Vibration And Buckling Of Circular Cylindrical Panels By Higher Order Shear Deformation Theory

Mr. Shrikant M. Harle, Dr. A. V. Asha

ABSTRACT:

The present study deals with the stability and vibration analysis of cross-ply cylindrical panels by a higher-order but simple shear deformation theory as suggested by Reddy and Liu. The theory is based on a displacement field in which the displacements of the middle surface are expanded as cubic functions of the thickness coordinate and the transverse displacement is assumed to be constant through the thickness. This displacement field leads to the parabolic distribution of the transverse shear stresses (and zero transverse normal strain) and therefore no shear correction factors are used. The analysis is also based on the assumption that the thickness to radius ratio of shell panel is small compared to unity and hence negligible. The eigenvalues, and hence, the frequency parameters and buckling parameter are calculated by using a standard computer program. To check the derivation and computer program, the frequencies for homogenous, isotropic circular cylindrical panels are calculated which compare very well with earlier available results.

Keywords: Stability, vibration, buckling, shell panels, higher order theory

1. INTRODUCTION:

An increasing number of structural designs, especially in the aerospace, automobiles and petrochemicals industries, are extensively utilizing fiber composite laminated plates and shell panels as structural elements. Structural components, including shell panels, are increasingly being fabricated as laminates, each lamina (or layer) consisting of parallel fibers (e.g. glass, boron, graphite) embedded in a matrix material (e.g. epoxy resin). The laminated orthotropic shell panel belongs to the composite shell panels category. It may be of an arbitrary number of bonded layers, each of which possesses different elastic properties, thickness, etc. The study of the static and dynamic behavior of such shell panels is very important in structures like pressure vessels, aircrafts, storage tanks and manned and unmanned spacecrafts and submersibles.
A higher order shear deformation theory of elastic shell was developed for shell laminated of orthotropic layers by Reddy and Liu [23]. The theory was a modification of the Sander’s theory and accounts for parabolic distribution of the transverse shear strains through the thickness of the shell and tangential stress-free boundary conditions on the boundary surfaces of the shell. The Navier-type exact solutions for bending and natural vibration were presented for cylindrical and spherical shell under simply supported boundary conditions.

A higher order shear deformation theory was used by Reddy and Phan [22] to determine the natural frequencies and buckling loads of elastic plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotary inertia. Exact solutions for simply supported plates were obtained and the results were compared with the exact solutions of three dimensional elastic theories, the first order shear deformation theory and the classical plate theory.

A mixed shear flexible finite element, with relaxed continuity, was developed for the geometrically linear and non-linear analysis of layered anisotropic plates by Putcha and Reddy [17]. The element formulation was based on the refined higher order theory which satisfied the zero transverse shear stress boundary conditions on the top and bottom faces of the plate and requires no shear correction coefficients. The mixed finite element developed consists of eleven degrees of freedom per node which include three displacements, two rotations and six moment resultants. The element was evaluated for its accuracy in the analysis of the stability and vibration of anisotropic rectangular plates with different lamination schemes and boundary conditions.

The present study is carried out to determine the natural frequency of vibration and stability analysis of laminated cross-ply cylindrical panels that are simply supported. The formulation has been done using higher order shear deformation theory as proposed by Reddy and Liu. The transverse displacement is assumed to be constant through the thickness. The thickness coordinate multiplied by the curvature is assumed to be small in comparison to unity and hence negligible.

The governing equations have been developed. These equations are then reduced to the equations of motion for cylindrical panel and the Navier solution has been obtained for cross-ply laminated composite cylindrical panels. The resulting equations are suitably nondimensionalised. The eigen value problem is to be solved to obtain the free vibration frequencies.
2. THEORY AND FORMULATION:

The shell panel under consideration is composed of a finite number of orthotropic layers of uniform thickness. Let ‘n’ denote the number of layers in the shell panel and \( h_k \) and \( h_{k+1} \) be the top and bottom \( \zeta \)-coordinates of the \( k^{th} \) lamina. The following displacement field is assumed:

\[
\begin{align*}
\ddot{u}(\alpha, \beta, \zeta, t) &= (1 + k_1 \zeta)u + \zeta \varphi_1 + \zeta^2 \psi_1 + \zeta^3 \theta_1, \\
\ddot{v}(\alpha, \beta, \zeta, t) &= (1 + k_2 \zeta)v + \zeta \varphi_2 + \zeta^2 \psi_2 + \zeta^3 \theta_2, \\
\ddot{w}(\alpha, \beta, \zeta, t) &= w
\end{align*}
\] (1)

Where \( t \) is the time, \((u, v, w)\) are the displacements along the \((\alpha, \beta, \zeta)\) coordinates, \((u, v, w)\) are the displacements of a point on the middle surface and \( \varphi_1 \) and \( \varphi_2 \) are the rotations at \( \zeta=0 \) of normal to the mid-surface with respect to \( \alpha \) and \( \beta \) axes, respectively. The displacement fields in equations are so chosen that the transverse shear strains will be quadratic functions of the thickness coordinate, \( \zeta \), and the transverse normal strain will be zero.

