

# Virtual Experimental Modal Analysis of a Cantilever Beam

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**Abstract**— Experimental modal analysis (EMA) is an accepted methodology for identifying modal parameters of complex system assemblies, but large channel counts and multiple references are usually required to perform a thorough modal survey of such structures. In these cases, modal testing can become very costly and time consuming. Moreover, success in accurately defining all the modes of interest from experimental data depends on a variety of factors, including how and where the excitation is applied, how and where measurement data is acquired, and how boundary conditions or other environmental conditions are simulated. This presentation suggest a methodology for virtual experimental modal analysis (VEMA).

## I. INTRODUCTION

Experimental modal analysis has grown steadily in popularity since the advent of the digital FFT spectrum analyser in the early 1970's. Today, impact testing (or bump testing) has become widespread as a fast and economical means of finding the modes of vibration of a machine or structure. Modes are used as a simple and efficient means of characterizing resonant vibration. The majority of structures can be made to resonate. That is, under the proper conditions, a structure can be made to vibrate with excessive, sustained, oscillatory motion. Resonant vibration is caused by an interaction between the inertial and elastic properties of the materials within a structure. Resonant vibration is often the cause of, or at least a contributing factor to many of the vibration related problems that occur in structures and operating machinery.

To better understand any structural vibration problem, the resonances of a structure need to be identified and quantified. A common way of doing this is to define the structure's modal parameters. Modes (or resonances) are inherent properties of a structure. Resonances are determined by the material properties (mass, stiffness, and damping properties), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape. If either the material properties or the boundary conditions of a structure change, its modes will change. At or near the natural frequency of a mode, the overall vibration shape (operating deflection shape) of a machine or structure will tend to be dominated by the mode shape of the resonance.

The initial step in experimental modal analysis is to obtain the response of a structure to a certain kind of excitation. This might be accomplished under operational conditions or from modal testing within a controlled test environment. Once the response measurement has been completed, the next step to be performed is the identification of the modal parameters of the structure from the measured data. For this task a broad range of modal analysis techniques have been developed over the past decades. The decision which technique to apply in a given situation is to be based on diverse considerations, the most important of which are: Is the excitation known (which is generally the case in experimental modal analysis) or unknown (which is generally the case in operational modal analysis where the response of the structure to an ambient excitation under operational conditions is measured)? Is the data available just in the frequency domain or also in the time domain? Does a reasonable estimation of the parameters exist as a starting point for an iterative procedure to improve the initial guess? Are the structural modes well separated or closely spaced? Is it reasonable to assume a lightly damped structure?

There are several criteria for the classification of modal analysis techniques. He and Fu distinguish between single-degree-of-freedom (SDoF) and multi-degree-of-freedom (MDoF) methods [7]. Natke classifies modal analysis techniques as either phase resonance methods or phase separation methods [8]. Phase resonance methods are based on an adaption of the excitation in such a way that only one single mode is excited which simplifies the post-processing of the thus obtained data quite a lot but is experimentally very extensive. Phase separation methods on the other hand shift most of the effort to the post-processing of the data as quite elaborate and computationally expensive numerical methods are required to extract reliable modal information from data comprising the coexistent contribution of several modes. The most widely-used classification however distinguishes between frequency domain methods and time domain methods. The following list gives an overview of some popular modal analysis techniques as presented in [7],[8] and [9].

### 1 - Frequency domain methods

- Peak-picking method
- Circle-fit method
- Inverse FRF method

## II - Time domain methods

- Least-squares time domain method
- Ibrahim time domain (ITD) method
- Random Decrement (RD) method
- ARMA Time Series method
- Least-squares Complex Exponential (LSCE) method

## III - Analysis methods

- Least-square method
- Dobson's method

The Fourier Transformation or Fourier Transform is a very fundamental mathematical instrument of signal analysis. Originally developed by Fourier for the solution of differential equations in the area of heat conduction, it soon was applied to totally different problems throughout the various disciplines of science and engineering. Initially developed as an analytical instrument and therefore constrained to the application to analytically defined and integrable functions, the real power of the Fourier Transform could only be exploited with the advent of modern computers.

