Vibration And Buckling Of Circular Cylindrical Panels By Higher Order Shear Deformation Theory

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ABSTRACT:

The present study deals with the stability and vibration analysis of cross-ply cylindrical panels by a higher-order but simple shear deformation theory as suggested by Reddy and Liu. The theory is based on a displacement field in which the displacements of the middle surface are expanded as cubic functions of the thickness coordinate and the transverse displacement is assumed to be constant through the thickness. This displacement field leads to the parabolic distribution of the transverse shear stresses (and zero transverse normal strain) and therefore no shear correction factors are used. The analysis is also based on the assumption that the thickness to radius ratio of shell panel is small compared to unity and hence negligible. The eigenvalues, and hence, the frequency parameters and buckling parameter are calculated by using a standard computer program. To check the derivation and computer program, the frequencies for homogenous, isotropic circular cylindrical panels are calculated which compare very well with earlier available results.

Keywords: Stability, vibration, buckling, shell panels, higher order theory

1. INTRODUCTION:

An increasing number of structural designs, especially in the aerospace, automobiles and petrochemicals industries, are extensively utilizing fiber composite laminated plates and shell panels as structural elements. Structural components, including shell panels, are increasingly being fabricated as laminates, each lamina (or layer) consisting of parallel fibers (e.g. glass, boron, graphite) embedded in a matrix material (e.g. epoxy resin). The laminated orthotropic shell panel belongs to the composite shell panels category. It may be of an arbitrary number of bonded layers, each of which possesses different elastic properties, thickness, etc. The study of the static and dynamic behavior of such shell panels is very important in structures like pressure vessels, aircrafts, storage tanks and manned and unmanned spacecrafts and submersibles.

A higher order shear deformation theory of elastic shell was developed for shell laminated of orthotropic layers by Reddy and Liu [23]. The theory was a modification of the Sander's theory and accounts for parabolic distribution of the transverse shear strains through the thickness of the shell and tangential stress-free boundary conditions on the boundary surfaces of the shell. The Navier-type exact solutions for bending and natural vibration were presented for cylindrical and spherical shell under simply supported boundary conditions.

A higher order shear deformation theory was used by Reddy and Phan [22] to determine the natural frequencies and buckling loads of elastic plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotary inertia. Exact solutions for simply supported plates were obtained and the results were compared with the exact solutions of three dimensional elastic theories, the first order shear deformation theory and the classical plate theory.

A mixed shear flexible finite element, with relaxed continuity, was developed for the geometrically linear and non-linear analysis of layered anisotropic plates by Putcha and Reddy [17]. The element formulation was based on the refined higher order theory which satisfied the zero transverse shear stress boundary conditions on the top and bottom faces of the plate and requires no shear correction coefficients. The mixed finite element developed consists of eleven degrees of freedom per node which include three displacements, two rotations and six moment resultants. The element was evaluated for its accuracy in the analysis of the stability and vibration of anisotropic rectangular plates with different lamination schemes and boundary conditions.

The present study is carried out to determine the natural frequency of vibration and stability analysis of laminated cross-ply cylindrical panels that are simply supported. The formulation has been done using higher order shear deformation theory as proposed by Reddy and Liu. The transverse displacement is assumed to be constant through the thickness. The thickness coordinate multiplied by the curvature is assumed to be small in comparison to unity and hence negligible.

The governing equations have been developed. These equations are then reduced to the equations of motion for cylindrical panel and the Navier solution has been obtained for cross-ply laminated composite cylindrical panels. The resulting equations are suitably nondimensionalised. The eigen value problem is to be solved to obtain the free vibration frequencies.

2. THEORY AND FORMULATION:

The shell panel under consideration is composed of a finite number of orthotropic layers of uniform thickness. Let 'n' denote the number of layers in the shell panel and h_k and h_{k+1} be the top and bottom ζ-coordinates of the k^{th} lamina. The following displacement field is assumed:

$$\bar{u}(\alpha,\beta,\zeta,t) = (1+k_1\zeta)u + \zeta\varphi_1 + \zeta^2\Psi_1 + \zeta^3\theta_1 \ ,$$

$$\bar{v}(\alpha,\beta,\zeta,t) = (1+k_2\zeta)v + \zeta\varphi_2 + \zeta^2\Psi_2 + \zeta^3\theta_2 \ , \qquad \bar{w}(\alpha,\beta,\zeta,t) = w \qquad (1)$$

Where t is the time, ($\bar{u}$, $\bar{v}$, $\bar{w}$) are the displacements along the (α, β, ζ) coordinates, (u, v, w) are the displacements of a point on the middle surface and φ_1 and φ_2 are the rotations at ζ=0 of normal to the mid-surface with respect to α and β axes, respectively. The displacement fields in equations are so chosen that the transverse shear strains will be quadratic functions of the thickness coordinate, ζ, and the transverse normal strain will be zero.

The functions Ψ_i and θ_i are determined using the condition that the transverse shear stresses, σ_4 and σ_5 vanish on the top and bottom surfaces of the shell panel:

$$\sigma_4\left(\alpha,\beta,\pm\frac{h}{2},t\right) = 0,\ \sigma_5\left(\alpha,\beta,\pm\frac{h}{2},t\right) = 0 \qquad (2)$$

For shell panels laminated of orthotropic layers, the conditions are equivalent to the requirement that the corresponding strains be zero on these surfaces. The transverse shear strains of a shell panel with two principal radii of curvature are given by

$$\varepsilon_4 = \frac{\partial\bar{u}}{\partial\zeta} + \frac{1}{A_1}\frac{\partial\bar{w}}{\partial\alpha} - k_1\bar{u},\quad \varepsilon_5 = \frac{\partial\bar{v}}{\partial\zeta} + \frac{1}{A_1}\frac{\partial\bar{w}}{\partial\beta} - k_2\bar{v} \qquad (3)$$

Substituting $\bar{u}$, $\bar{v}$, $\bar{w}$ in the above equations, and neglecting the term multiplied by $k_1\zeta$ and $k_2\zeta$ we have,

$$\varepsilon_4 = \varphi_1 + 2\zeta\Psi_1 + 3\zeta^2\theta_1 + \frac{1}{A_1}\frac{\partial w}{\partial\alpha},\ \varepsilon_5 = \varphi_2 + 2\zeta\Psi_2 + 3\zeta^2\theta_2 + \frac{1}{A_2}\frac{\partial w}{\partial\beta}, \qquad \varepsilon_4\left(\alpha,\beta,\pm\frac{h}{2},t\right) = 0,$$
$$\varepsilon_5\left(\alpha,\beta,\pm\frac{h}{2},t\right) = 0 \qquad (4)$$

Setting Ψ_1 and Ψ_2 to zero,

$$\theta_1 = \frac{-4}{3h^2}\left(\varphi_1 + \frac{1}{A_1}\frac{\partial w}{\partial\alpha}\right),\ \theta_2 = \frac{-4}{3h^2}\left(\varphi_2 + \frac{1}{A_2}\frac{\partial w}{\partial\alpha}\right) \qquad (5)$$

Substituting equations (5) into equations (4),

$$\bar{u} = (1+k_1\zeta)u + \zeta\varphi_1 + \zeta^3\frac{4}{3h^2}\left(-\varphi_1 - \frac{1}{A_1}\frac{\partial w}{\partial\alpha}\right)\ ,\bar{v} = (1+k_2\zeta)v + \zeta\varphi_2 + \zeta^3\frac{4}{3h^2}\left(-\varphi_2 - \frac{1}{A_2}\frac{\partial w}{\partial\beta}\right) \qquad (6)$$

Substituting above equations into the strain-displacement relations referred to an orthogonal curvilinear coordinate system, we get

$$\varepsilon_1 = \varepsilon_1^0 + \zeta K_1^0 + \zeta^3 k_1^2,\ \varepsilon_2 = \varepsilon_2^0 + \zeta K_2^0 + \zeta^3 k_2^2,\ \ \varepsilon_4 = \varepsilon_4^0 + \zeta^2 k_4^1,\ \ \varepsilon_5 = \varepsilon_5^0 + \zeta^2 k_5^1,$$
$$\varepsilon_6 = \varepsilon_6^0 + \zeta K_6^0 + \zeta^3 k_6^2 \qquad (7)$$