The functions \( \Psi_i \) and \( \theta_i \) are determined using the condition that the transverse shear stresses, \( \sigma_4 \) and \( \sigma_5 \) vanish on the top and bottom surfaces of the shell panel:

\[
\sigma_4 \left( \alpha, \beta, \pm \frac{h}{2}, t \right) = 0, \quad \sigma_5 \left( \alpha, \beta, \pm \frac{h}{2}, t \right) = 0
\] (2)

For shell panels laminated of orthotropic layers, the conditions are equivalent to the requirement that the corresponding strains be zero on these surfaces. The transverse shear strains of a shell panel with two principal radii of curvature are given by

\[
\varepsilon_4 = \frac{\partial u}{\partial \zeta} + \frac{1}{A_1} \frac{\partial w}{\partial \alpha} - k_1 \ddot{u}, \quad \varepsilon_5 = \frac{\partial v}{\partial \zeta} + \frac{1}{A_1} \frac{\partial w}{\partial \beta} - k_2 \ddot{v}
\] (3)

Substituting \( \ddot{u}, \ddot{v}, \ddot{w} \) in the above equations, and neglecting the term multiplied by \( k_1 \zeta \) and \( k_2 \zeta \) we have,

\[
\varepsilon_4 = \varphi_1 + 2\zeta \Psi_1 + 3\zeta^2 \theta_1 + \frac{1}{A_1} \frac{\partial w}{\partial \alpha}, \varepsilon_5 = \varphi_2 + 2\zeta \Psi_2 + 3\zeta^2 \theta_2 + \frac{1}{A_2} \frac{\partial w}{\partial \beta},
\] \[\varepsilon_4 \left( \alpha, \beta, \pm \frac{h}{2}, t \right) = 0, \quad \varepsilon_5 \left( \alpha, \beta, \pm \frac{h}{2}, t \right) = 0
\] (4)

Setting \( \Psi_1 \) and \( \Psi_2 \) to zero,

\[
\theta_1 = -\frac{4}{3h^2} (\varphi_1 + \frac{1}{A_1} \frac{\partial w}{\partial \alpha}), \quad \theta_2 = -\frac{4}{3h^2} (\varphi_2 + \frac{1}{A_2} \frac{\partial w}{\partial \alpha})
\] (5)

Substituting equations (5) into equations (4),

\[
\ddot{u} = (1 + k_1 \zeta)u + \zeta \varphi_1 + \zeta^3 \frac{4}{3h^2} \left(-\varphi_1 + \frac{1}{A_1} \frac{\partial w}{\partial \alpha}\right), \quad \ddot{v} = (1 + k_2 \zeta)v + \zeta \varphi_2 + \zeta^3 \frac{4}{3h^2} \left(-\varphi_2 - \frac{1}{A_2} \frac{\partial w}{\partial \beta}\right)
\] (6)
Substituting above equations into the strain-displacement relations referred to an orthogonal curvilinear coordinate system, we get

\[ \varepsilon_1 = \varepsilon_1^0 + \zeta K_1^0 + \zeta^3 k_1^2, \varepsilon_2 = \varepsilon_2^0 + \zeta^2 k_2^2, \varepsilon_4 = \varepsilon_4^0 + \varepsilon^2 k_4^1, \varepsilon_5 = \varepsilon_5^0 + \zeta^2 k_5^2, \varepsilon_6 = \varepsilon_6^0 + \zeta K_6^0 + \zeta^3 k_6^2 \]

(7)

Where,

\[ \varepsilon_1^0 = \frac{1}{A_1} \left[ \frac{\partial u}{\partial \alpha} + \frac{1}{A_2} \frac{\partial A_1}{\partial \beta} v + A_1 k_1 w \right], \quad \varepsilon_2^0 = \frac{1}{A_2} \left[ \frac{\partial v}{\partial \beta} + \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} u + A_2 k_2 w \right] \]

\[ K_1^0 = \frac{1}{A_1} \left[ k_1 \frac{\partial u}{\partial \alpha} + \frac{1}{A_2} \frac{\partial k_1}{\partial \alpha} v + \frac{1}{A_2} \frac{\partial A_1}{\partial \beta} \Phi_1 + \frac{1}{A_1} \frac{\partial A_1}{\partial \beta} \Phi_2 \right] \]

\[ K_2^0 = \frac{1}{A_2} \left[ k_2 \frac{\partial v}{\partial \beta} + \frac{1}{A_1} \frac{\partial k_2}{\partial \beta} v + \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} \Phi_1 + \frac{1}{A_2} \frac{\partial A_2}{\partial \alpha} \Phi_2 \right] \]

\[ K_1^2 = -\frac{4}{3h^2} \left( \frac{1}{A_1} \left[ \frac{\partial \Phi_1}{\partial \alpha} + \frac{1}{A_2} \frac{\partial A_1}{\partial \beta} \Phi_2 + \frac{1}{A_2} \frac{\partial w}{\partial \alpha} \right] + \frac{1}{A_2} \frac{\partial A_1}{\partial \beta} \frac{\partial w}{\partial \alpha} \right) \]

\[ K_2^2 = -\frac{4}{3h^2} \left( \frac{1}{A_2} \left[ \frac{\partial \Phi_2}{\partial \beta} + \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} \Phi_1 + \frac{1}{A_1} \frac{\partial w}{\partial \beta} \right] + \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} \frac{\partial w}{\partial \beta} \right) \]

\[ \varepsilon_4^0 = \Phi_2 + \frac{1}{A_2} \frac{\partial w}{\partial \beta}, \varepsilon_5^0 = \Phi_1 + \frac{1}{A_1} \frac{\partial w}{\partial \alpha}, \quad \kappa_4 = -\frac{4}{h^2} (\Phi_2 + \frac{1}{A_2} \frac{\partial w}{\partial \beta}), \quad \kappa_5 = -\frac{4}{h^2} (\Phi_1 + \frac{1}{A_1} \frac{\partial w}{\partial \alpha}) \]

\[ \varepsilon_6^0 = \left[ \frac{1}{A_1} \frac{\partial v}{\partial \alpha} + \frac{1}{A_2} \frac{\partial u}{\partial \beta} - \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \beta} w - \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha} v \right] \]

(8)