Andrew S. Elliott, Mark H. Richardson [1] have introduced a new method for simulating an Experimental Modal Analysis (EMA) using a combination of two commercially available software tools, ADAMS and ME'scope and named this new simulation method Virtual Experimental Modal Analysis, or VEMA. The primary advantage of this completely analytical approach is that many important questions regarding modal testing of very complex structures can be addressed before the actual testing is done.

Brian J. Schwarz, Mark H. Richardson [2], gives a review on all the main topics associated with experimental modal analysis, including making FRF measurements with a FFT analyser modal excitation techniques and modal parameter estimation from a set of FRFs (curve fitting).

Peter Avitabile [3], presents a non-mathematical presentation of Experimental Modal Analysis, which give an idea on modal analysis, Frequency Response Functions, difference between shaker test and impact test, effect of window. He describes structural vibration and the use of some of the available tools for the solution of structural dynamic problems.

Experimental modal analysis (EMA) is an accepted methodology for identifying modal parameters of complex system assemblies, but large channel counts and multiple references are usually required to perform a thorough modal survey of such structures. In these cases, modal testing can become very costly and time consuming. Moreover, success in accurately defining all the modes of interest from experimental data depends on a variety of factors.

- How and where the excitation is applied.
- How and where measurement data is acquired.
- How boundary conditions or other environmental conditions are given.

The chances of performing a successful EMA are significantly improved if answers to the following critical questions are known beforehand:

- What type of excitation signals should be used.
- What analysis frequency range should be used.
- How much frequency resolution is needed.
- How the boundary conditions are accounted.
- How do operating conditions, such as internally generated forces, excite the modes.

A key advantage of the VEMA approach is that it simulates multi-channel data acquisition. Multiple transducer and excitation locations and directions can be easily introduced into the model. Transducer sensitivities and signal processing can also be simulated. All of the excitation and response data can be simultaneously sampled, for as many channels as desired at any desired rate. A set of simulated test data can be acquired in only a few minutes, whereas acquisition of the actual data in a laboratory environment might take hours or even days.

### A. Frequency Range and Resolution (FRR)

One of the fundamental questions in any EMA is, What frequency range and resolution is needed to adequately estimate the parameters of the fundamental modes. In many real EMAs, the easiest way to answer this question is to impact the structure with an instrumented hammer, measure a few Frequency Response Functions (FRFs), and examine the FRFs for resonance peaks. Also, a preliminary look at the mode shapes is helpful if a sufficient number of FRFs are measured over the surface of the structure so that one shape can be distinguished from another.

### B. Frequency Response Function (FRF)

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of a mechanical structure. Experimental modal parameters (frequency, damping, and mode shape) are also obtained from a set of FRF measurements.

The FRF describes the input-output relationship between two points on a structure as a function of frequency, as shown in Fig 3.1. Since both force and motion are vector quantities, they have directions associated with them. Therefore, an FRF is actually defined between a single input DOF (point & direction), and a single output DOF.

An FRF is a measure of how much displacement, velocity, or acceleration response a structure has at an output DOF, per unit of excitation force at an input DOF.

Fig. 1 also indicates that an FRF is defined as the ratio of the Fourier transform of an output response ( $X(\omega)$ ) divided by the Fourier transform of the input force ( $F(\omega)$ ) that caused the output.

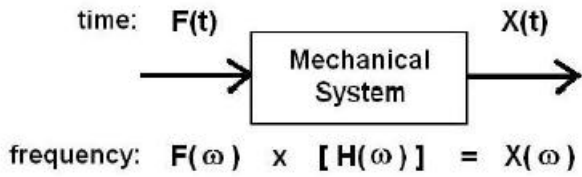


Fig. 1. Block Diagram of FRF

An FRF is a complex valued function of frequency that is displayed in various formats, as shown in Fig. 2.