Where,

$$\varepsilon_1^0 = \frac{1}{A_1}\left[\frac{\partial u}{\partial \alpha} + \frac{1}{A_2}\frac{\partial A_1}{\partial \beta} v + A_1 k_1 w\right], \qquad \varepsilon_2^0 = \frac{1}{A_2}\left[\frac{\partial v}{\partial \beta} + \frac{1}{A_1}\frac{\partial A_2}{\partial \alpha} u + A_2 k_2 w\right]$$

$$K_1^0 = \frac{1}{A_1}\left[k_1\frac{\partial u}{\partial \alpha} + \frac{\partial k_1}{\partial \alpha} u + \frac{1}{A_2}\frac{\partial A_1}{\partial \beta} k_2 v + \frac{\partial \varphi_1}{\partial \alpha} + \frac{1}{A_2}\frac{\partial A_1}{\partial \beta}\varphi_2\right]$$

$$K_2^0 = \frac{1}{A_2}\left[k_2\frac{\partial v}{\partial \beta} + \frac{\partial k_2}{\partial \beta} v + \frac{1}{A_1}\frac{\partial A_2}{\partial \alpha} k_1 u + \frac{\partial \varphi_2}{\partial \beta} + \frac{1}{A_2}\frac{\partial A_2}{\partial \alpha}\varphi_1\right]$$

$$K_1^2 = \frac{-4}{3h^2}\left\{\frac{1}{A_1}\left[\frac{\partial \varphi_1}{\partial \alpha} + \frac{1}{A_2}\frac{\partial A_1}{\partial \beta}\varphi_2 + \frac{\partial}{\partial \alpha}\left(\frac{1}{A_1}\frac{\partial w}{\partial \alpha}\right) + \frac{1}{A_2^2}\frac{\partial A_1}{\partial \beta}\frac{\partial w}{\partial \beta}\right]\right\}$$

$$K_2^2 = \frac{-4}{3h^2}\left\{\frac{1}{A_2}\left[\frac{\partial \varphi_2}{\partial \beta} + \frac{1}{A_1}\frac{\partial A_2}{\partial \alpha}\varphi_1 + \frac{\partial}{\partial \beta}\left(\frac{1}{A_2}\frac{\partial w}{\partial \beta}\right) + \frac{1}{A_1^2}\frac{\partial A_2}{\partial \alpha}\frac{\partial w}{\partial \alpha}\right]\right\}$$

$$\varepsilon_4^0 = \varphi_2 + \frac{1}{A_2}\frac{\partial w}{\partial \beta},\ \varepsilon_5^0 = \varphi_1 + \frac{1}{A_1}\frac{\partial w}{\partial \alpha},\ \ k_4^1 = \frac{-4}{h^2}\left(\varphi_2 + \frac{1}{A_2}\frac{\partial w}{\partial \beta}\right),\ \ k_5^1 = \frac{-4}{h^2}\left(\varphi_1 + \frac{1}{A_1}\frac{\partial w}{\partial \alpha}\right)$$

$$\varepsilon_6^0 = \left[\frac{1}{A_1}\frac{\partial v}{\partial \alpha} + \frac{1}{A_2}\frac{\partial u}{\partial \beta} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \beta} w - \frac{1}{A_2 A_1}\frac{\partial A_2}{\partial \alpha} v\right]$$

$$K_6^0 = \left[\frac{1}{A_1}\left\{k_2\frac{\partial v}{\partial \alpha} + \frac{\partial k_2}{\partial \alpha} v - \frac{1}{A_2}\frac{\partial A_1}{\partial \beta} k_1 u + \frac{\partial \varphi_2}{\partial \alpha} - \frac{1}{A_2}\frac{\partial A_1}{\partial \beta}\varphi_1\right\} + \frac{1}{A_2}\left\{k_1\frac{\partial u}{\partial \beta} + \frac{\partial k_1}{\partial \beta} u - \frac{1}{A_1}\frac{\partial A_2}{\partial \alpha} k_2 v + \frac{\partial \varphi_1}{\partial \beta} - \frac{1}{A_1}\frac{\partial A_2}{\partial \alpha}\varphi_2\right\}\right] \qquad (8)$$

For generalized plane stress condition, the elastic moduli $\left(Q_{ij}^k\right)$ are related to the engineering constants as follows:

$$Q_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}}\ ,\ Q_{12} = \frac{E_1.\upsilon_{21}}{1-\upsilon_{12}\upsilon_{21}} = \frac{E_2.\upsilon_{12}}{1-\upsilon_{12}\upsilon_{21}}\ ,\ Q_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}},\ Q_{44} = G_{23},\ Q_{55} = G_{13}$$
$$Q_{66} = G_{12}, \frac{E_1}{E_2} = \frac{\upsilon_{12}}{\upsilon_{21}} \qquad (9)$$

Following are the expressions for stress resultants and stress couples:

$$N_i = A_{ij}\varepsilon_j^0 + B_{ij}K_j^0 + E_{ij}K_j^2,\quad M_i = B_{ij}\varepsilon_j^0 + D_{ij}K_j^0 + F_{ij}K_j^2\ ,\quad P_i = E_{ij}\varepsilon_j^0 + F_{ij}K_j^0 + H_{ij}K_j^2$$

$$Q_1 = A_{4j}\varepsilon_j^0 + D_{4j}K_j^1,\ Q_2 = A_{5j}\varepsilon_j^0 + D_{5j}K_j^1,\ \overline{K}_1 = D_{4j}\varepsilon_j^0 + F_{4j}K_j^1,\ \overline{K}_2 = D_{5j}\varepsilon_j^0 + F_{5j}K_j^1 \qquad (10)$$

where A_{ij} , B_{ij}, etc. are the laminate stiffnesses

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right) = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} Q_{ij}^{k} \left(1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^6\right) d\zeta$$

For I, j = 1, 2, 4, 5, 6, h_k and h_{k+1} are the distances measured from the middle surface of the shell panel

Following are the equations of equilibrium obtained:

$$\delta u: \frac{\partial(A_2N_1)}{\partial\alpha} + \frac{\partial(A_1N_6)}{\partial\beta} - N_2\frac{\partial A_2}{\partial\alpha} + N_6\frac{\partial A_1}{\partial\beta} + k_1\left[\frac{\partial(A_2M_1)}{\partial\alpha} + M_6\frac{\partial A_1}{\partial\beta} - M_2\frac{\partial A_2}{\partial\alpha} + \frac{\partial(A_1M_6)}{\partial\beta}\right] + q_1A_1A_2 = \left[\bar{I}_1\ddot{u} + \bar{I}_2\ddot{\phi}_1 - \bar{I}_3\frac{1}{A_1}\frac{\partial\ddot{w}}{\partial\alpha}\right]A_1A_2$$

$$\delta v = \frac{\partial(A_2N_6)}{\partial\alpha} + \frac{\partial(A_1N_2)}{\partial\beta} - N_1\frac{\partial A_1}{\partial\beta} + N_6\frac{\partial A_2}{\partial\alpha} + k_2\left[\frac{\partial(A_2M_6)}{\partial\alpha} - M_1\frac{\partial A_1}{\partial\beta} + M_2\frac{\partial A_1}{\partial\beta} + \frac{\partial(A_2M_6)}{\partial\alpha}\right] + q_2A_1A_2 = \left[\bar{I}'_1\ddot{v} + \bar{I}'_2\ddot{\phi}_2 - \bar{I}'_3\frac{1}{A_2}\frac{\partial\ddot{w}}{\partial\beta}\right]A_1A_2$$

$$\delta w = \frac{\partial(A_2Q_1)}{\partial\alpha} + \frac{\partial(A_1Q_2)}{\partial\beta} - A_1A_2k_1N_1 - A_1A_2k_2N_2 + \frac{4}{3h^2}\left[\frac{\partial}{\partial\alpha}\left(\frac{1}{A_1}\frac{\partial(A_2P_1)}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{1}{A_2}\frac{\partial(A_1P_2)}{\partial\beta}\right) + \frac{\partial}{\partial\beta}\left(\frac{1}{A_2}\frac{\partial(A_2P_6)}{\partial\alpha}\right) + \frac{\partial}{\partial\alpha}\left(\frac{1}{A_1}\frac{\partial(A_1P_6)}{\partial\beta}\right) - \frac{\partial}{\partial\beta}\left(\frac{P_1}{A_2}\frac{\partial A_1}{\partial\beta}\right) - \frac{\partial}{\partial\alpha}\left(\frac{P_2}{A_1}\frac{\partial A_2}{\partial\alpha}\right) + \frac{\partial}{\partial\alpha}\left(\frac{P_6}{A_1}\frac{\partial A_1}{\partial\beta}\right) + \frac{\partial}{\partial\beta}\left(\frac{P_6}{A_2}\frac{\partial A_2}{\partial\alpha}\right)\right] - \frac{4}{h^2}\left[\frac{\partial(A_2\bar{K}_1)}{\partial\alpha} + \frac{\partial(A_1\bar{K}_2)}{\partial\beta}\right] + \left(\frac{\partial}{\partial\alpha}\right)\left(n_1\frac{\partial w}{\partial\alpha}\right) + \left(\frac{\partial}{\partial\beta}\right)\left(n_2\frac{\partial w}{\partial\beta}\right) + q_nA_1A_2 = I_1\ddot{w}A_1A_2 + \left\{\bar{I}_3\frac{\partial(A_2\ddot{u})}{\partial\alpha} + \bar{I}_5\frac{\partial(A_2\ddot{\varphi}_1)}{\partial\alpha} + \bar{I}_3{}'\frac{\partial(A_1\ddot{v})}{\partial\beta} + \bar{I}_5\frac{\partial(A_1\ddot{\varphi}_2)}{\partial\beta} - \frac{16}{9}\frac{I_7}{h^4}\left[\frac{\partial}{\partial\alpha}\left(\frac{A_2}{A_1}\frac{\partial\ddot{w}}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A_1}{A_2}\frac{\partial\ddot{w}}{\partial\beta}\right)\right]\right\}$$