For generalized plane stress condition, the elastic moduli \(Q_i^j\) are related to the engineering constants as follows:

\[ Q_{11} = \frac{E_1}{1-v_{12} v_{21}}, \quad Q_{12} = \frac{E_1 v_{21}}{1-v_{12} v_{21}} = \frac{E_2 v_{12}}{1-v_{12} v_{21}}, \quad Q_{22} = \frac{E_2}{1-v_{12} v_{21}}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \]

\[ Q_{66} = G_{12}, \quad E_{1} = \frac{v_{12}}{v_{21}} \]

(9)

Following are the expressions for stress resultants and stress couples:

\[ N_i = A_i \varepsilon_i^0 + B_i K_i^0 + E_i K_i^2, \quad M_i = B_i \varepsilon_i^0 + D_i K_i^0 + F_i K_i^2, \quad P_i = E_i \varepsilon_i^0 + F_i K_i^0 + H_i K_i^2 \]

\[ Q_1 = A_i \varepsilon_i^0 + D_i K_i^1, \quad Q_2 = A_i \varepsilon_i^0 + D_i K_i^1, \quad K_1 = D_i \varepsilon_i^0 + F_i K_i^1, \quad K_2 = D_i \varepsilon_i^0 + F_i K_i^1 \]

(10)
where \( A_{ij} \), \( B_{ij} \), etc. are the laminate stiffnesses

\[
(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} Q_{ij}^k \left( 1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^6 \right) d\zeta
\]

For \( I, j = 1, 2, 4, 5, 6 \), \( h_k \) and \( h_{k+1} \) are the distances measured from the middle surface of the shell panel

Following are the equations of equilibrium obtained:

\[
\delta u = \frac{\partial (A_{2N_1})}{\partial \alpha} + \frac{\partial (A_{1N_6})}{\partial \beta} - N_2 \frac{\partial A_{2}}{\partial \alpha} + N_6 \frac{\partial A_{1}}{\partial \beta} + k_1 \left[ \frac{\partial (A_{2N_1})}{\partial \alpha} + \frac{M_6 \frac{\partial A_{1}}{\partial \alpha} - \frac{\partial (A_{1M_6})}{\partial \beta} \right] + q_1 A_{12} = \left[ \bar{T}_1 \dot{u} + \bar{T}_2 \dot{\phi}_1 - \bar{T}_3 \frac{1}{A_1} \frac{\partial \bar{w}}{\partial \beta} \right] A_{12}
\]

\[
\delta v = \frac{\partial (A_{2N_6})}{\partial \beta} + \frac{\partial (A_{1N_2})}{\partial \alpha} - N_1 \frac{\partial A_{1}}{\partial \beta} + N_6 \frac{\partial A_{2}}{\partial \alpha} + k_2 \left[ \frac{\partial (A_{2N_6})}{\partial \beta} - M_1 \frac{\partial A_{1}}{\partial \alpha} + M_2 \frac{\partial A_{1}}{\partial \beta} + \frac{\partial (A_{1M_6})}{\partial \alpha} \right] + q_2 A_{12} = \left[ \bar{T}_1 \dot{v} + \bar{T}_2 \dot{\phi}_2 - \bar{T}_3 \frac{1}{A_2} \frac{\partial \bar{w}}{\partial \beta} \right] A_{12}
\]

\[
\delta w = \frac{\partial (A_{1Q_1})}{\partial \alpha} + \frac{\partial (A_{2Q_2})}{\partial \beta} - A_{1} A_{2} k_{1} N_{1} - A_{1} A_{2} k_{2} N_{2} + \frac{4}{3 h^2} \left[ \frac{\partial}{\partial \alpha} \left( \frac{1}{A_1} \frac{\partial (A_{2P_1})}{\partial \alpha} \right) + \frac{\partial}{\partial \beta} \left( \frac{1}{A_2} \frac{\partial (A_{2P_1})}{\partial \beta} \right) \right] + \frac{\partial}{\partial \beta} \left( \frac{1}{A_2} \frac{\partial (A_{2P_2})}{\partial \alpha} \right) + \frac{\partial}{\partial \alpha} \left( \frac{1}{A_1} \frac{\partial (A_{2P_2})}{\partial \beta} \right) - \frac{4}{A_2} \left[ \frac{\partial (A_{2P_1})}{\partial \beta} + \frac{\partial (A_{1P_2})}{\partial \alpha} \right] \left( n_1 \frac{\partial \alpha}{\partial \alpha} + n_2 \frac{\partial \beta}{\partial \beta} \right) + q_n A_{12} = \left[ \bar{T}_1 \dot{w} A_1 A_2 + \left[ \bar{T}_3 \frac{1}{A_2} \frac{\partial \bar{w}}{\partial \beta} \right] A_{12} \right]
\]

\[
\delta \phi_1 = \frac{\partial (M_{1A_{1}})}{\partial \alpha} + \frac{\partial (M_{6A_{1}})}{\partial \beta} - M_2 \frac{\partial A_{2}}{\partial \alpha} + M_6 \frac{\partial A_{1}}{\partial \beta} + 4 \frac{3 h^2}{A_2} \left[ \frac{\partial (A_{2P_1})}{\partial \alpha} + \frac{\partial (A_{1P_2})}{\partial \beta} - P_2 \frac{\partial A_{2}}{\partial \alpha} + P_6 \frac{\partial A_{1}}{\partial \beta} \right] - \frac{4}{h^2} R_{1} A_{12} + \frac{4}{h^2} \bar{R}_{1} A_{12} = \left[ \bar{T}_2 \ddot{\phi}_1 - \bar{T}_4 \ddot{\phi}_1 - \bar{T}_5 \frac{1}{A_1} \frac{\partial \bar{w}}{\partial \beta} \right] A_{12}
\]

