2395	D_4 0.1380
$C_{0.5}$ 0.0064	D_5 -0.0201
C_0 1.0000	
C_{-1} -1.9261	
C_{-2} 2.0038	
C_{-3} -1.0755	
C_{-4} 0.2294	

F. Thermal Diffusivity

In heat transfer, the ability of any material to conduct thermal energy relative to its ability to store it is measured as thermal diffusivity. In the one dimensional heat equation derivation by Cannon [13] from Fourier's law and conservation of energy, it is seen that the thermal diffusivity is given by:

$$\alpha = k/\,(\rho\, Cp)$$

III. SENSITIVITY ANALYSIS

A. Graphical representation

The graphs showing the variation of air properties with respect to the temperatures are shown below and a comparison with the values available from the heat and mass transfer data hand book by C P Kothandaraman and S Subramanyan is done for the validation of all the correlations presented on this paper. The trend of errors is also plotted in the graphs to clearly understand the range of temperatures over which the correlations are reliable.

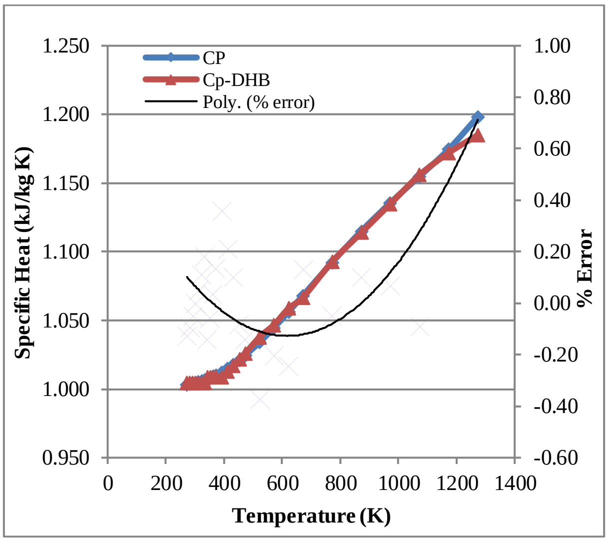