$$\delta\varphi_1 = \frac{\partial(M_1A_2)}{\partial\alpha} + \frac{\partial(M_6A_1)}{\partial\beta} - M_2\frac{\partial A_2}{\partial\alpha} + M_6\frac{\partial A_1}{\partial\beta} - \frac{4}{3h^2}\left[\frac{\partial(A_2P_1)}{\partial\alpha} + \frac{\partial(A_1P_6)}{\partial\beta} - P_2\frac{\partial A_2}{\partial\alpha} + P_6\frac{\partial A_1}{\partial\beta}\right] - Q_1A_1A_2 + \frac{4}{h^2}\bar{K}_1A_1A_2 = \left[\bar{I}_2\ddot{u} + \bar{I}_4\ddot{\phi}_1 - \bar{I}_5\frac{1}{A_1}\frac{\partial\ddot{w}}{\partial\alpha}\right]A_1A_2$$

$$\delta\varphi_2 = \frac{\partial(M_6A_2)}{\partial\alpha} + \frac{\partial(M_2A_1)}{\partial\beta} - M_1\frac{\partial A_1}{\partial\beta} + M_6\frac{\partial A_2}{\partial\alpha} - \frac{4}{3h^2}\left[\frac{\partial(A_2P_6)}{\partial\alpha} + \frac{\partial(A_1P_2)}{\partial\beta} - P_1\frac{\partial A_1}{\partial\beta} + P_6\frac{\partial A_2}{\partial\alpha}\right] - Q_2A_1A_2 + \frac{4}{h^2}\bar{K}_2A_1A_2 = \left[\bar{I}'_2\ddot{v} + \bar{I}_4\ddot{\phi}_2 - \bar{I}_5\frac{1}{A_2}\frac{\partial\ddot{w}}{\partial\beta}\right]A_1A_2 \quad (11)$$

q_1, q_2, q_n can be defined as the transverse loads

The inertias $\bar{I}_1$ and $\bar{I}'_1$ (i=1, 2, 3, 4, 5) are defined by the equations,

$$\bar{I}_1 = I_1 + 2k_1I_2,\ \ \bar{I}'_1 = I_1 + 2k_2I_2, \bar{I}_2 = I_2 + k_1I_3 - \frac{4}{3h^2}I_4 - \frac{4k_1}{3h^2}I_5,\ I_2 + k_2I_3 - \frac{4}{3h^2}I_4 - \frac{4k_2}{3h^2}I_5$$

$$\bar{I}_3 = \frac{4}{3h^2}I_4 + \frac{4k_1}{3h^2}I_5, \bar{I}_3{}' = \frac{4}{3h^2}I_4 + \frac{4k_1}{3h^2}I_5,\ \ \bar{I}_4 = I_3 - \frac{8}{3h^2}I_5 + \frac{16}{9h^2}I_7,\ \ \bar{I}_5 = \frac{4}{3h^2}I_5 + \frac{16}{9h^4}I_7 \quad (12)$$

Where $\quad (I_1, I_2, I_3, I_4, I_5, I_7) = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} \rho^k \left(1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^6\right) d\zeta$

Where (k) is the density of the material of the k[th] layer.

Considering the line integrals while integrating by parts the displacement gradients in the Hamilton principle, the boundary conditions at an edge α=constant, are obtained as follows:

$$[(N_1+k_1M_1),u];[(N_6+k_2M_6),v];[\{Q_1+\frac{4}{3h^2}\frac{1}{A_2}\frac{\partial P_6}{\partial\beta}-\frac{4}{h^2}\bar{K}_1+\frac{4}{3h^2}\frac{1}{A_1A_2}\left\{\frac{\partial(A_2P_1)}{\partial\alpha}+\frac{\partial(A_1P_6)}{\partial\beta}-P_2\frac{\partial A_2}{\partial\alpha}+P_6\frac{\partial A_1}{\partial\beta}\right\}-\left(\bar{I}_3\ddot{u}+\bar{I}_5\ddot{\phi}_1-\frac{16}{9}\frac{I_7}{h^2}\frac{1}{A_1}\frac{\partial w}{\partial\alpha}\right)\},w];\ [(M_1-\frac{4}{3h^2}P_1),\varphi_1];\ [(M_6-\frac{4}{3h^2}P_6),\varphi_2];\ [P_1,\frac{1}{A_1}\frac{\partial w}{\partial\alpha}] \quad (13)$$

For the cylindrical shell panel configuration shown in the figure, the coordinates are given by α = x/R, β = β, the Lame's parameters $A_1 = A_2 = R$ and the principal curvatures $Pho\ (k_1) = 0\ and\ Pho\ (k_2) = \frac{1}{R}$, where 'R' is the radius of the mid-surface of the cylindrical shell panel. Then the equations of motion in terms of the stress resultants and stress couples are obtained from equations.

The strain-displacement relations of equations reduce to

$$\varepsilon_1^0=\frac{1}{R}\frac{\partial u}{\partial\alpha},\qquad \varepsilon_2^0=\frac{1}{R}\left[\frac{\partial v}{\partial\beta}+w\right],\ K_1^0=\frac{1}{R}\frac{\partial\varphi_1}{\partial\alpha},\qquad K_2^0=\frac{1}{R}\left[\frac{1}{R}\frac{\partial v}{\partial\beta}+\frac{\partial\varphi_2}{\partial\beta}\right]$$

$$K_1^2=\frac{-4}{3h^2}\frac{1}{R}\left[\frac{1}{R}\frac{\partial\varphi_1}{\partial\alpha}+\frac{1}{R}\frac{\partial^2 w}{\partial\alpha^2}\right],\qquad K_2^2=\frac{-4}{3h^2}\frac{1}{R}\left[\frac{1}{R}\frac{\partial\varphi_2}{\partial\beta}+\frac{1}{R}\frac{\partial^2 w}{\partial\beta^2}\right],\varepsilon_4^0=\varphi_2+\frac{1}{R}\frac{\partial w}{\partial\beta},$$

$$\varepsilon_5^0=\varphi_1+\frac{1}{R}\frac{\partial w}{\partial\alpha},\qquad K_4^1=\frac{-4}{h^2}\left[\varphi_2+\frac{1}{R}\frac{\partial w}{\partial\beta}\right],\qquad K_5^1=\frac{-4}{h^2}\left[\varphi_1+\frac{1}{R}\frac{\partial w}{\partial\alpha}\right],\varepsilon_6^0=\frac{1}{R}\left[\frac{\partial v}{\partial\alpha}+\frac{\partial u}{\partial\beta}\right]$$

$$K_6^0=\frac{1}{R}\left[\frac{1}{R}\frac{\partial v}{\partial\beta}+\frac{\partial\varphi_2}{\partial\alpha}+\frac{\partial\varphi_1}{\partial\beta}\right],\ K_6^2=\frac{-4}{3h^2}\frac{1}{R}\left[\frac{\partial\varphi_2}{\partial\alpha}+\frac{2}{R}\frac{\partial^2 w}{\partial\alpha\partial\beta}+\frac{\partial\varphi_1}{\partial\beta}\right] \quad (14)$$

For cylindrical shell panel configuration, the equations of equilibrium take the form

$$\frac{\partial N_1}{\partial\alpha}+\frac{\partial N_6}{\partial\beta}+q_1R=\left[\bar{I}_1\ddot{u}+\bar{I}_2\ddot{\phi}_1-\bar{I}_3\frac{1}{R}\frac{\partial\ddot{w}}{\partial\alpha}\right]R$$

$$\frac{\partial N_6}{\partial\alpha}+\frac{\partial N_2}{\partial\beta}+\frac{1}{R}\frac{\partial M_6}{\partial\alpha}+\frac{1}{R}\frac{\partial M_2}{\partial\beta}+q_2R=\left[\bar{I}'_1\ddot{v}+\bar{I}'_2\ddot{\phi}_2-\bar{I}'_3\frac{1}{R}\frac{\partial\ddot{w}}{\partial\beta}\right]R$$

$$\frac{\partial Q_1}{\partial\alpha}+\frac{\partial Q_2}{\partial\beta}+\left(\frac{\partial}{\partial\alpha}\right)\left(n_1\frac{\partial w}{\partial\alpha}\right)+\left(\frac{\partial}{\partial\beta}\right)\left(n_2\frac{\partial w}{\partial\beta}\right)-N_2+\frac{4}{3h^2}\frac{1}{R}\left[\frac{\partial^2 P_1}{\partial\alpha^2}+\frac{\partial^2 P_2}{\partial\beta^2}+\frac{2\partial^2 P_6}{\partial\alpha\partial\beta}\right]-\frac{4}{h^2}\left[\frac{\partial\bar{K}_1}{\partial\alpha}+\frac{\partial\bar{K}_2}{\partial\beta}\right]+q_nR=\left(\bar{I}_3\frac{\partial\ddot{u}}{\partial\alpha}+\bar{I}_5\frac{\partial\ddot{\varphi}_1}{\partial\alpha}+\bar{I}'_3\frac{\partial\ddot{v}}{\partial\beta}+\bar{I}_5\frac{\partial\ddot{\varphi}_2}{\partial\beta}-\frac{16}{9}\frac{I_7}{h^4}\frac{1}{R}\left(\frac{\partial^2\ddot{w}}{\partial\alpha^2}+\frac{\partial^2\ddot{w}}{\partial\beta^2}\right)\right)+I_1\ddot{w}R$$

$$\frac{\partial M_1}{\partial\alpha}+\frac{\partial M_6}{\partial\beta}-\frac{4}{3h^2}\left(\frac{\partial P_1}{\partial\alpha}+\frac{\partial P_6}{\partial\beta}\right)-Q_1R+\frac{4}{h^2}R\bar{k}_1=\left[\bar{I}_2\ddot{u}+\bar{I}_4\ddot{\phi}_1-\bar{I}_5\frac{1}{R}\frac{\partial\ddot{w}}{\partial\alpha}\right]R$$

$$\frac{\partial M_6}{\partial \alpha}+\frac{\partial M_2}{\partial \beta}-\frac{4}{3h^2}\left(\frac{\partial P_6}{\partial \alpha}+\frac{\partial P_2}{\partial \beta}\right)-Q_2R+\frac{4}{h^2}R\bar{k}_2=\left[\bar{I}'_2\ddot{v}+\bar{I}_4\ddot{\phi}_2-\bar{I}_5\frac{1}{R}\frac{\partial \ddot{w}}{\partial \beta}\right]R \qquad (15)$$