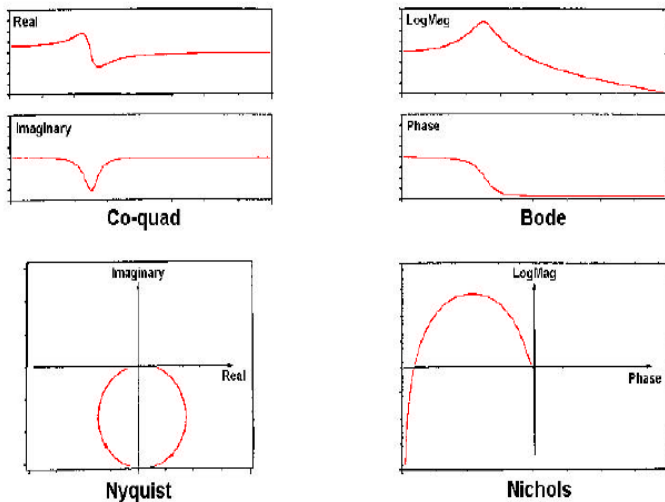


Fig. 2. Alternate Formats of FRF

### C. Vibration is Easier to Understand in Terms of Modes

Fig. 3 points out a reason why vibration is easier to understand in terms of modes of vibration. It is a plot of the Log Magnitude of an FRF measurement (the solid curve), but several resonance curves are also plotted as dotted lines below the FRF magnitude. Each of these resonance curves is the structural response due to a single mode of vibration.

The overall structural response (the solid curve) is in fact, the summation of resonance curves. In other words, the overall response of a structure at any frequency is a summation of responses due to each of its modes. It is also evident that close to the frequency of one of the resonance peaks, the response of one mode will dominate the frequency response.

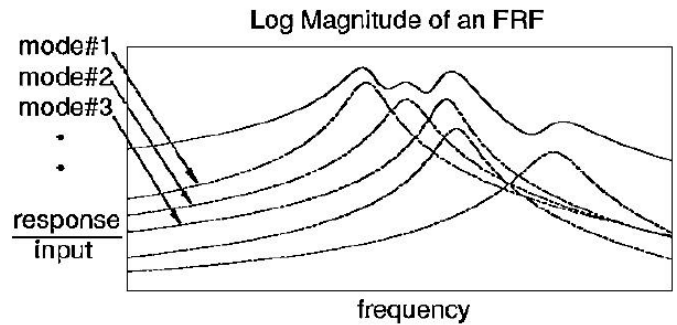


Fig. 3. Response as Summation of Modal Responses

### D. Fast Fourier Transform (FFT)

Any waveform is actually just the sum of a series of simple sinusoids of different frequencies, amplitudes, and phases. A Fourier series is that series of sine waves and we use Fourier analysis or spectrum analysis to deconstruct a signal into its individual sine wave components. The result is vibration amplitude as a function of frequency, which lets us perform analysis in the *frequency domain* (or spectrum) to gain a deeper understanding of our vibration profile. Most vibration analysis will typically be done in the frequency domain. Fourier analysis converts a signal from its original domain (often time) to a representation in the frequency domain. FFT representation of a mixture of Sin wave is as shown in Fig. 4.

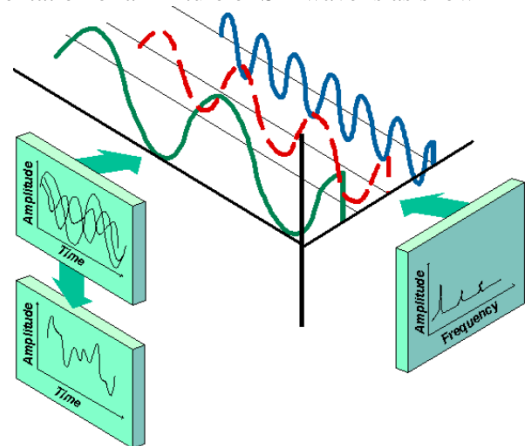


Fig. 4. FFT representation of a mixture of Sin wave

## II. METHODOLOGY

### A. Theoretical Route of Vibration Analysis

The theoretical route is as shown in Fig. 4. The usual procedure for getting the natural frequencies and mode shapes is by modelling the system, followed by meshing the system, then followed by Eigen value analysis.