\[
\delta \phi_2 = \frac{\partial (M_{2A_{2}})}{\partial \alpha} + \frac{\partial (M_{6A_{2}})}{\partial \beta} - M_1 \frac{\partial A_{1}}{\partial \alpha} + M_6 \frac{\partial A_{2}}{\partial \beta} - 4 \frac{3 h^2}{A_2} \left[ \frac{\partial (A_{2P_1})}{\partial \alpha} + \frac{\partial (A_{1P_2})}{\partial \beta} - P_1 \frac{\partial A_{1}}{\partial \alpha} + P_6 \frac{\partial A_{2}}{\partial \beta} \right] - \frac{4}{h^2} R_{2} A_{12} + \frac{4}{h^2} \bar{R}_{2} A_{12} = \left[ \bar{T}_2 \ddot{\phi}_2 - \bar{T}_4 \ddot{\phi}_2 - \bar{T}_5 \frac{1}{A_2} \frac{\partial \bar{w}}{\partial \beta} \right] A_{12}
\]

\[
q_1, q_2, q_n \text{ can be defined as the transverse loads}
\]

The inertias \( \bar{T}_1 \) and \( \bar{T}_3 \) (\( i=1, 2, 3, 4 \)) are defined by the equations.

\[
\bar{T}_1 = I_1 + 2 k_2 l_2, \quad \bar{T}_3 = \frac{4}{3 h^2} I_4 + \frac{4 k_1}{3 h^2} I_5, \quad \bar{T}_4 = l_3 - \frac{8}{9 h^2} I_5 + \frac{16}{9 h^2} I_7, \quad \bar{T}_5 = \frac{4}{3 h^2} I_5 + \frac{16}{9 h^2} I_7
\]

\[
(I_1, I_2, I_3, I_4, I_5, I_7) = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} \rho \left( 1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^6 \right) d\zeta
\]

Where \( (k) \) is the density of the material of the \( k \)th layer.
Considering the line integrals while integrating by parts the displacement gradients in the Hamilton principle, the boundary conditions at an edge $\alpha=\text{constant}$, are obtained as follows:

\[
[(N_1+1\kappa M_1, u);[(N_6+1\kappa M_6), v]; [(Q_1+11 \kappa \frac{4}{3h^2} A_2 \frac{\partial P_6}{\partial \beta} - \frac{4}{h} R\bar{K}_1 + \frac{4}{3h^2} A_1 A_2 \left\{(\frac{\partial (A \varphi_1)}{\partial x} + \frac{\partial (A \varphi_6)}{\partial \beta} - P_2 \frac{\partial A_2}{\partial x} + \\right\} - \left(\bar{I}_3 \ddot{\varphi} + \bar{I}_5 \ddot{\varphi_1} - \frac{16}{9} h^2 \frac{1}{A_1} \frac{\partial w}{\partial x}\right), w]; [(M_1+11 \kappa \frac{4}{3h^2} P_1), \varphi_1]; [(M_6+11 \kappa \frac{4}{3h^2} P_6), \varphi_2]; [P_1, 1 \frac{\partial w}{\partial x}] (13)
\]

For the cylindrical shell panel configuration shown in the figure, the coordinates are given by $\alpha = x/R$, $\beta = \beta$, the Lame’s parameters $A_1 = A_2 = R$ and the principal curvatures $Pho (k_1) = 0$ and $Pho (k_2) = \frac{1}{R}$, where ‘R’ is the radius of the mid-surface of the cylindrical shell panel. Then the equations of motion in terms of the stress resultants and stress couples are obtained from equations.

The strain-displacement relations of equations reduce to

\[
\begin{align*}
\epsilon_1^0 &= \frac{1}{R} \frac{\partial u}{\partial \alpha}, \quad \epsilon_2^0 = \frac{1}{R} \left[\frac{\partial v}{\partial \beta} + w\right], \quad K_1^0 = \frac{1}{R} \frac{\partial \varphi_1}{\partial \alpha}, \quad K_2^0 = \frac{1}{R} \left[\frac{1}{R} \frac{\partial v}{\partial \beta} + \frac{\partial \varphi_2}{\partial \beta}\right] \\
K_2^1 &= \frac{-4}{3h^2} R \left[\frac{\partial \varphi_1}{\partial \alpha} + \frac{\partial^2 w}{\partial \alpha^2}\right], \quad K_2^2 = \frac{-4}{3h^2} R \left[\frac{\partial \varphi_2}{\partial \beta} + \frac{\partial^2 w}{\partial \beta^2}\right], \epsilon_4^0 = \varphi_2 + \frac{1}{R} \frac{\partial w}{\partial \beta}, \\
K_2^3 &= \frac{1}{R} \left[\frac{\partial \varphi_1}{\partial \alpha} + \frac{\partial \varphi_2}{\partial \beta} + \frac{\partial \varphi_1}{\partial \beta}\right], \quad K_2^4 = \frac{-4}{h^2} \left[\varphi_2 + \frac{1}{R} \frac{\partial w}{\partial \beta}\right], \quad K_2^5 = \frac{1}{h^2} \left[\varphi_1 + \frac{1}{R} \frac{\partial w}{\partial \beta}\right], \epsilon_6^0 = \frac{1}{R} \left[\frac{\partial v}{\partial \alpha} + W\right] \\
K_2^6 &= \frac{1}{R} \left[\frac{\partial \varphi_1}{\partial \alpha} + \frac{\partial \varphi_2}{\partial \beta} + \frac{\partial \varphi_1}{\partial \beta}\right], \quad K_2^7 = \frac{-4}{3h^2} R \left[\frac{\partial \varphi_2}{\partial \alpha} + \frac{2}{R} \frac{\partial^2 w}{\partial \alpha \beta} + \frac{\partial \varphi_1}{\partial \beta}\right] \\
(14)
\end{align*}
\]