Since the solution for the equations of motion is done by using the Navier solution, therefore such a solution exists only for a specially orthotropic shell panel for which the following laminate stiffnesses are zero;

$$D_{i6}=A_{i6}=B_{i6}=E_{i6}=0,\quad A_{45}=D_{45}=0$$

The equations of motion in terms of the displacement hence reduce to

$$A_{11}\frac{\partial^2 u}{\partial \alpha^2}+A_{66}\frac{\partial^2 u}{\partial \beta^2}\left[\left\{(A_{12}+A_{66})+\frac{1}{R}(B_{12}+B_{66})\right\}\frac{\partial^2 v}{\partial \alpha\, \partial \beta}\right]+\left[A_{12}\frac{\partial w}{\partial \alpha}-\frac{4}{3}\frac{E_{11}}{Rh^2}\frac{\partial^3 w}{\partial \alpha^3}-\frac{4}{3}\frac{}{Rh^2}(E_{12}+2E_{66})\frac{\partial^3 w}{\partial \alpha\, \partial \beta^2}\right]+\left[\left(B_{11}-\frac{4}{3}\frac{E_{11}}{h^2}\right)\frac{\partial^2 \varphi_1}{\partial \alpha^2}+\left(B_{66}-\frac{4}{3}\frac{E_{66}}{h^2}\right)\frac{\partial^2 \varphi_1}{\partial \beta^2}\right]+\left[\left\{(B_{12}+B_{66})-\frac{4}{3}\frac{}{h^2}(E_{12}+E_{66})\right\}\frac{\partial^2 \varphi_2}{\partial \alpha\, \partial \beta}\right]\quad =R^2\left[\bar{I}_1\ddot{u}+\bar{I}_2\ddot{\phi}_1-\frac{\bar{I}_3}{\mathrm{R}}\frac{\partial w}{\partial \alpha}\right]$$

$$\left[\left\{\frac{1}{R}(A_{12}+A_{66})+\frac{1}{R^2}(B_{12}+B_{66})\right\}\frac{\partial^2 u}{\partial \alpha\, \partial \beta}\right]+\left[\frac{1}{R}\left(A_{66}+\frac{2B_{66}}{R}+\frac{D_{66}}{R^2}\right)\frac{\partial^2 v}{\partial \alpha^2}+\frac{1}{R}\left(A_{22}+\frac{2B_{22}}{R}+\frac{D_{22}}{R^2}\right)\frac{\partial^2 v}{\partial \beta^2}\right]+\left[\frac{1}{R}\left(A_{22}+\frac{B_{22}}{R}\right)\frac{\partial w}{\partial \beta}\right]-\frac{4}{3h^2R^2}\left(2E_{66}+E_{12}+\frac{2F_{66}}{R}+\frac{F_{12}}{R}\right)\frac{\partial^3 w}{\partial \alpha^2\, \partial \beta}-\frac{4}{3h^2R^2}\left(E_{22}+\frac{F_{22}}{R}\right)\frac{\partial^3 w}{\partial \beta^3}+\left[\left\{\frac{1}{R}\left(B_{66}+B_{12}+\frac{D_{66}}{R}+\frac{D_{12}}{R}\right)-\frac{4}{3h^2R}\left(E_{66}+E_{12}+\frac{F_{66}}{R}+\frac{F_{12}}{R}\right)\right\}\frac{\partial^2 \varphi_1}{\partial \alpha\, \partial \beta}\right]+\left[\left\{\frac{1}{R}\left(B_{66}+\frac{D_{66}}{R}\right)-\frac{4}{3h^2R}\left(E_{66}+\frac{F_{66}}{R}\right)\right\}\frac{\partial^2 \varphi_2}{\partial \alpha^2}+\left\{\frac{1}{R}\left(B_{22}+\frac{D_{22}}{R}\right)-\frac{4}{3h^2R}\left(E_{22}+\frac{F_{22}}{R}\right)\right\}\frac{\partial^2 \varphi_2}{\partial \beta^2}\right]=R^2\left[\bar{I}'_1\ddot{v}-\frac{\bar{I}_3{}'}{R}\frac{\partial \ddot{w}}{\partial \beta}+\bar{I}'_2\ddot{\phi}_2\right]$$

$$\left[-\frac{A_{21}}{R}\frac{\partial u}{\partial \alpha}+\frac{4}{3h^2R^2}\left\{E_{11}\frac{\partial^3 u}{\partial \alpha^3}+(2E_{66}+E_{12})\frac{\partial^3 u}{\partial \alpha\, \partial \beta^2}\right\}\right]+\left[-\frac{1}{R}\left(A_{22}+\frac{B_{22}}{R}\right)\frac{\partial v}{\partial \beta}+\frac{4}{3h^2R^2}\left\{\left(E_{22}+\frac{F_{22}}{R}\right)\frac{\partial^3 v}{\partial \beta^3}+\left(2E_{66}+E_{12}+\frac{F_{12}}{R}+\frac{2F_{66}}{R}\right)\frac{\partial^3 v}{\partial \alpha^2\, \partial \beta}\right\}\right]+\left[-\frac{A_{22}w}{R}+\left(\frac{A_{55}}{R}-\frac{8}{h^2}\frac{D_{55}}{R}+\frac{8}{3h^2}\frac{E_{12}}{R^2}+\frac{16}{Rh^4}F_{55}\right)\frac{\partial^2 w}{\partial \alpha^2}+\left(\frac{A_{44}}{R}-\frac{8}{h^2}\frac{D_{44}}{R}+\frac{8}{3h^2}\frac{E_{22}}{R^2}+\frac{16}{Rh^4}\frac{D_{44}}{R}\right)\frac{\partial^2 w}{\partial \beta^2}\right]-\left(\frac{16}{9h^4}\frac{H_{11}}{R^3}\frac{\partial^4 w}{\partial \alpha^4}\right)-\left(\frac{16}{9h^4}\frac{H_{22}}{R^3}\frac{\partial^4 w}{\partial \beta^4}\right)-\left(\frac{32}{9h^4R^3}(2H_{66}+H_{12})\right)\frac{\partial^4 w}{\partial \alpha^2\, \partial \beta^2}+\left[\left\{A_{55}-\frac{8}{h^2}D_{55}-\frac{B_{12}}{R}+\frac{4E_{12}}{3h^2R}+\frac{16}{h^4}F_{55}\right\}\frac{\partial \varphi_1}{\partial \alpha}\right]+\left\{\frac{4}{3h^2}\frac{F_{12}}{R^2}-\frac{16}{9h^4}\frac{H_{12}}{R^2}+\frac{8}{3h^2}\frac{F_{66}}{R^2}-\frac{32}{9h^4}\frac{H_{66}}{R^2}\right\}\frac{\partial^3 \varphi_1}{\partial \alpha\, \partial \beta^2}+\left[\left\{A_{44}-\frac{8}{h^2}D_{44}-\frac{B_{22}}{R}+\frac{4}{3h^2}\frac{E_{22}}{R}+\frac{16F_{44}}{h^4}\right\}\frac{\partial \varphi_2}{\partial \beta}+\left\{\frac{4}{3h^2}\frac{F_{12}}{R^2}-\right.\right.$$

$$\frac{16}{9h^4}\frac{H_{12}}{R^2} + \frac{8}{3h^2}\frac{F_{66}}{R^2} - \frac{32}{9h^4}\frac{H_{66}}{R^2}\Big\}\frac{\partial^3\varphi_2}{\partial\beta\,\partial\alpha^2}\Big] + \Big\{\frac{4}{3h^2}\frac{F_{22}}{R^2} - \frac{16}{9h^4}\frac{H_{22}}{R^2}\Big\}\frac{\partial^3\varphi_2}{\partial\beta^3} = I_1\ddot{w}R^2 + \bar{I}_3\frac{\partial\ddot{u}}{\partial\alpha} + \bar{I}_3{}'\frac{\partial\ddot{v}}{\partial\beta} - \frac{16}{9h^4}\frac{I_7}{R}\Big(\frac{\partial^2\ddot{w}}{\partial\alpha^2} + \frac{\partial^2\ddot{w}}{\partial\beta^2}\Big) + \bar{I}_5\frac{\partial\ddot{\varphi}_1}{\partial\alpha} + \bar{I}_5\frac{\partial\ddot{\varphi}_2}{\partial\beta}$$

