Viscosity Effect on Vortices in Magnetized Dusty Plasmas

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Abstract

*Recently Nebbat and Annou proposed a model that explains the behavior of the vortex structure that is considered a consequence of an instability in dusty plasmas [Phys. Plasmas,**17**, 093702 (2010); Phys. Plasmas,**19**, 093705 (2012).]. As the dust numerical density increases the momentum loss due to grain-grain collisions is substantial and the shear viscosity is to be coped with. The model is augmented accordingly and a comparison is conducted. The effect of viscosity is found depending on the size grain.*

1. Introduction

In a dusty plasma that contains electrons, ions and massive and highly charged grains, the latter rearrange themselves collectively to give rise to numerous nonlinear solutions such as, voids, vortices, solitons and cnoidal waves. The vortices are formed under certain conditions when a rotational motion around a core, is triggered. These non-linear structures that have been observed in many laboratories [1-3] are investigated by way of various theoretical models to explain their characteristics [4-6]. However, the viscosity factor is neglected in the above mentioned models, as the fluid is taken tenuous, although in most experiments the gas pressure is not that low. As a matter of fact, an experimental investigation of shear viscosity in dusty plasma medium conducted by Gavrikov *et al.*[7] reveals that the shear viscosity increases with increasing gas pressure. In dust free media such as fluids, when taking into account the viscosity parameter [8-14] , Olonius *et al.*[15] studied vortices and found that these structures exist for longer periods of time when the influence of viscosity is weak, otherwise the viscosity leads to the decay of the structure. Indeed, according to Babkin's *et al.*[16] findings the intensity as well as the velocity of the vortex are reduced in viscous medium. The contribution of the shear viscosity in one component plasmas has been analyzed also by Murillo[17]. In dusty plasmas, experiments conducted by Agarwal *et al.*in unmagnetized plasmas confirmed that when the gas pressure increases, the friction due to viscosity increases and the peripheral velocity decreases [18]. Similarly, in magneto-plasmas, the same inclination is emphasized, i. e., when pressure increases the peripheral velocity of dust grains decreases [19].

In this paper the model in Ref.4 is extended, as the viscosity is introduced, the results are compared with those in Ref. [4]. The paper is organized as follows. In Sec.I, the problem is introduced whereas in Sec. II it is formulated. In Sec.III, the results are discussed and the Section IV is devoted to the concluding remarks.

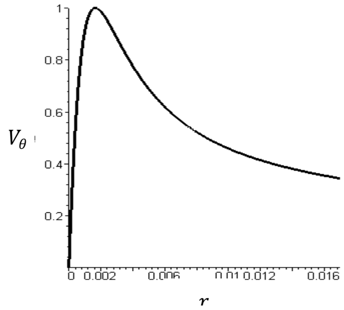

Fig.1: Peripheral velocity V_θ vs position r for $r_d = 20\ \mu m$ without viscosity

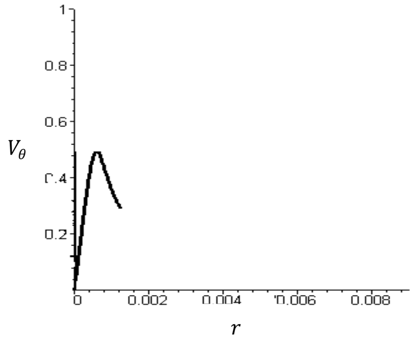

Fig.2: Peripheral velocity V_θ vs position r for $r_d = 20\ \mu m$ with viscosity

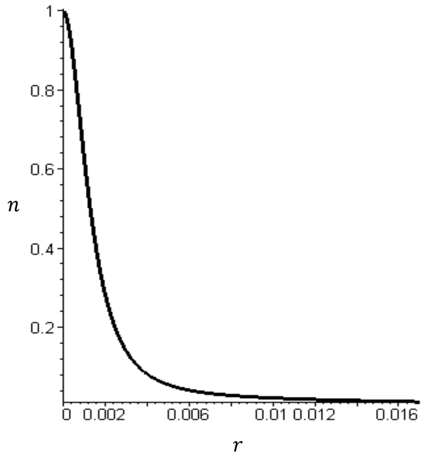

Fig.3: Dust numerical density n vs position r for $r_d = 20\ \mu m$ without viscosity