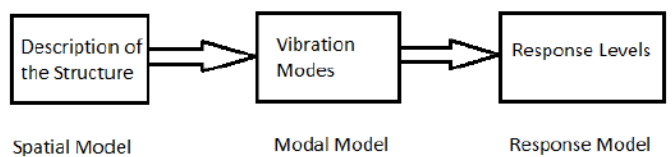


Fig. 4. Theoretical Route of Vibration Analysis

### B. Experimental Route of Vibration Analysis

The experimental route is as shown in Fig. 5. The procedure for getting the natural frequencies and mode shapes is by modeling of the system, followed by meshing the system, then followed by response analysis of shaker test and impact test. Least square curve fitting technique is used get the natural frequencies and mode shapes.

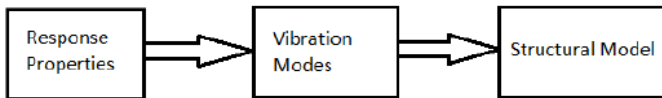


Fig. 5. Experimental Route of Vibration Analysis

### C. Modal Analysis of Cantilever Beam

Modal analysis of a cantilever beam was done in both ANSYS and MATLAB and a comparison of the natural frequencies were done.

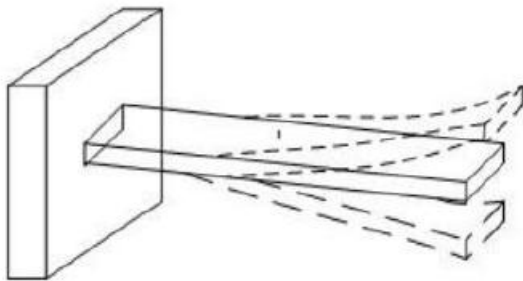


Fig. 6. Structural View of Cantilever Beam

TABLE I. DETAILS OF THE BEAM

DIMENSIONS	Length (l)	1000 mm
	Breadth (b)	20 mm
	Height (h)	1.5 mm
PROPERTIES	Elastic Modulus (E)	210 * 10 <sup>9</sup> Pa
	Density (ρ)	7800 kg/m <sup>3</sup>
	Moment of Inertia (I)	5.625 * 10 <sup>-12</sup> m <sup>4</sup>

$$\text{Moment of Inertia, } I = (b \cdot h^3) / 12$$

A 3D analysis of the cantilever beam was done in ANSYS and the element taken was 3D-2Node 188 beam element. The 3D-2Node 188 beam element is a 3D element with 2 nodes and each node is having 6 degree of freedom (DOF)(ie.. rotation about x,y,z axis and translation along x,y,z axis). The beam was divided into 10 sections of equal length, with each element having 2 nodes, thus resulting in a total of 11 nodes. At node 1 All DOFs were constrained.

A 2D analysis of the cantilever beam was done using MATLAB program. The beam was assumed to have only 2 DOFs per node (ie.. rotation about z-axis and translation along y-axis) The beam was divided into 10 sections of equal length. Thus each element is having 2 nodes and so a total of 11 nodes where required. Since each node is having 2 DOFs, so the total DOF of the system was 22. At node 1 all DOFs were constrained.

### D. Modal Analysis Using Ansys

Procedure is as follows:

- Step 1:Pre-processor→ Element type→ Add→ Beam→ 2node188→ OK
- Step 2:Pre-processor→ Modelling→ Create→ Keypoints→ In Active CS  
X= 1  
Pre-processor→ Modelling→ Create→ Lines→ Straight Line→ OK
- Step 3:Pre-processor→ Meshing→ Mesh Tools→ Set Lines→OK  
Number of element division = 10  
Mesh Tools→ Mesh→ OK
- Step 4:PlotCtrls→ Numbering→ Node Numbers→ On→ OK
- Step 5:Pre-processor→ Material Properties→ Material Models→ Structural→ Linear→ Elastic→ Isotropic  
Elastic Modulus = 210 \* 10<sup>9</sup>  
Poissons Ratio = 0.3  
Density = 7800
- Step 6:Pre-processor→ Sections→ Beam→ Common Sections→  
Breadth = 20 \* 10<sup>-3</sup>  
Height = 1.5 \* 10<sup>-3</sup>  
Apply→ OK
- Step 7:Solutions→ Define Loads→ Apply→ Structural→ Displacement→ On Nodes→ Select Node 1→ OK  
DOFs to be constrained→ All DOF  
Value of Displacement = 0→ OK
- Step 8:Solution→ Analysis Type→ New Analysis→ Modal→ OK
- Step 9:Solution→ Analysis Type→ Analysis Options→ Mode Extraction Method→ Block Lanczos  
Number of Modes to extract = 10  
Number of Modes to expand = 10
- Step 10: Solution→ Solve→ Current LS→ OK
- Step 11: General Postproc→ Result Summary