$$\Big[\Big\{\frac{B_{11}}{R} - \frac{4}{3h^2}\frac{E_{11}}{R}\Big\}\frac{\partial^2 u}{\partial\alpha^2} + \Big\{\frac{B_{66}}{R} - \frac{4}{3h^2}\frac{E_{66}}{R}\Big\}\frac{\partial^2 u}{\partial\beta^2}\Big] + \Big[\Big\{\frac{B_{12}}{R} + \frac{D_{12}}{R^2} + \frac{B_{66}}{R} + \frac{D_{66}}{R^2} - \frac{4}{3h^2}\Big(\frac{E_{12}}{R} + \frac{F_{12}}{R^2} + \frac{E_{66}}{R} + \frac{F_{66}}{R^2}\Big)\Big\}\frac{\partial^2 v}{\partial\alpha\,\partial\beta}\Big] + \Big[\Big\{\frac{B_{12}}{R} - \frac{4}{3h^2}\frac{E_{12}}{R} - A_{55} + \frac{8}{h^2}D_{55} - \frac{16}{h^4}F_{55}\Big\}\frac{\partial w}{\partial\alpha}\Big] + \Big\{-\frac{4}{3h^2}\Big(\frac{F_{11}}{R^2} - \frac{4}{3h^2}\frac{H_{11}}{R^2}\Big)\Big\}\frac{\partial^3 w}{\partial\alpha^3} + \Big\{-\frac{4}{3h^2}\Big(\frac{F_{12}}{R^2} + \frac{2F_{66}}{R^2} - \frac{4}{3h^2}\frac{H_{12}}{R^2} - \frac{8}{3h^2}\frac{H_{66}}{R^2}\Big)\Big\}\frac{\partial^3 w}{\partial\alpha\,\partial\beta^2} + \Big\{\Big[-RA_{55} + \frac{8R}{h^2}D_{55} - \frac{16}{h^4}RF_{55}\Big]\varphi_1 + \Big[\frac{D_{11}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{11}}{R} - \frac{4}{3h^2}\frac{H_{11}}{R}\Big)\Big]\frac{\partial^2\varphi_1}{\partial\alpha^2} + \Big[\frac{D_{66}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{66}}{R} - \frac{4}{3h^2}\frac{H_{66}}{R}\Big)\Big]\frac{\partial^2\varphi_1}{\partial\beta^2} + \Big[\Big\{\frac{D_{12}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{12}}{R} + \frac{2F_{66}}{R} - \frac{4}{3h^2}\frac{H_{12}}{R} - \frac{4}{3h^2}\frac{H_{66}}{R}\Big) + \frac{D_{66}}{R}\Big\}\frac{\partial^2\varphi_2}{\partial\alpha\,\partial\beta}\Big]\Big\} = \Big[\bar{I}_2\ddot{u} - \frac{\bar{I}_5}{R}\frac{\partial\ddot{w}}{\partial\alpha} + \bar{I}_4\ddot{\phi}_1\Big]R$$

$$\Big[\Big\{\frac{B_{66}}{R} + \frac{B_{12}}{R} - \frac{4}{3h^2}\Big(\frac{E_{66}}{R} + \frac{E_{12}}{R}\Big)\Big\}\frac{\partial^2 u}{\partial\alpha\,\partial\beta}\Big] + \Big[\Big\{\frac{B_{66}}{R} + \frac{D_{66}}{R^2} - \frac{4}{3h^2}\Big(\frac{E_{66}}{R} + \frac{F_{66}}{R^2}\Big)\Big\}\frac{\partial^2 v}{\partial\alpha^2} + \Big\{\frac{B_{22}}{R} + \frac{D_{22}}{R^2} - \frac{4}{3h^2}\Big(\frac{E_{22}}{R} + \frac{F_{22}}{R^2}\Big)\Big\}\frac{\partial^2 v}{\partial\beta^2}\Big] + \Big[\Big\{\frac{B_{22}}{R} - \frac{4}{3h^2}\frac{E_{22}}{R} - A_{44} + \frac{8}{h^2}D_{44} - \frac{16}{h^4}F_{44}\Big\}\frac{\partial w}{\partial\beta}\Big] + \Big\{-\frac{4}{3h^2}\Big(\frac{2F_{66}}{R^2} + \frac{F_{12}}{R^2} - \frac{4}{3h^2R^2}[2H_{66} + H_{12}]\Big)\Big\}\frac{\partial^3 w}{\partial\alpha^2\,\partial\beta} + \Big\{-\frac{4}{3h^2}\Big(\frac{F_{22}}{R^2} - \frac{4}{3h^2}\frac{H_{22}}{R^2}\Big)\Big\}\frac{\partial^3 w}{\partial\beta^3} + \Big[\Big\{\frac{D_{66}}{R} + \frac{D_{12}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{66}}{R} + \frac{2F_{12}}{R} - \frac{4}{3h^2}\Big(\frac{H_{66}}{R} + \frac{H_{12}}{R}\Big)\Big)\Big\}\frac{\partial^2\varphi_1}{\partial\alpha\,\partial\beta}\Big] + \Big[\Big\{-RA_{44} + \frac{8R}{h^2}D_{44} - \frac{16}{h^4}RF_{44}\Big\}\varphi_2\Big] + \Big\{\frac{D_{66}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{66}}{R} - \frac{4}{3h^2}\frac{H_{66}}{R}\Big)\Big\}\frac{\partial^2\varphi_2}{\partial\alpha^2} + \Big\{\frac{D_{22}}{R} - \frac{4}{3h^2}\Big(\frac{D_{22}}{R} - \frac{4}{3h^2}\Big(\frac{2F_{22}}{R} - \frac{4}{3h^2}\frac{H_{22}}{R}\Big)\Big)\frac{\partial^2\varphi_2}{\partial\beta^2}\Big\} = \Big[\bar{I}'_2\ddot{v} - \frac{\bar{I}_5}{R}\frac{\partial\ddot{w}}{\partial\beta} + \bar{I}_4\ddot{\phi}_2\Big]R \qquad (16)$$

The boundary conditions for a simply supported cylindrical shell panel are given by

$N_1 = 0$, $v = 0$, $w = 0$, $M_1 = 0$, $\varphi_2 = 0$ At $\alpha = 0$ and $\alpha = L/R$

Following the Navier solution procedure, the following form which satisfies the boundary condition in equations (2.9.1) is assumed for the stability analysis

$$u = U\cos\lambda_m\alpha\,\sin n\beta\,,\; v = V\sin\lambda_m\alpha\,\cos n\beta\,,\; w = W\sin\lambda_m\alpha\,\cos n\beta$$
$$\varphi_1 = \bar{\varphi}_1\cos\lambda_m\alpha\,\sin n\beta\,,\quad \varphi_2 = \bar{\varphi}_2\sin\lambda_m\alpha\,\cos n\beta \qquad (17)$$

Following the Navier solution procedure, the following form which satisfies the boundary condition in above equations is assumed for the free vibration analysis

$$u = U\ \cos\lambda_m\alpha\ \sin n\beta\, e^{i\varpi t}\ ,v = V\ \sin\lambda_m\alpha\ \cos n\beta\, e^{i\varpi t}\ ,\varphi_2 = \bar{\varphi}_2\ \sin\lambda_m\alpha\ \cos n\beta\, e^{i\varpi t}$$

$$w = W\ \sin\lambda_m\alpha\ \cos n\beta\, e^{i\varpi t},\ \varphi_1 = \bar{\varphi}_1\ \cos\lambda_m\alpha\ \sin n\beta\, e^{i\varpi t} \qquad (18)$$

For convenience, the elements of the above matrices are suitably non-dimensionalised as follows:

$$u = \bar{U}h,\ U = \bar{U}h,\ v = \bar{V}h,\ V = \bar{V}h, w = \bar{W}h,\ W = \bar{W}h, \varphi_1 = \bar{\varphi}_1,\ \varphi_2 = \bar{\varphi}_2, A_{ij} = \bar{A}_{ij}Q_2h,$$

$$B_{ij} = \bar{B}_{ij}Q_2h^2, D_{ij} = \bar{D}_{ij}Q_2h^3, E_{ij} = \bar{E}_{ij}Q_2h^4, F_{ij} = \bar{F}_{ij}Q_2h^5, H_{ij} = \bar{H}_{ij}Q_2h^7, I_1 = I_1'\rho^{(1)}h$$

$$I_2 = I_2'\rho^{(1)}h^2,\quad I_3 = I_3'\rho^{(1)}h^3,\quad I_4 = I_4'\rho^{(1)}h^4,\quad I_5 = I_5'\rho^{(1)}h^5, I_7 = I_7'\rho^{(1)}h^7$$

(I_1', I_2', I_3'................are the non-dimensionalised quantities)

$$\bar{I}_1 = \tilde{I}_1\rho^{(1)}h,\ \bar{I}_1' = \left[I_1' + 2\frac{h}{R}I_2'\right]\rho^{(1)} = \tilde{I}'_1\rho^{(1)}h,\ \bar{I}_2 = \left[I_2' - \frac{4}{3}I_4'\right]\rho^{(1)}h^2 = \tilde{I}_2\rho^{(1)}h^2$$

$$\bar{I}_2' = [I_2' + \frac{h}{R}I_3' - \frac{4}{3}I_4' - \frac{4}{3}\frac{h}{R}I_5']\rho^{(1)}h^2 = \tilde{I}_2'\rho^{(1)}h^2,\ \bar{I}_3 = \left[\frac{4}{3}I_4'\right]\rho^{(1)}h^2 = \tilde{I}_3\rho^{(1)}h^2$$

$$\bar{I}_3' = \left(\frac{4}{3}I_4' + \frac{4}{3}\frac{h}{R}I_5'\right)\rho^{(1)}h^2 = \tilde{I}_3'\rho^{(1)}h^2, \bar{I}_4 = [I_3' - \frac{8}{3}I_5' + \frac{16}{9}I_7']\rho^{(1)}h^3 = \tilde{I}_4\rho^{(1)}h^3$$

$$\bar{I}_5 = [\frac{4}{3}I_5' - \frac{16}{9}I_7']\rho^{(1)}h^3 = \tilde{I}_5\rho^{(1)}h^3 \qquad (19)$$