For cylindrical shell panel configuration, the equations of equilibrium take the form

\[
\begin{align*}
\frac{\partial N_1}{\partial \alpha} + \frac{\partial N_6}{\partial \beta} + q_1 R &= \left[\bar{I}_1 \ddot{u} + \bar{I}_2 \ddot{\varphi}_1 - \bar{I}_3 \frac{1}{R} \frac{\partial \bar{w}}{\partial \alpha}\right] R \\
\frac{\partial N_6}{\partial \alpha} + \frac{\partial N_2}{\partial \beta} + \frac{1}{R} \frac{\partial M_1}{\partial \alpha} + \frac{1}{R} \frac{\partial M_2}{\partial \beta} + q_2 R &= \left[\bar{I}_1 \ddot{v} + \bar{I}_2 \ddot{\varphi}_2 - \bar{I}_3 \frac{1}{R} \frac{\partial \bar{w}}{\partial \beta}\right] R \\
\frac{\partial Q_1}{\partial \alpha} + \frac{\partial Q_2}{\partial \beta} + \left(\frac{\partial}{\partial \alpha}\left(\frac{1}{R} \frac{\partial N_1}{\partial \alpha}\right) \right) + \left(\frac{\partial}{\partial \beta}\left(\frac{1}{R} \frac{\partial N_2}{\partial \beta}\right) \right) - N_2 + \frac{4}{3h^2} R \left[\frac{\partial^2 P_1}{\partial \alpha^2} + \frac{\partial^2 P_2}{\partial \alpha \beta} + \frac{2}{R} \frac{\partial^2 P_6}{\partial \alpha \beta}\right] - \frac{4}{h^2} \left[\frac{\partial K_1}{\partial \alpha} + \frac{\partial K_2}{\partial \beta}\right] \\
q_{\alpha} R &= \left(\bar{I}_3 \ddot{u} + \bar{I}_5 \frac{\partial \varphi_1}{\partial \alpha} + \bar{I}_3 \frac{\partial \bar{v}}{\partial \beta} + \bar{I}_5 \frac{\partial \varphi_2}{\partial \beta} - \frac{16}{9} h^2 \frac{1}{R} \left(\frac{\partial^2 \bar{w}}{\partial \alpha^2} + \frac{\partial^2 \bar{w}}{\partial \beta^2}\right)\right) + I_1 \ddot{R} R \\
\frac{\partial M_1}{\partial \alpha} + \frac{\partial M_6}{\partial \beta} - \frac{4}{3h^2} \left(\frac{\partial P_1}{\partial \alpha} + \frac{\partial P_6}{\partial \beta}\right) - Q_1 R + \frac{4}{h^2} \bar{R} \bar{K}_1 &= \left[\bar{I}_2 \ddot{u} + \bar{I}_4 \ddot{\varphi}_1 - \bar{I}_5 \frac{1}{R} \frac{\partial \bar{w}}{\partial \alpha}\right] R
\end{align*}
\]
$$\frac{\partial M_6}{\partial \alpha} + \frac{\partial M_2}{\partial \beta} - \frac{4}{3h^2} \left( \frac{\partial P_6}{\partial \alpha} + \frac{\partial P_2}{\partial \beta} \right) - Q_2 R + \frac{4}{h^2} R \tilde{k}_2 = \left[ \bar{T}_2 \ddot{v} + \bar{T}_4 \ddot{\phi}_2 - \bar{I}_5 \frac{1}{R} \frac{\partial \bar{w}}{\partial \beta} \right] R \quad (15)$$

Since the solution for the equations of motion is done by using the Navier solution, therefore such a solution exists only for a specially orthotropic shell panel for which the following laminate stiffnesses are zero;

$$D_{16} = A_{16} = B_{16} = E_{16} = 0, \quad A_{45} = D_{45} = 0$$

The equations of motion in terms of the displacement hence reduce to

$$A_{11} \frac{\partial^2 u}{\partial \alpha^2} + A_{66} \frac{\partial^2 u}{\partial \beta^2} \left[ \left( A_{12} + A_{66} \right) + \frac{1}{R} \left( B_{12} + B_{66} \right) \right] \frac{\partial^2 v}{\partial \alpha \partial \beta} + \left[ A_{12} \frac{\partial w}{\partial \alpha} - \frac{4}{3h^2} h \frac{\partial^3 \varphi_1}{\partial \alpha^3} - \frac{4}{3h^2} \left( E_{12} + 2E_{66} \right) \frac{\partial^3 \varphi_2}{\partial \alpha \partial \beta} \right] \frac{\partial^2 w}{\partial \beta^2} + \left[ \left( A_{12} + B_{66} \right) - \frac{4}{3h^2} \left( E_{12} + 2E_{66} \right) \right] \frac{\partial^2 \varphi_2}{\partial \alpha \partial \beta} = R^2 \left[ \bar{T}_1 \ddot{u} + \bar{T}_2 \ddot{\phi}_2 - \bar{I}_3 \frac{\partial \bar{w}}{\partial \alpha} \right]$$
The boundary conditions for a simply supported cylindrical shell panel are given by

\[ N_1 = 0, \ v = 0, \ w = 0, \ M_1 = 0, \ \varphi_2 = 0 \quad \text{At} \ \alpha = 0 \quad \text{and} \quad \alpha = L/R \]

Following the Navier solution procedure, the following form which satisfies the boundary condition in equations (2.9.1) is assumed for the stability analysis

\[ u = U \cos \lambda_m \alpha \sin n \beta, \ v = V \sin \lambda_m \alpha \cos n \beta, \ w = W \sin \lambda_m \alpha \cos n \beta \]

\[ \varphi_1 = \tilde{\varphi}_1 \cos \lambda_m \alpha \sin n \beta, \ \varphi_2 = \tilde{\varphi}_2 \sin \lambda_m \alpha \cos n \beta \]