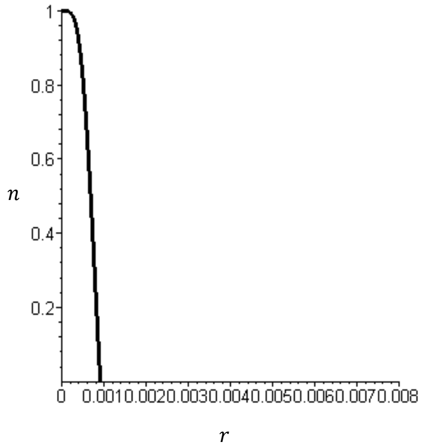

Fig.4: Dust numerical density n vs position r for $r_d = 20\ \mu m$ with viscosity

2.Formulation

We consider a dusty plasma containing electrons, ions, and massive negatively charged dust grains. Due to viscosity, the grains momentum equation is rewritten as follows,

$$m_d\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -Ze\left(\vec{E} + \frac{\vec{V}\times\vec{B}}{c}\right) - \frac{\vec{\nabla}P}{n} + \vec{F_i} + \vec{F}_{int} + \eta\vec{\nabla}.\vec{\nabla}\vec{V} \quad (1)$$

where $n, \vec{V}, T, Z, m, D, \vec{F_i}$ and $\vec{F}_{int}$ are respectively, dust density, dust velocity, dust temperature, dust charge, grain mass and dust diffusion coefficient., ion drag force, and the interaction between grains. The ion drag force is approached by Khrapak's ion drag force

$$F_i = 8\sqrt{2\pi}/3\ r_d^2\ n_i\ m_i\ v_{thi}\ v_i\ \left[1 + 1/2\ z\,\tau + 1/4\ z^2\ \tau^2\ \Lambda\right]$$

with Λ is the Coulomb logarithm, r_d the grain radii and $\tau = T_e/T_i = 1/\tau_i$, we note that the ion velocity is very high, therefore, the magnetic field effect on this force may be neglected, and the interaction force is

$$F_{int} = (Z^2e^2/\alpha^2 n^{-2/3})exp(-\alpha n^{-1/3}/\lambda_D),$$

where d is the inter-grain distance and $\alpha = (3/4\pi)^{1/3}$.
The viscosity η is determined by the Einstein-Stokes formula written as,

$$\eta = \frac{T_d}{8.1\ r_d D}.$$

The dust continuity equation is written as,

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{V}) = D\nabla^2 n - \frac{ZeD}{T}\vec{\nabla}.(n\vec{E}) \quad (2)$$

The ion continuity equation is given by,

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}.(n_i\vec{V_i}) = \delta_i \quad (3)$$

where, $\delta_i = -a_i n_i n + \alpha_e n_a n_e$, accounts for the loss and gain of particles that are due to ion attachment and ionization.
The self-consistent electric field is given by Poisson's equation,

$$\vec{\nabla}.\vec{E} = 4\pi e(n_i - n_e - Zn) \quad (4)$$

The electron dynamics is described by the momentum equation,

$$e\vec{E} = -\frac{\vec{\nabla}P_e}{n_e} \quad (5)$$

The vorticity vector $\vec{\Omega} = \vec{\nabla}\times\vec{V}$ satisfies Helmholtz equation,

$$\frac{d}{dt}\left(\frac{\vec{\Omega}}{n}\right) = \eta\nabla^2\vec{\Omega}, \quad (6)$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V}.\vec{\nabla}$.