3. NUMERICAL RESULTS

In the case of vibration analysis, $\bar{w}^2$ is the eigenvalue for free vibration and $\bar{n}_1$ for the buckling load and {X} is the eigenvector. For convenience the above equation is non-dimensionalised. Then if the equation is premultiplied by $[\bar{M}]^{-1}$, one obtains the following standard eigenvalue problem,

$$[\bar{H}]\{\bar{X}\} = \bar{\omega}^2\{\bar{X}\}\ , [\bar{H}]\{\bar{X}\} = \bar{n}_1\{\bar{X}\}\ , \bar{\omega}^2 = \frac{\omega^2R^2I_1}{A_{22}}\ , \bar{n}_1 = \frac{n_1R}{Q_{22}h^2}\ ,\ [\bar{H}] = [\bar{M}]^{-1}[\bar{C}]$$

In table 1, a two-layer cross-ply cylindrical shell panel has been analysed. The frequency parameter is $\Omega^2 = \bar{\omega}^2 \rho_0 L_s{}^2 / A_{11}$ where $L_s = 2\pi R$. The results given by Dong and Tso's theory and present theory are in very close agreement for all cases.

Table 1: Comparison of frequency parameter of a two layer cross-ply cylindrical shell panel ($E_1/E_2 = 40$; $G_{12}/E_2 = 0.5$; $h/R = 0.01$; $G_{23}/E_2 = 0.6$; $\upsilon_{12} = 0.25$; $G_{13} = G_{12}$; $\boldsymbol{\rho}=1$)

L/R	n	Dong & Tso	Present Theory
1.0	1	2.106	2.106
	2	1.344	1.344
	3	0.9587	0.9587
	4	0.7493	0.7493
	5	0.6419	0.6420
	6	0.6130	0.6130
2.0	1	1.073	1.073
	2	0.6710	0.6710
	3	0.4710	0.4710
	4	0.3773	0.3773
	5	0.3629	0.3630
	6	0.4160	0.4162
5.0	1	0.4212	0.4212
	2	0.2470	0.2470
	3	0.1733	0.1734
	4	0.1818	0.1819
	5	0.2516	0.2517
	6	0.3567	0.3569

The dimensionless natural frequencies $\bar{\omega}^* = \bar{\omega} L_s{}^2 \sqrt{\frac{\rho_0}{E_2 h^3}}$ for various values of R/h ratios for two-layered cylindrical panels having aspect ratio S/L =1 are given in table 2. In all cases, the results given by Dong and Tso's theory and present theory are quite agreeable.

Table 2 Comparison of frequency parameter of a two-layered circular cylindrical panel

$E_1/E_2=40.0$, $\upsilon_{12}=0.25$, $G_{12}/E_2=1$, $G_{23}/E_2=0.6$, $G_{13}=G_{12}$

Φ (radians)	R/h	Dong & Tso	Present Theory
0.16	312.2	14.04	14.04
0.32	156.25	19.14	19.14
0.16	62.5	11.16	11.24
0.32	31.25	11.32	11.39

The variation of non-dimensional frequency with various parameters is now studied. The material properties of the first layer are given in Table 3.

Table 3: Material Properties (first layer) of laminated cylindrical panels

E_1 (x 10^6psi)	E_2 (x 10^6psi)	υ_{12}	G_{12} (x 10^6psi)	G_{23} (x 10^6psi)	G_{13} (x 10^6psi)	ρ
25.0	1.0	0.25	0.5	0.2	0.5	1.0

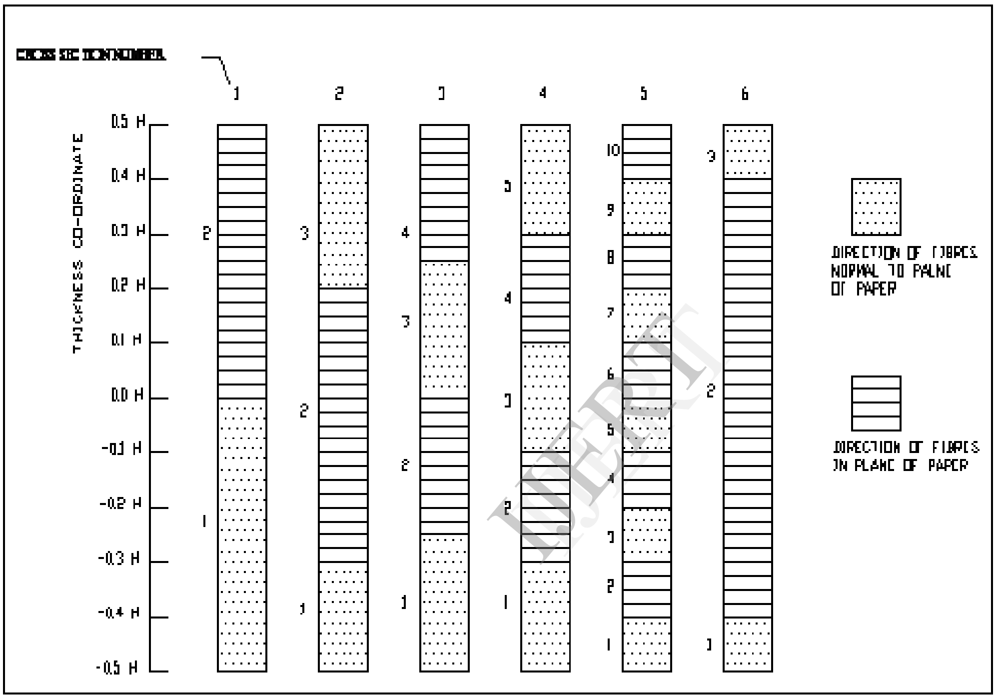

Fig. 1 Section Properties for different layer of panels

In Table 4, the values of non-dimensional frequencies for the various cross-sections for m =1 and various included angles Φ are shown.

Table 4: Non-dimensional frequency parameters of cross-ply circular cylindrical panels

Φ (radian)	Number of Layers	n =1	n = 2	n =3	n = 4
0.5235	2	0.47	1.098	2.031	3.082
	3	0.6203	1.08	1.75	2.47
	5	0.6209	1.33	2.269	3.249
	6	0.6011	1.1	1.822	2.591
	10	0.6802	1.72	3.53	5.89
1.048	2	0.205	0.313	0.623	1.026

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	3	0.247	0.237	0.581	0.905
	5	0.23	0.416	0.803	1.25
	6	0.226	0.336	0.621	0.971
	10	0.26	0.44	0.925	1.6
1.57	2	0.172	0.157	0.291	0.494
	3	0.185	0.166	0.28	0.454
	5	0.178	0.203	0.395	0.651
	6	0.169	0.167	0.301	0.493
	10	0.212	0.207	0.41	0.72
2.094	2	0.157	0.109	0.168	0.284
	3	0.163	0.115	0.164	0.267
	5	0.16	0.13	0.23	0.389
	6	0.149	0.112	0.175	0.291
	10	0.195	0.136	0.229	0.401
2.618	2	0.145	0.092	0.11	0.18
	3	0.149	0.0956	0.111	0.174
	5	0.146	0.101	0.15	0.254
	6	0.136	0.091	0.117	0.19
	10	0.18	0.116	0.148	0.253

Table 6: Frequency parameters of cross-ply circular cylindrical panels for different material property

Material property 1: $E_1/E_2 = 25.0$, $\upsilon_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.2$, $G_{13} = G_{12}$

Material property 2: $E_1/E_2 = 40.0$, $\upsilon_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.6$, $G_{13} = G_{12}$

φ (radian)	Number of Layers	Material property 1	Material property 2
0.5235	2	0.473	0.431
	3	0.657	0.619
	5	0.604	0.582
	6	0.6029	0.584
	10	0.731	0.7045

Table 6: Frequency parameters of cross-ply circular cylindrical panels for different ratio of L/h

φ (radian)	Number of Layers	L/h = 20	L/h =100
0.5235	2	0.268	0.157
	3	0.390	0.169
	5	0.357	0.165
	6	0.363	0.155
	10	0.418	0.186

Table 7: Lowest frequency parameters of various cross-ply circular cylindrical panels

(m =1, n =1)

φ (radian)	Number of Layers	Frequency parameter
0.5235	2	0.473
	3	0.657
	5	0.604
	6	0.6029
	10	0.6802
1.048	2	0.205
	3	0.247
	5	0.230
	6	0.226
	10	0.2609
1.57	2	0.172
	3	0.185
	5	0.178
	6	0.169
	10	0.212
2.094	2	0.157
	3	0.163
	5	0.16
	6	0.149
	10	0.195
2.618	2	0.145
	3	0.149
	5	0.146
	6	0.136
	10	0.18

The frequency envelopes of a two-layer, three-layer, four layer, five layer and ten-layer cross-ply circular cylindrical panel are plotted for m =1 and for various shallowness angles φ for a thickness to radius ratio of 0.05 and are shown in figure3.