(17)
Following the Navier solution procedure, the following form which satisfies the boundary condition in above equations is assumed for the free vibration analysis

\[
\begin{align*}
u &= U \cos \lambda_m \alpha \sin n \beta e^{i \omega t}, \\
v &= V \sin \lambda_m \alpha \cos n \beta e^{i \omega t}, \\
\varphi_2 &= \bar{\varphi}_2 \sin \lambda_m \alpha \cos n \beta e^{i \omega t} \\
w &= W \sin \lambda_m \alpha \cos n \beta e^{i \omega t}, \\
\varphi_1 &= \bar{\varphi}_1 \cos \lambda_m \alpha \sin n \beta e^{i \omega t}
\end{align*}
\]

For convenience, the elements of the above matrices are suitably non-dimensionalised as follows:

\[
\begin{align*}
u &= \bar{U} h, \\
\bar{U} &= \bar{V} h, \\
\bar{V} &= \bar{W} h, \\
\bar{W} &= \bar{W} h, \\
\varphi_1 &= \bar{\varphi}_1, \\
\varphi_2 &= \bar{\varphi}_2, \\
A_{ij} &= \bar{A}_{ij} Q_2 h, \\
B_{ij} &= \bar{B}_{ij} Q_2 h^2, \\
D_{ij} &= \bar{D}_{ij} Q_2 h^3, \\
E_{ij} &= \bar{E}_{ij} Q_2 h^4, \\
F_{ij} &= \bar{F}_{ij} Q_2 h^5, \\
H_{ij} &= \bar{H}_{ij} Q_2 h^7, \\
I_1 &= I_1 \rho (1) h, \\
I_2 &= I_2 \rho (1) h^2, \\
I_3 &= I_3 \rho (1) h^3, \\
I_4 &= I_4 \rho (1) h^4, \\
I_5 &= I_5 \rho (1) h^5, \\
I_6 &= I_6 \rho (1) h^6, \\
I_7 &= I_7 \rho (1) h^7
\end{align*}
\]

\[ (I'_1, I'_2, I'_3, \ldots) \text{ are the non-dimensionalised quantities} \]

\[
\begin{align*}
I'_1 &= I'_1 \rho (1) h, \\
\bar{I}'_1 &= \left( I'_2 + 2 \frac{h}{R} I'_2 \right) \rho (1) h = I'_1 \rho (1) h, \\
\bar{I}'_2 &= \left( I'_2 + \frac{h}{R} I'_3 - \frac{4}{3} I'_4 - \frac{4}{3} \frac{h}{R} I'_5 \right) \rho (1) h^2 = \bar{I}'_2 \rho (1) h^2, \\
\bar{I}'_3 &= \left( \frac{4}{3} I'_4 + \frac{4}{3} \frac{h}{R} I'_5 \right) \rho (1) h^2 = \bar{I}'_3 \rho (1) h^2, \\
\bar{I}'_4 &= \left[ I'_4 - \frac{9}{3} I'_5 + \frac{16}{9} \frac{h}{R} I'_7 \right] \rho (1) h^3 = \bar{I}'_4 \rho (1) h^3 \\
\bar{I}'_5 &= \left[ \frac{4}{3} I'_5 - \frac{16}{9} I'_7 \right] \rho (1) h^3 = \bar{I}'_5 \rho (1) h^3
\end{align*}
\]

\[
\bar{I}'_5 = \left[ \frac{4}{3} I'_5 - \frac{16}{9} I'_7 \right] \rho (1) h^3 = \bar{I}'_5 \rho (1) h^3
\]

\[
\begin{align*}
3. \text{ NUMERICAL RESULTS}
\end{align*}
\]

In the case of vibration analysis, $\bar{w}^2$ is the eigenvalue for free vibration and $\bar{\mathbf{n}}_1$ for the buckling load and $\{X\}$ is the eigenvector. For convenience the above equation is non-dimensionalised. Then if the equation is premultiplied by $[\bar{M}]^{-1}$, one obtains the following standard eigenvalue problem,

\[
[\bar{H}] \{\bar{X}\} = \bar{\omega}^2 \{\bar{X}\} ,
\]

\[
[\bar{H}] \{\bar{X}\} = \bar{n}_1 \{\bar{X}\} \quad \bar{\omega}^2 = \frac{\omega^2 R^2 l_1}{A_{22}}, \\
\bar{n}_1 = \frac{n_1 R}{Q_2 h^2}, \\
[H] = [M]^{-1} [\bar{C}]
\]
In Table 1, a two-layer cross-ply cylindrical shell panel has been analysed. The frequency parameter is \( \Omega^2 = \bar{\omega}^2 \rho_0 L_s^2 / A_{11} \) where \( L_s = 2\pi R \). The results given by Dong and Tso’s theory and present theory are in very close agreement for all cases.

**Table 1: Comparison of frequency parameter of a two layer cross-ply cylindrical shell panel** (\( E_1/E_2 = 40; G_{12}/E_2 = 0.5; h/R = 0.01; G_{23}/E_2 = 0.6; \nu_{12} = 0.25; G_{13} = G_{12}; \rho = 1 \))

<table>
<thead>
<tr>
<th>L/R</th>
<th>n</th>
<th>Dong &amp; Tso</th>
<th>Present Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1</td>
<td>2.106</td>
<td>2.106</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.344</td>
<td>1.344</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.9587</td>
<td>0.9587</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.7493</td>
<td>0.7493</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.6419</td>
<td>0.6420</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.6130</td>
<td>0.6130</td>
</tr>
<tr>
<td>2.0</td>
<td>1</td>
<td>1.073</td>
<td>1.073</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6710</td>
<td>0.6710</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.4710</td>
<td>0.4710</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.3773</td>
<td>0.3773</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.3629</td>
<td>0.3630</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.4160</td>
<td>0.4162</td>
</tr>
<tr>
<td>5.0</td>
<td>1</td>
<td>0.4212</td>
<td>0.4212</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2470</td>
<td>0.2470</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.1733</td>
<td>0.1734</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1818</td>
<td>0.1819</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.2516</td>
<td>0.2517</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.3567</td>
<td>0.3569</td>
</tr>
</tbody>
</table>

The dimensionless natural frequencies \( \bar{\omega}^* = \bar{\omega} L_s^2 \sqrt{\rho_0 / E_2 h^3} \) for various values of R/h ratios for two-layered cylindrical panels having aspect ratio S/L = 1 are given in Table 2. In all cases, the results given by Dong and Tso’s theory and present theory are quite agreeable.