As motion is solid, the peripheral velocity and density depend on the radial dimension.
The system of equations (1)-(6) is normalized and rewritten as follows,

$$\frac{\partial n_i}{\partial r} = -\frac{a_i n_{i0} e\chi^2 n_i n}{\mu Z T_e E} + \frac{\alpha_e n_{i0} n_a e\chi^2 n_e}{\mu T_e E} - \frac{4\pi e^2 n_{i0}\chi^2}{T_e E} n_i(n_i - n_e - n), \quad (7)$$

$$\frac{\partial n_e}{\partial r} = -n_e E, \qquad (8)$$

$$\frac{\partial \Psi_r}{\partial r} = \Psi_r \left\{ \frac{\omega_{pd}\chi^2 V_r}{D} + \frac{ZeT_d\chi}{m_d D\left(\omega_{pd}\chi V_r - \frac{\eta}{m_d \chi r}\right) D} - \frac{1}{r} + \frac{ZT_e E}{T_d} \right\} +$$

$$n \left\{ \frac{4\pi Z e^2 D n_{i0}\chi^2 (n_i - n_e - n)}{T_i \tau_d} + \frac{\omega_{pd}\chi^2 V_r}{r} + \left(\left(-\frac{ZT_e\chi}{m_d} + \frac{8\sqrt{2\pi} r_d^2 n_{i0} V_{thi} T_e A_\tau \chi}{3\nu_{in} m_d} \right) E + m_d \xi \chi^2 V_\theta + \frac{(Ze)^2 \chi^2 e^{\frac{-\alpha \left({}^{n_{i0}}/_{Z} \right)^{-1/3}}{\lambda_D} n^{-1/3}}}{\alpha^2 m_d \left({}^{n_{i0}}/_{Z} \right)^{-2/3} n^{-2/3}} + \left(\frac{m_d c\, \xi}{B} \right)^2 \frac{\chi V_\theta^2}{r} - \frac{\eta \chi \omega_{pd} V_r}{r^2} \right) \left(\frac{1}{\omega_{pd}\chi V_r - \frac{\eta}{m_d \chi r}} \right) \right\},$$

(9)

$$\frac{\partial V_r}{\partial r} = \left\{ -\left(\frac{m_i}{m_d} \right)^2 \frac{T_e E}{e\chi\xi} - V_\theta - \frac{T_d \Psi_r}{m_d \chi \xi n} + \frac{8\sqrt{2\pi} r_d^2 n_{i0} V_{thi} A_\tau T_e E}{3\nu_{in} m_d \omega_{pd} \chi} + \frac{(Ze)^2 e^{\frac{-\alpha \left({}^{n_{i0}}/_{Z} \right)^{-1/3}}{\lambda_D} n^{-1/3}}}{\alpha^2 \omega_{pd} m_d \left({}^{n_{i0}}/_{Z} \right)^{-2/3} n^{-2/3}} - \frac{\eta V_r}{m_d \chi r^2} + \left(\frac{m_d c}{B} \right)^2 \frac{\xi V_\theta^2}{Ze\chi r} \right\} \left\{ \frac{1}{\omega_{pd}\chi V_r - \frac{\eta}{m_d \chi r}} \right\}, \qquad (10)$$

$$\frac{\partial V_\theta}{\partial r} = \Omega - \frac{V_\theta}{r}, \qquad (11)$$

$$\frac{\partial E}{\partial r} = 4\pi e^2 n_{i0} \chi^2 (n_i - n_e - n) - \frac{E}{r} \qquad (12)$$

$$\frac{\partial \Omega}{\partial r} = \frac{\Omega}{n\left(1 - \frac{\eta}{\chi r}\right)} \Psi_r, \qquad (13)$$

$$\frac{\partial n}{\partial r} = \Psi_r, \qquad (14)$$

where, $\chi = r_d \tau_i^2 \left(\frac{m_d}{m_i} \right)$, $\xi = \frac{\omega_{pd}}{Ze}$, $\alpha = \left(\frac{3}{4\pi} \right)^{1/3}$, $\tau_i = \frac{T_i}{T_e}$, $\Psi_r = \frac{\partial n}{\partial r}$, and $A_\tau = 1 + \frac{Z\tau}{2} + \frac{Z^2 \tau^2 \Lambda}{2}$.