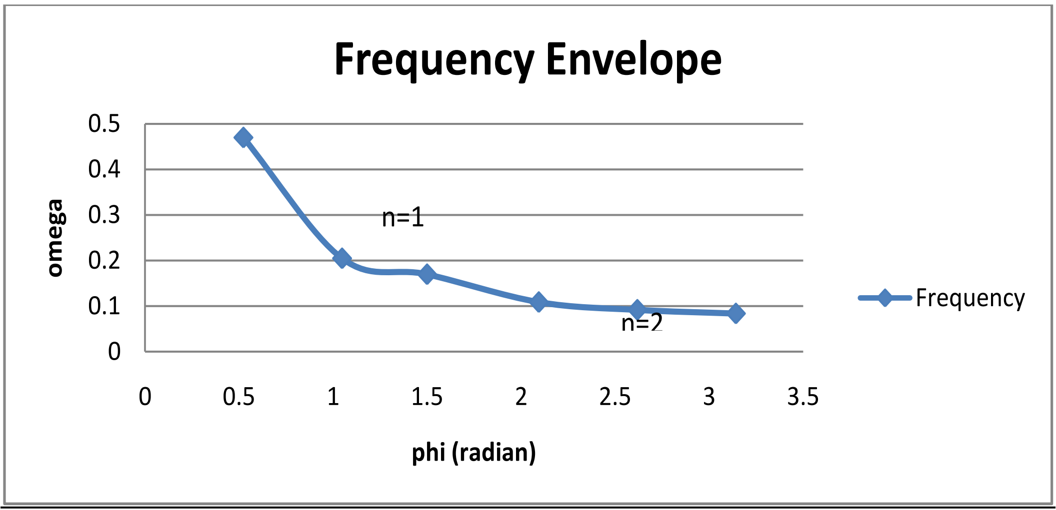

Fig. 2 Frequency envelope for two layer cross-ply circular cylindrical panel

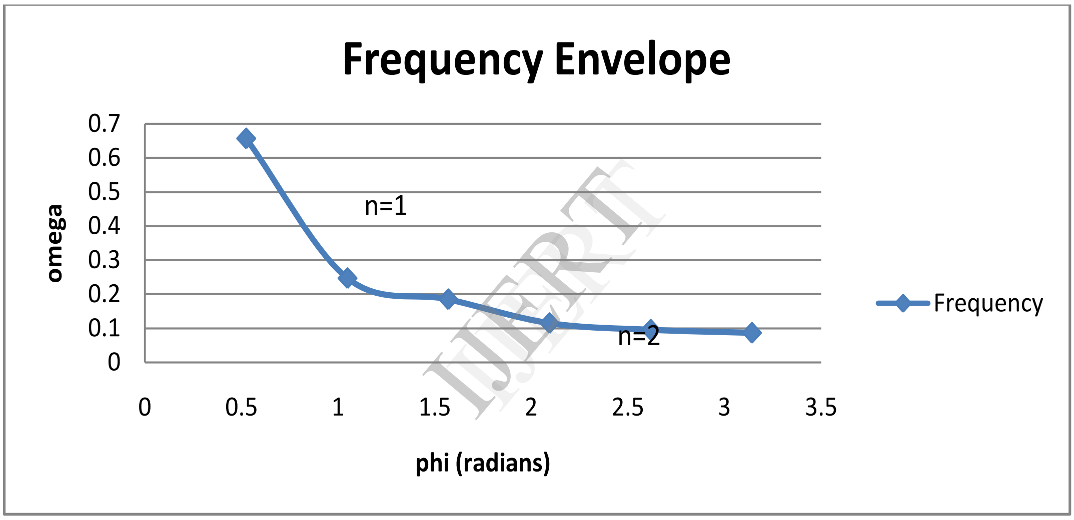

Fig. 3 Frequency envelope for three layer cross-ply circular cylindrical panel

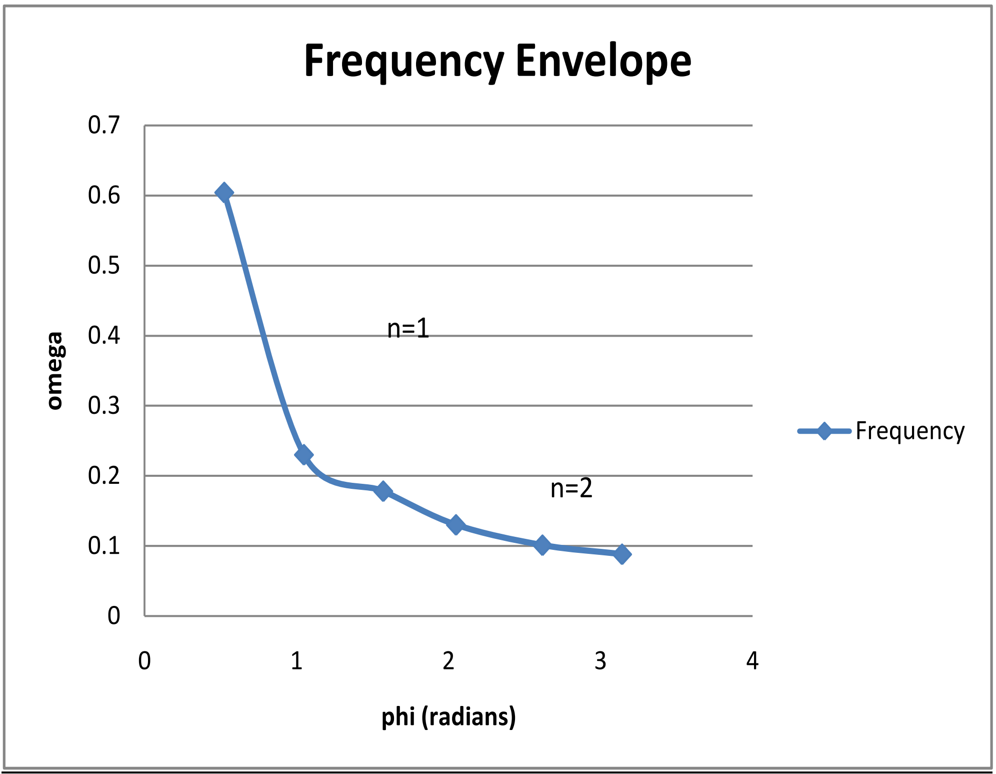

Fig. 4 Frequency envelope for four layer cross-ply circular cylindrical panel

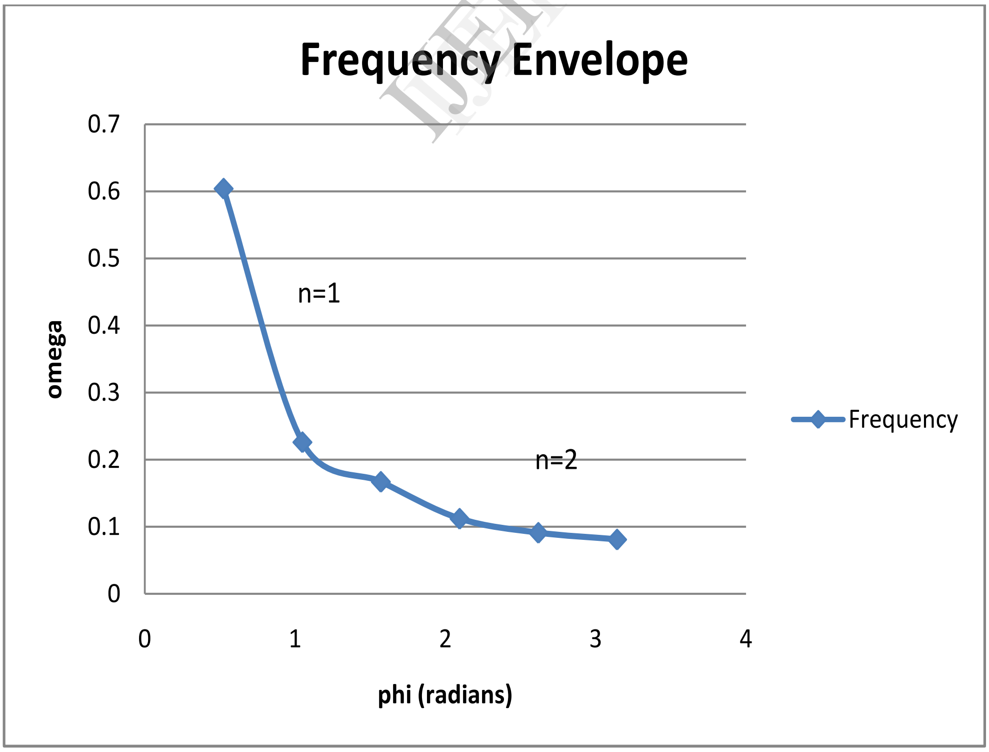

Fig. 5 Frequency envelope for five layer cross-ply circular cylindrical panel

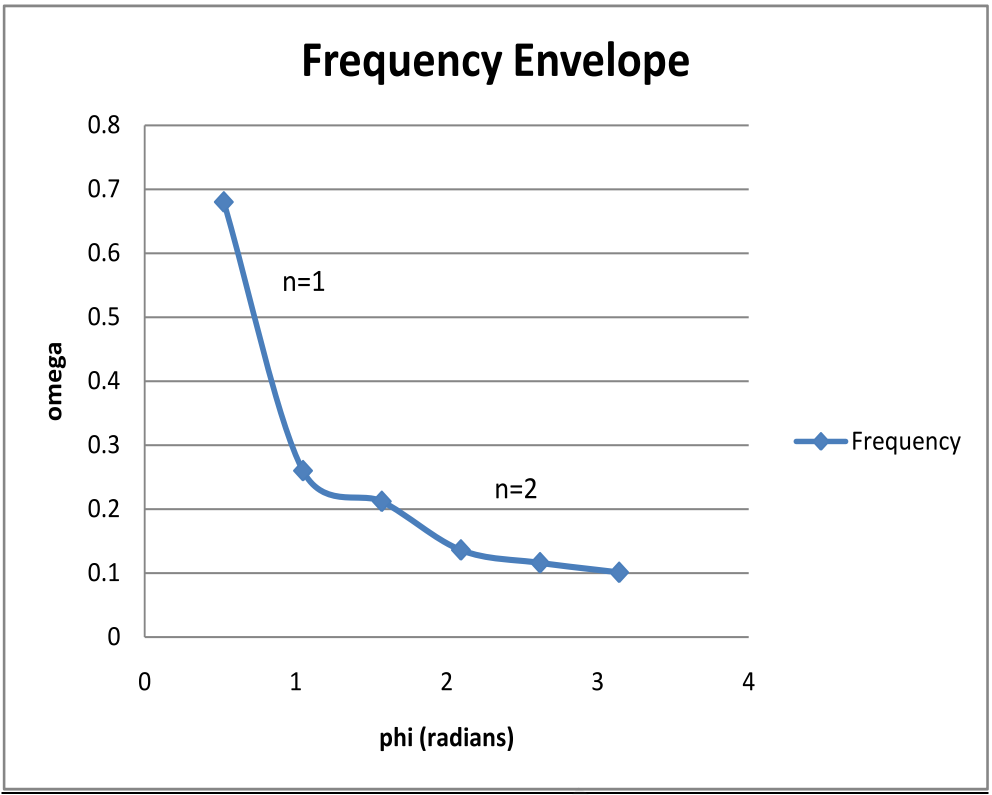

Fig. 6 Frequency envelope for ten layer cross-ply circular cylindrical panel

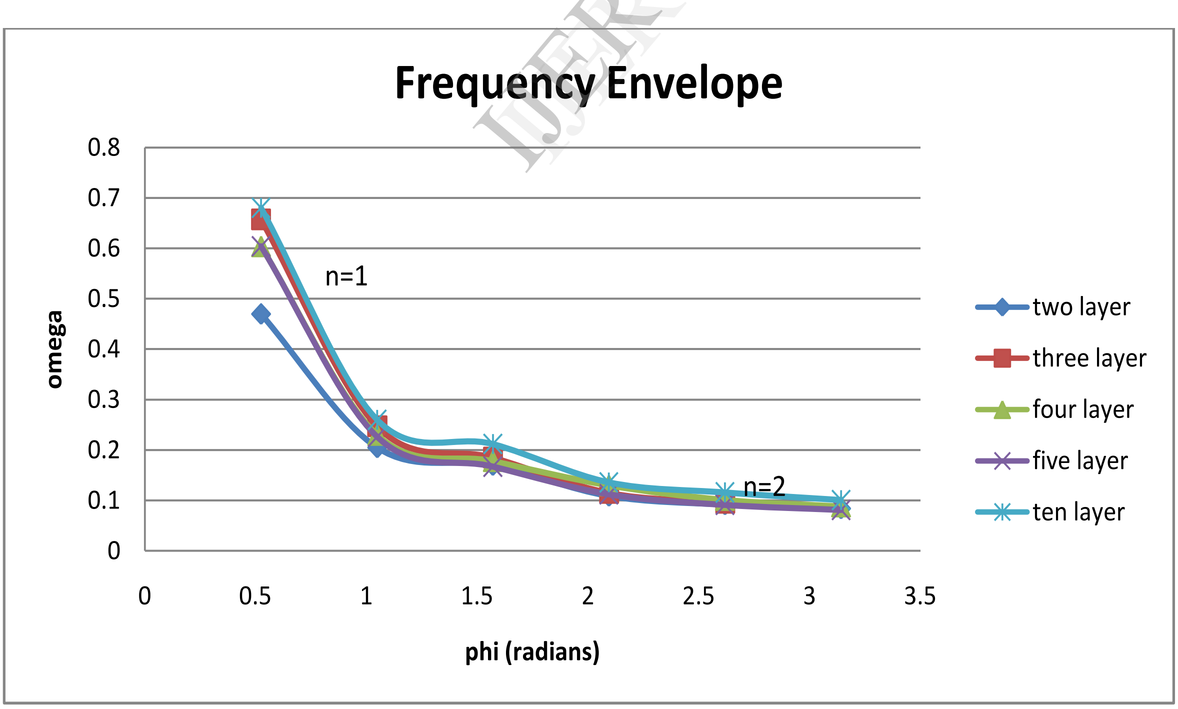

Fig. 7 Frequency envelope for cross-ply circular cylindrical panel

The non-dimensionalised critical buckling loads for cross-ply laminated plates are presented in the form of table. The following table shows the comparison of results of present analysis and Reddy [22] for the non-dimensionalised buckling parameter of two layer, three layer and four layer of simply supported plates. The geometrical and material properties used are $E_1/E_2 = 40$; $G_{12}/E_2 = 0.5$; $h/R = 0.01$; $G_{23}/E_2 = 0.6$; $\upsilon_{12} = 0.25$; $G_{13} = G_{12}$; $\boldsymbol{\rho}=1$

Table 8: Comparison of buckling parameter of cross-ply laminated plates:

a/h	Two Layers 0°/90°			Three Layers 0°/90°/0°			Four Layers 0°/90°/90°/0°		
	Reddy	Present	Error in %	Reddy	Present	Error in %	Reddy	Present t	Error in %
50	12.569	12.494	-0.6	34.936	35.873	2.68	35.100	35.277	0.5
100	12.614	12.531	-0.66	35.602	35.959	1.0	35.645	35.369	-0.77

Error in Percentage = 100 * (Present Theory –Reddy Theory)/ Reddy theory

From the results presented in the above table, it is clear that results of present theory in good agreement with those obtained by Reddy theory.

The non-dimensional buckling loads for different R/h values for cylindrical panels of L/h = 500 are tabulated in Table 9. It is seen that the values of the buckling load increase as the r/h ratio decreases.

Table 9: Non-dimensional Buckling load of cross-ply circular cylindrical panels for l/h = 100

R/h	Two Layer 0°/90°	Three Layer 0°/90°/0°	Four Layer 0°/90°/90°/0°
1000	21.508	44.13	50.55
500	25.53	59.15	61.09
400	31.89	73.94	76.34
200	34.81	147.84	152.49