**Table 2 Comparison of frequency parameter of a two-layered circular cylindrical panel**

\( E_1/E_2 = 40.0, \nu_{12} = 0.25, G_{12}/E_2 = 1, G_{23}/E_2 = 0.6, G_{13} = G_{12} \)

<table>
<thead>
<tr>
<th>( \Phi ) (radians)</th>
<th>R/h</th>
<th>Dong &amp; Tso</th>
<th>Present Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>312.2</td>
<td>14.04</td>
<td>14.04</td>
</tr>
<tr>
<td>0.32</td>
<td>156.25</td>
<td>19.14</td>
<td>19.14</td>
</tr>
<tr>
<td>0.16</td>
<td>62.5</td>
<td>11.16</td>
<td>11.24</td>
</tr>
<tr>
<td>0.32</td>
<td>31.25</td>
<td>11.32</td>
<td>11.39</td>
</tr>
</tbody>
</table>
The variation of non-dimensional frequency with various parameters is now studied. The material properties of the first layer are given in Table 3.

**Table 3: Material Properties (first layer) of laminated cylindrical panels**

<table>
<thead>
<tr>
<th></th>
<th>$E_1$ (x 10^6 psi)</th>
<th>$E_2$ (x 10^6 psi)</th>
<th>$\nu_{12}$</th>
<th>$G_{12}$ (x 10^6 psi)</th>
<th>$G_{23}$ (x 10^6 psi)</th>
<th>$G_{13}$ (x 10^6 psi)</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.0</td>
<td>1.0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In Table 4, the values of non-dimensional frequencies for the various cross-sections for $m = 1$ and various included angles $\Phi$ are shown.

**Table 4: Non-dimensional frequency parameters of cross-ply circular cylindrical panels**

<table>
<thead>
<tr>
<th>$\Phi$ (radian)</th>
<th>Number of Layers</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5235</td>
<td>2</td>
<td>0.47</td>
<td>1.098</td>
<td>2.031</td>
<td>3.082</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.6203</td>
<td>1.08</td>
<td>1.75</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.6209</td>
<td>1.33</td>
<td>2.269</td>
<td>3.249</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.6011</td>
<td>1.1</td>
<td>1.822</td>
<td>2.591</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.6802</td>
<td>1.72</td>
<td>3.53</td>
<td>5.89</td>
</tr>
<tr>
<td>1.048</td>
<td>2</td>
<td>0.205</td>
<td>0.313</td>
<td>0.623</td>
<td>1.026</td>
</tr>
</tbody>
</table>
Table 6: Frequency parameters of cross-ply circular cylindrical panels for different material property

Material property 1: $E_1/E_2 = 25.0$, $\nu_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.2$, $G_{13} = G_{12}$

Material property 2: $E_1/E_2 = 40.0$, $\nu_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.6$, $G_{13} = G_{12}$

<table>
<thead>
<tr>
<th>$\phi$ (radian)</th>
<th>Number of Layers</th>
<th>Material property 1</th>
<th>Material property 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5235</td>
<td>2</td>
<td>0.473</td>
<td>0.431</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.657</td>
<td>0.619</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.604</td>
<td>0.582</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.6029</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.731</td>
<td>0.7045</td>
</tr>
</tbody>
</table>

Table 6: Frequency parameters of cross-ply circular cylindrical panels for different ratio of $L/h$

<table>
<thead>
<tr>
<th>$\phi$ (radian)</th>
<th>Number of Layers</th>
<th>$L/h = 20$</th>
<th>$L/h = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5235</td>
<td>2</td>
<td>0.268</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.390</td>
<td>0.169</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.357</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.363</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.418</td>
<td>0.186</td>
</tr>
</tbody>
</table>
Table 7: Lowest frequency parameters of various cross-ply circular cylindrical panels

(m =1, n =1)

<table>
<thead>
<tr>
<th>φ (radian)</th>
<th>Number of Layers</th>
<th>Frequency parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5235</td>
<td>2</td>
<td>0.473</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.657</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.604</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.6029</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.6802</td>
</tr>
<tr>
<td>1.048</td>
<td>2</td>
<td>0.205</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.247</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.226</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.2609</td>
</tr>
<tr>
<td>1.57</td>
<td>2</td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.185</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.169</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.212</td>
</tr>
<tr>
<td>2.094</td>
<td>2</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.195</td>
</tr>
<tr>
<td>2.618</td>
<td>2</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The frequency envelopes of a two-layer, three-layer, four layer, five layer and ten-layer cross-ply circular cylindrical panel are plotted for m =1 and for various shallowness angles φ for a thickness to radius ratio of 0.05 and are shown in figure 3.
Fig. 2 Frequency envelope for two layer cross-ply circular cylindrical panel

Fig. 3 Frequency envelope for three layer cross-ply circular cylindrical panel
Fig. 4 Frequency envelope for four layer cross-ply circular cylindrical panel