Fig.1. Specific heat vs Temperature

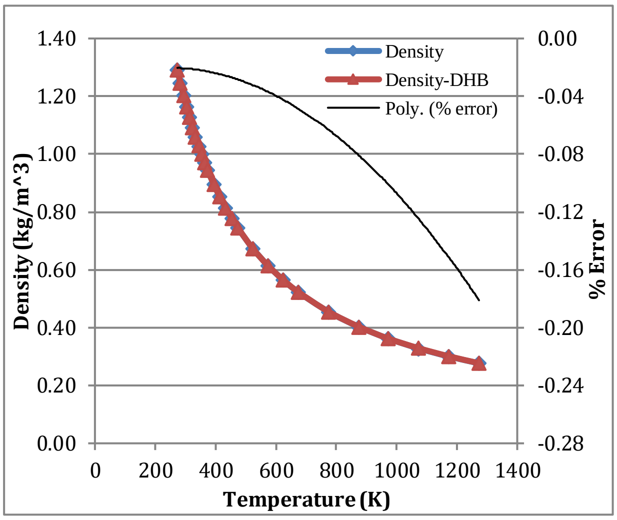

Fig. 2. Density vs Temperature

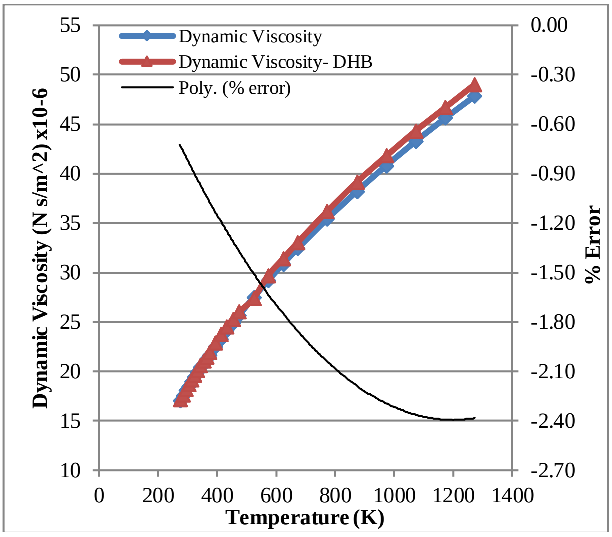

Fig.3. Dynamic viscosity vs Temperature

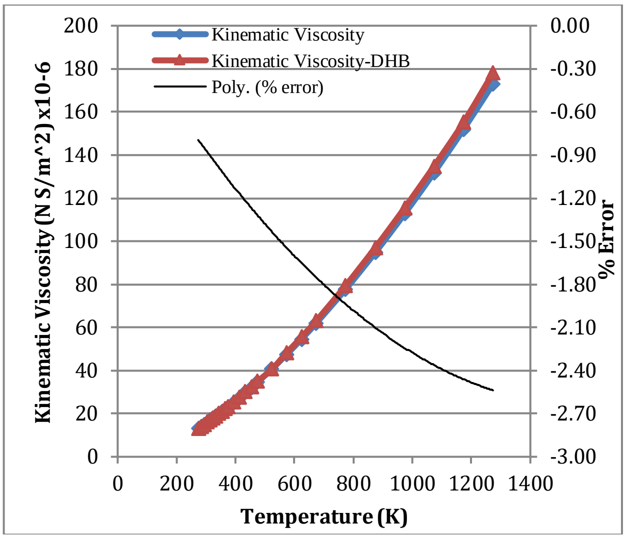

Fig.4. Kinematic viscosity vs Temperature

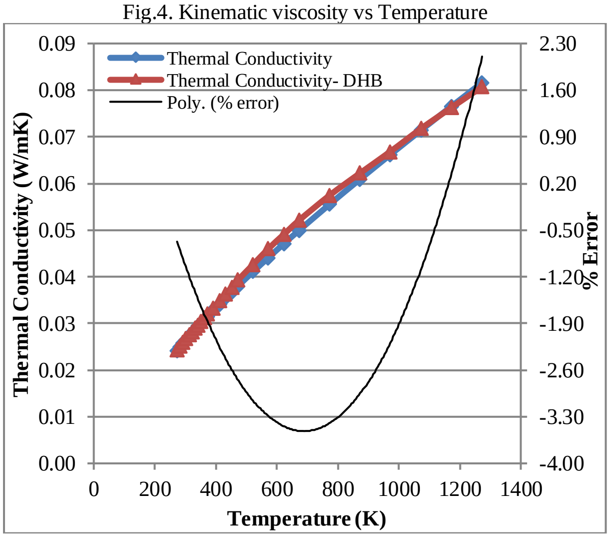

Fig.5. Thermal Conductivity vs Temperature

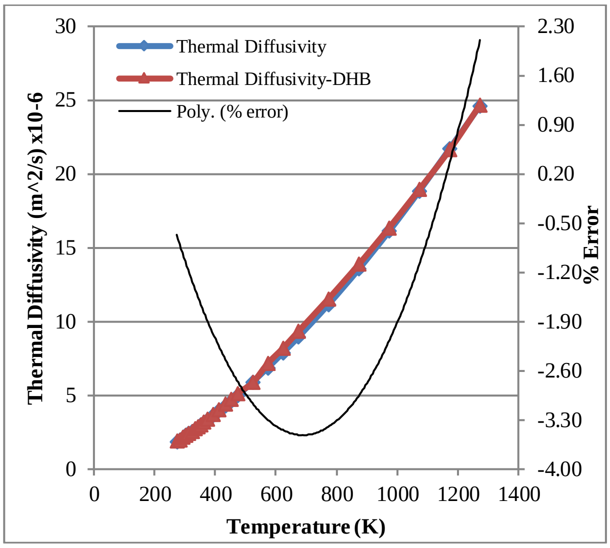

Fig.6. Thermal Diffusivity vs Temperature

IV. OBSERVATIONS

The comparative studies shown above have red curve corresponding to the values obtained from heat transfer data hand book, the blue curve corresponds to the values obtained from correlation suggested in the paper and the black curve shows the trend of error with respect to change in temperature values.

From Figure 1 it is observed that the errors are within ±0.2% for the temperature range of 273 K to 1100 K and go higher up to 1.2%beyond elevated temperatures of 1100 K. Similarly, for other properties the error limits are summarized in the table2 below.