The position is normalized by $\frac{r_d \tau_i^2 m}{m_i}$, the electric field by $\frac{T_e m_i}{e r_d \tau_i^2 m}$, and time by ω_{pd}^{-1}, whereas the density of dust, electrons and ions is normalized by $\frac{n_{i0}}{Z}$, n_{i0}, and n_{i0} respectively, the vorticity by $\frac{m_i c \omega_{pd}^2}{ZeB r_d \tau_i^2}$, and the peripheral velocity by $\frac{m c \omega_{pd}}{ZeB}$.

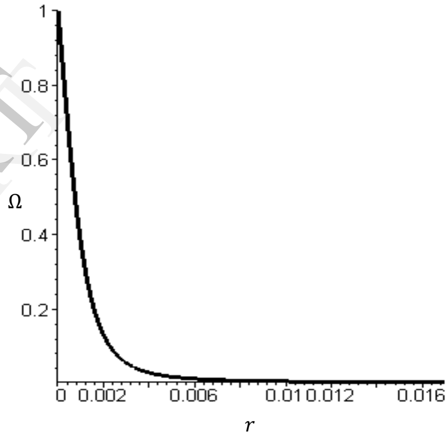

Fig.5: Vorticity Ω vs position r for $r_d = 20\ \mu m$ without viscosity

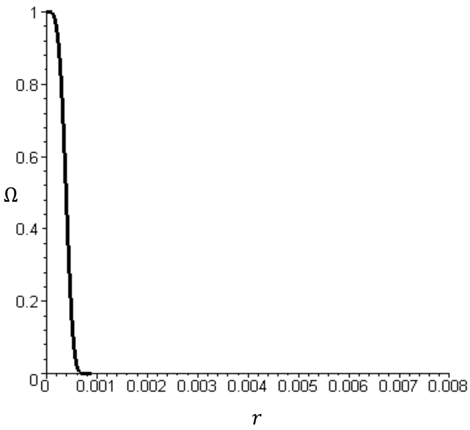

Fig.6: Vorticity Ω vs position r for $r_d = 20\ \mu m$ with viscosity

3.Discussion

We consider an argon plasma with the following parameters, viz., $T_e = 3\, eV$, $n_i = 10^{11} cm^{-3}$, $T_i = 0.1\, eV$, $n_0 = 1.8 \times 10^6 cm^{-3}$, $T_d = 0.025 eV$, $\rho = 2.2\, gcm^{-3}$ (mass density of the grain) and $B = 0.2\, T$. The vortex is characterized by its velocity peripheral that increases from the center to reach a maximum and decreases beyond. The velocity with respect to position r for $r_d = 20\, \mu m$ is plotted in Fig. 1 in absence of viscosity and in Fig. 2 with shear viscosity accounted for; it is clear that in case viscosity is neglected, we retrieve earlier work findings (c.f. Ref.[4]). The result reveals that the shear viscosity decreases the peripheral velocity, i. e., the shear viscosity introduces friction between grains, in agreement with experimental work in Refs.[6,18]. Furthermore, the shear viscosity reduces the vortex size. In Figs. 3 and 4, the density distribution with respect to position r for $r_d = 20\, \mu m$ is plotted respectively in absence and with shear viscosity, where it is revealed that the shear viscosity generates a collapse of the grain cloud toward the center that is the grains are pushed inwardly. In Figs. 5and 6, the vorticity is plotted respectively in absence and with shear viscosity for $r_d = 20\, \mu m$, where it is seen a reduction of vorticity caused by viscosity. In presence of shear viscosity the peripheral velocity is also plotted in Fig. 7 for different values of grain radius, viz, $r_d = 20\, \mu m, r_d = 10\, \mu m, r_d = 2.44\, \mu m$; the results reveal the decrease of the maximum value of the peripheral velocity when the grain size increases, contrary to the result found in Ref.[4] for the non-viscous magnetoplasma where the maximum value of the peripheral velocity stays constant for different sizes of the grain. Moreover, the comparison between d_{c1},d_{c2} and d_{c3}which are the vortex core sizes for grain radii respectively equal to $r_d = 2.4\, \mu m, r_d = 10\, \mu m$, and$r_d = 20\, \mu m$ gives $d_{c1} > d_{c2} > d_{c3}$, that is the vortex core decreases for grain size increase. Also, the same figure shows that closer to center, the peripheral velocity is the same for all grain sizes. In Fig. 8, density is plotted versus position r for different values of the grain radius, viz, $r_d = 20\, \mu m, r_d = 10\, \mu m, r_d = 2.44\, \mu m$; it appears that the collapse of the grains cloud increases when the grain radius increases, viz., a result found earlier in non-viscous plasmas[4]. In addition, for the same condition of Figs. 4 and 5, the vorticity is plotted in Fig. 9 for different values of grain radius, where it is found that vorticity is higher for smaller grains in size, in accordance with the findings of Ref.[4].