Fig. 5 Frequency envelope for five layer cross-ply circular cylindrical panel
The non-dimensionalised critical buckling loads for cross-ply laminated plates are presented in the form of table. The following table shows the comparison of results of present
analysis and Reddy [22] for the non-dimensionalised buckling parameter of two layer, three layer and four layer of simply supported plates. The geometrical and material properties used are $E_1/E_2 = 40; G_{12}/E_2 = 0.5; h/R = 0.01; G_{23}/E_2 = 0.6; \nu_{12} = 0.25; G_{13} = G_{12}; \rho = 1$

### Table 8: Comparison of buckling parameter of cross-ply laminated plates:

<table>
<thead>
<tr>
<th>a/h</th>
<th>Two Layers 0°/90°</th>
<th>Error in %</th>
<th>Three Layers 0°/90°/0°</th>
<th>Error in %</th>
<th>Four Layers 0°/90°/90°/0°</th>
<th>Error in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reddy</td>
<td>Present</td>
<td></td>
<td>Reddy</td>
<td>Present</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>12.569</td>
<td>12.494</td>
<td>-0.6</td>
<td>34.936</td>
<td>35.873</td>
<td>2.68</td>
</tr>
<tr>
<td>100</td>
<td>12.614</td>
<td>12.531</td>
<td>-0.66</td>
<td>35.602</td>
<td>35.959</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Error in Percentage = 100 * (Present Theory – Reddy Theory)/ Reddy theory

From the results presented in the above table, it is clear that results of present theory in good agreement with those obtained by Reddy theory.

The non-dimensional buckling loads for different R/h values for cylindrical panels of L/h = 500 are tabulated in Table 9. It is seen that the values of the buckling load increase as the r/h ratio decreases.

### Table 9: Non-dimensional Buckling load of cross-ply circular cylindrical panels for l/h = 100

<table>
<thead>
<tr>
<th>R/h</th>
<th>Two Layer 0°/90°</th>
<th>Three Layer 0°/90°/0°</th>
<th>Four Layer 0°/90°/90°/0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>21.508</td>
<td>44.13</td>
<td>50.55</td>
</tr>
<tr>
<td>500</td>
<td>25.53</td>
<td>59.15</td>
<td>61.09</td>
</tr>
<tr>
<td>400</td>
<td>31.89</td>
<td>73.94</td>
<td>76.34</td>
</tr>
<tr>
<td>200</td>
<td>34.81</td>
<td>147.84</td>
<td>152.49</td>
</tr>
</tbody>
</table>

In Table 10, the non-dimensional buckling loads for cylindrical panels with R/h = 500 and different material properties are shown. It is seen that as the $E_1/E_2$ ratio is increased, the buckling load increases.
Table 10: Non-dimensional buckling load of cylindrical panels for different material properties

Material property 1: \( \frac{E_1}{E_2} = 25.0, \nu_{12} = 0.25, \frac{G_{12}}{E_2} = 1, \frac{G_{23}}{E_2} = 0.2, G_{13} = G_{12} \)

Material property 2: \( \frac{E_1}{E_2} = 40.0, \nu_{12} = 0.25, \frac{G_{12}}{E_2} = 1, \frac{G_{23}}{E_2} = 0.6, G_{13} = G_{12} \)

<table>
<thead>
<tr>
<th>R/h</th>
<th>No. of Layers</th>
<th>Material Property1</th>
<th>Material Property2</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>2 ([0^\circ/90^\circ])</td>
<td>17.70</td>
<td>25.53</td>
</tr>
<tr>
<td></td>
<td>3 ([0^\circ/90^\circ/0^\circ])</td>
<td>38.016</td>
<td>59.15</td>
</tr>
<tr>
<td></td>
<td>4 ([0^\circ/90^\circ/90^\circ/0^\circ])</td>
<td>37.411</td>
<td>61.09</td>
</tr>
</tbody>
</table>

Table 11: Non-dimensional buckling load of circular cylindrical panels for different ratio of l/h

<table>
<thead>
<tr>
<th>l/h</th>
<th>Two layers ([0^\circ/90^\circ])</th>
<th>Three layers ([0^\circ/90^\circ/0^\circ])</th>
<th>Four Layers ([0^\circ/90^\circ/90^\circ/0^\circ])</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>25.53</td>
<td>59.15</td>
<td>61.09</td>
</tr>
<tr>
<td>200</td>
<td>44.35</td>
<td>150.59</td>
<td>176.57</td>
</tr>
</tbody>
</table>

In Table 11, the non-dimensional buckling loads have been calculated for different l/h ratios (varying thickness). It is seen that as the l/h ratio decreases, the buckling load decreases.

Table 12: Non-dimensional buckling load of circular cylindrical panel for different cross-section (l/h=100, R/h=500)

<table>
<thead>
<tr>
<th>l/h=100, R/h=500</th>
<th>Two layers</th>
<th>Three layers</th>
<th>Four layers</th>
<th>Five Layers</th>
<th>Ten Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25.53</td>
<td>59.15</td>
<td>61.09</td>
<td>56.66</td>
<td>66.45</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

As the number of layers increases, the non-dimensional frequency increases. As \( \Phi \) increases, the value of the non-dimensional frequency parameter decreases in all cases. As the degree of orthotropy increases, the value of the frequency parameter decreases. As the thickness decreases, the frequency parameter decreases owing to a decrease in the stiffness matrix. As the L/h ratio increases, the frequency parameter decreases. As \( \frac{E_1}{E_2} \) ratio increases the frequency parameter decreases. As the thickness decreases buckling parameter increases. As the number of layers increases the buckling parameter increases. As the R/h ratio decreases buckling parameter increases. As \( \frac{E_1}{E_2} \) ratio increases the buckling parameter...
As the l/h ratio increases the buckling parameter increases. The above conclusions establish that the present theory, though a simple one including the shear deformation effects, gives comparable results.

REFERENCES