In Table 10, the non-dimensional buckling loads for cylindrical panels with R/h = 500 and different material properties are shown**.** It is seen that as the E_1/E_2 ratio is increased, the buckling load increases.

Table 10: Non-dimensional buckling load of cylindrical panels for different material properties

Material property 1: $E_1/E_2 = 25.0$, $\upsilon_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.2$, $G_{13} = G_{12}$

Material property 2: $E_1/E_2 = 40.0$, $\upsilon_{12} = 0.25$, $G_{12}/E_2 = 1$, $G_{23}/E_2 = 0.6$, $G_{13} = G_{12}$

R/h	No. of Layers	Material Property1	Material Property2
500	2 [0°/90°]	17.70	25.53
	3 [0°/90°/0°]	38.016	59.15
	4 [0°/90°/90°/0°]	37.411	61.09

Table 11: Non-dimensional buckling load of circular cylindrical panels for different ratio of l/h

l/h	Two layers [0°/90°]	Three layers [0°/90°/0°]	Four Layers [0°/90°/90°/0°]
100	25.53	59.15	61.09
200	44.35	150.59	176.57

In Table 11, the non-dimensional buckling loads have been calculated for different l/h ratios (varying thickness). It is seen that as the l/h ratio decreases, the buckling load decreases.

Table 12: Non-dimensional buckling load of circular cylindrical panel for different cross-section (l/h=100, R/h=500)

l/h=100,	Two layers	Three layers	Four layers	Five Layers	Ten Layers
R/h=500	25.53	59.15	61.09	56.66	66.45

4. CONCLUSIONS

As the number of layers increases, the non-dimensional frequency increases. As Φ increases, the value of the non-dimensional frequency parameter decreases in all cases. As the degree of orthotropy increases, the value of the frequency parameter decreases. As the thickness decreases, the frequency parameter decreases owing to a decrease in the stiffness matrix. As the L/h ratio increases, the frequency parameter decreases. As E_1/E_2 ratio increases the frequency parameter decreases. As the thickness decreases buckling parameter increases.
As the number of layers increases the buckling parameter increases. As the R/h ratio decreases buckling parameter increases. As E_1/E_2 ratio increases the buckling parameter

increases. As the l/h ratio increases the buckling parameter increases. The above conclusions establish that the present theory, though a simple one including the shear deformation effects, gives comparable results.

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