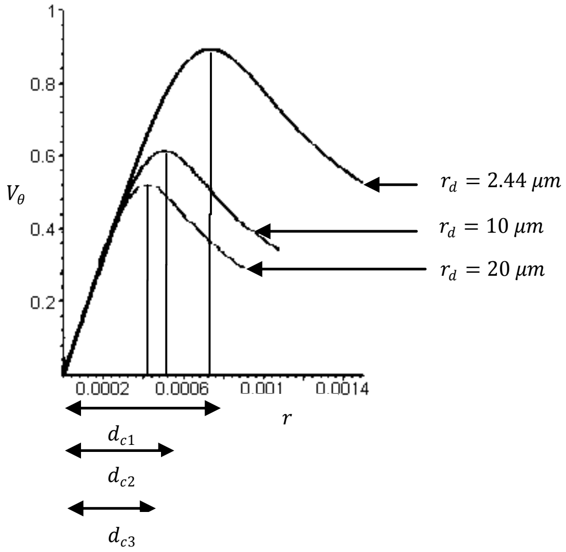

Fig.7: Peripheral velocity V_θ vs position r for different values of grain radius r_d with viscosity

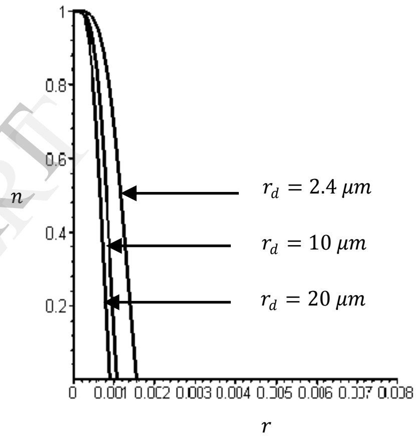

Fig.8: Dust numerical density n vs position r for different values of grain radius r_d with viscosity

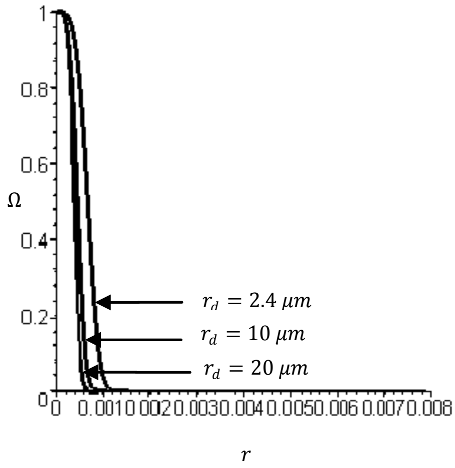

Fig.9: Vorticity Ω vs position r for different values of grain radius r_d with viscosity

4. Conclusion

Recently a model is proposed that explains the behavior of the vortex structure where it is considered a consequence of an instability in dusty plasmas. As the dust numerical density increases the momentum loss due to grain-grain collisions is substantial and the shear viscosity is to be taken care of. The model is then augmented and a comparison is conducted. The effect of viscosity is found to decrease the peripheral velocity in good agreement with Refs.[6, 18], along with the vortex core. Furthermore, the shear viscosity generates a collapse of the grain cloud towards the center. The vorticity, which indicates the rotational movement is less when the shear viscosity is introduced. In addition, the effect of the shear viscosity is found depending on the grain size, for the calculations show that when the grain size increases, the peripheral velocity decreases as well as the vortex core, but closer to center the peripheral velocity is the same for all grain sizes. Furthermore, is noted that vorticity shows a rapid decrease for bigger grains.

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