### E. Modal Analysis with MATLAB

Program is as follows:

```

% Variable descriptions:
% nel = number of elements
% nnel = number of nodes per element
% ndof = number of dofs per node
% nnode = total number of nodes in system
% sdof = total system dofs
% el = elastic modulus
% b = breadth
% h = height
% xi = moment of inertia of cross-section
% rho = mass density
% tleng = total length of beam
% leng = length of each element
% area = cross-sectional area
% k = element stiffness matrix
% m = element mass matrix
    
```

```
% kk = system stiffness matrix
% mm = system mass matrix
% index = a vector containing system dofs
associated with each element
% bcdof = a vector containing dofs
associated with boundary conditions
% bcval = a vector containing boundary
condition values associated with
the dofs in 'bcdof'

clear
clc

format long;
nel=10;
nnel=2
ndof=2

nnode=(nnel-1)*nel+1;
sdof=nnode*ndof

el=210e9;
b=20e-3;
h=1.5e-3;
xi=(b*(h^3))/12
rho=7800;

tleng=1
leng=tleng/nel;
area=b*h;
bcdof(1)=1;
bcdof(2)=2;

kk=zeros(sdof, sdof);
mm=zeros(sdof, sdof);
index=zeros(nnel*ndof, 1);

for iel=1:nel
    index=feeldof1(iel,nnel,ndof);
    % extract system dofs associated with
    element
    [k,m]=febeam1(el,xi,leng,area,rho,1);
    % computes element stiffness & mass
    matrix
    kk=feasmbll(kk,k,index); % assemble
    element stiffness matrices into
    system matrix
    mm=feasmbll(mm,m,index); % assemble
    element mass matrices into system
    matrix
end

[kn,mn]=feaplycs(kk,mm,bcdof); % apply
the boundary conditions.
fsol=eig(kn,mn); % solve the matrix
equation and print
fsol=sqrt(fsol);
freqhz=fsol/(2*pi);
sort(freqhz)
```

Functions written in the above programme are feeldof1, febeam1, feasmbll, feaplycs.

- feeldof1 function is defined as follows:

```
function [index]=feeldof1(iel,nnel,ndof)
% Purpose: Compute system dofs associated
with each element in one dimensional
problem
% Variable Description:
% index - system dof vector associated
with element "iel"
% iel - element number whose system dofs
are to be determined
% nnel - number of nodes per element
% ndof - number of dofs per node

edof = nnel*ndof;
start = (iel-1)*(nnel-1)*ndof;

for
    i=1:edof
        index(i)=start+i;
    end
```

- febeam1 function is defined as follows:

```
function [k,m]=febeam1(el,xi,leng,area,rho,ipt)
% Variable Description:
% k - element stiffness matrix (size of
4x4)
% m - element mass matrix (size of 4x4)
% el - elastic modulus
% xi - second moment of inertia of cross-
section
% leng - element length

% area - area of beam cross-section
% rho - mass density (mass per unit
volume)
% ipt = 1: consistent mass matrix

% stiffness matrix
c=el*xi/(leng^3);

k=c*[(12)      (6*leng)  (-12)      (6*leng);
      (6*leng) (4*leng^2) (-6*leng) (2*leng^2);
      (-12)   (-6*leng) (12)      (-6*leng);
      (6*leng) (2*leng^2)          (-6*leng)
(4*leng^2)];

% consistent mass matrix
if ipt==1
mm=rho*area*leng/420;

m=mm*[(156)      (22*leng)  (54)      (-13*leng);
      (22*leng) (4*leng^2) (13*leng) (-3*leng^2);
      (54)      (13*leng) (156)      (-22*leng);
```





#### IV. CONCLUSION

Modal analysis of a Cantilever beam was done in both ANSYS-16 and MATLAB 2008 and the first three natural frequencies obtained in both the platforms are almost same. Hence the MATLAB code is validated.

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