TABLE II. PROPERTY VS TEMPERATURE AND ERROR

Property	Temperature (K)	Error (%)
Specific Heat	273 – 1100 Above 1100	±0.2 Up to 1.2
Density	273 – 1300	-0.2 to 0.03
Dynamic Viscosity	273 – 1300	-0.5 to -2.5
Kinematic Viscosity	273 – 1300	-0.5 to -2.8
Thermal Conductivity	273 – 700 700 – 1300	-0.6 to -4.1 -4 to +1.1
Thermal Diffusivity	273 – 1300	-4.1 to 0.8

It is clear that the difference between values of air properties obtained using the correlations and the data hand book is negligible since the error, as shown above, is less than 5% in any case. The maximum error of about 4% is observed in the case of thermal conductivity and thermal diffusivity within a range of 650K to 750 K only.

V. CONCLUDING REMARKS

We have presented the correlations for specific heat, density, dynamic viscosity, kinematic viscosity, thermal conductivity & thermal diffusivity varying with temperature over the range of 273-1300K . Graphs have been plotted showing the variation of these air properties with respect to temperature. For validation of results the curves have been compared with the values given in heat and mass transfer data hand book by C P Kothandaraman and S Subramanayan. The correlations suggested in this paper are found satisfactory when compared with the data hand book air properties. Finally,a MATLAB code for students is given to strengthen their understanding of these thermodynamic properties.

Further study is needed to accommodate the impact of varying the pressure and chemical composition of air on the correlations presented.

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Appendix A

```
% Input Parameters


T=[100,200,300,400,500,600,700,800,900,1000]; % Kelvin


% Specific heat Cp

% For T value less than 1000K

a1 = 1.009950160e+04;

a2 = -1.968275610e+02;

a3 = 5.009155110e+00;

a4 = -5.761013730e-03;

a5 = 1.066859930e-05;

a6 = -7.940297970e-09;

a7 = 2.185231910e-12;
```

```
R = 0.287; % KJ/KgK

% Enthalpy

b1 = 6.462263190E+03;

b2 = -8.147411905E+00;


% Dynamic Viscosity

a = 1.47*10^(-6);

C = 113;


% Thermal Conductivity

C1= 0.2395;

C0 = 0.0064;

for i=1:10

Tr = T(i)/132.5;

 j=1;

  Cp(i) = R*(a1*T(i)^(-2) + a2*T(i)^(-1) + a3 + a4*T(i) + a5*T(i)^2 + a6*T(i)^3 + a7*T(i)^4); % Specifi Heat KJ/KgK


h(i) = R*T(i)*(-a1*T(i)^(-2) + a2*T(i)^(-1)*log(T(i)) + a3 + a4*T(i)/2 + a5*T(i)^2/3 + a6*T(i)^3/4 + a7*T(i)^4/5 + b1/T(i)); % Enthalpy KJ/Kg


s(i) = R*(-a1*T(i)^-2/2 - a2*T(i)^(-1) + a3*log(T(i)) + a4*T(i) + a5*T(i)^2/2 + a6*T(i)^3/3 + a7*T(i)^4/4 + b2); % Entropy KJ/KgK


rho(i) = 351.99/T(i)+ 344.84/T(i)^2;  % Density Kg/m^3


Mu(i) = 1.47*10^(-6)*(T(i)^(1.5))/(113+T(i));


Nu(i) = Mu(i)/rho(i); % Kinematic Viscosity


C2(i) = 1*Tr^(0);
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```
C3(i) = -1.9261*Tr^(-1);
C4(i) = 2.0038*Tr^(-2);
C5(i) = -1.0755*Tr^(-3);
C6(i) = 0.2294*Tr^(-4);
D1(i) = 0.4022*rho(i)/314.3;
D2(i) = 0.3566*rho(i)/314.3^2;
D3(i) = -0.1631*rho(i)/314.3^3;
D4(i) = 0.138*rho(i)/314.3^4;
D5(i) = -0.0201*rho(i)/314.3^5;
K0(i) = (C1*Tr) + C0*(Tr)^0.5 + C2(i)+C3(i)+C4(i)+C5(i)+C6(i);
K1(i) = D1(i)+D2(i)+D3(i)+D4(i)+D5(i);
C = 0.025978;
K(i) = C*(K0(i)+K1(i)); % Thermal Conductivity
 alpha(i) = K(i)/(rho(i)*Cp(i)*1000); % Thermal diffusivity

Property(i,1) = T(i);
Property(i,2) = Cp(i);
Property(i,3) = h(i);
Property(i,4) = s(i);
Property(i,5) = rho(i);
Property(i,6) = Mu(i);
Property(i,7) = Nu(i);
Property(i,8) = K(i);
Property(i,9) = alpha(i);

end
```