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Value distribution for q-shift difference polynomial of meromorphic function with maximal deficiency sum

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Abstract

The main purpose of this paper is to investigate the relationship between the characteristic function of a meromorphic function $f(z)$ involving maximal deficiency sum and that of q shift difference polynomial.We improve and generalize the results of Zhaojun Wu and S.S.Bhoosnurmath, R.S.Dyavanal et.al.

Keywords: difference operator,difference polynomial, maximal deficiency sum, meromorphic function,

1 Introduction(Definitions and Results)

In this paper, we shall use the standard notions in Nevanlinna's value distribution theory of meromorphic functions such as $T(r,f)$, $N(r,f)$ and $m(r,f)$(see [8, 17, 19]). By $S(r,f)$ we denote any quantity satisfying the condition $S(r,f) = o(T(r,f))$ as $r \to \infty$ possibly outside of an exceptional set E of finite linear measure. A meromorphic function $a \equiv a(z)$ is called a small function with respect to f if either $a \equiv \infty$ or $T(r,a) = S(r,f)$. We denote by $S(f)$ the collection of all small functions with respect to f. Clearly $\mathbb{C} \cup \{\infty\} \subset S(f)$ and $S(f)$ is a field over the set of complex numbers.
For $a \in \mathbb{C} \cup \{\infty\}$, the quantities $\delta(a,f)$ and $\Theta(a,f)$ defined as follows

$$\delta(a,f) = 1 - \lim_{r\to\infty} Sup \frac{N(r,a;f)}{T(r,f)}$$

and

$$\Theta(a,f) = 1 - \lim_{r\to\infty} Sup \frac{\overline{N}(r,a;f)}{T(r,f)}$$

are called the deficiency and the ramification index of $'a'$ for the function f. The classical second fundamental theorem of Nevanlinna theory asserts that the total deficiency of any meromorphic function $f(z)$ satisfies the inequality

$$\sum_{a\in\mathbb{C}} \delta(a,f) + \delta(\infty,f) \leq 2$$

The above inequality holds, then we say that f has maximal deficiency sum. The Valiron Mahonko identity states that if the function $R(z,f)$ is rational in f and has small merorphic coefficients, then

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$$T(r, R(z, f)) = \deg_f(R)T(r, f) + S(r, f) \tag{1.1}$$

Certain relationship between the characteristic function of a meromorphic function $f(z)$ with maximal deficiency sum and that of derivative $f'(z)$ plays a key role in the study of a conjecture of Nevanlinna (see [4]). The main contribution of this paper is to study the relationship between the characteristic function of a meromorphic function $f(z)$ with maximal deficiency sum and that of the exact difference $\Delta_c f = f(z+c) - f(z)$, where $c \neq 0$ (see [5]). In 1956, Shan and Singh [6] proved the following theorem.

Theorem A [6] Suppose that $f(z)$ is a transcendental meromorphic function of finite order and $\sum_{a\in\mathbb{C}} \delta(a, f) = 2$. Then

$$T(r, f') \sim 2T(r, f), \quad r \to +\infty.$$

After that, Edrei [7] and Weitsman [4] proved the following theorem, respectively.

Theorem B [4, 7] Suppose that $f(z)$ is a transcendental meromorphic function of finite order with maximal deficiency sum. Then

$$\lim_{r\to+\infty} \frac{T(r, f')}{T(r, f)} = 2 - \delta(\infty, f)$$

and

$$\lim_{r\to+\infty} \frac{N\left(r, \frac{1}{f'}\right)}{T(r, f')} = 0.$$

Under the condition of Theorem B, Singh and Gopalakrishna [8] proved that

$$\lim_{r\to+\infty} \frac{N(r, a)}{T(r, f)} = 1 - \delta(a, f)$$

holds for every $a \in \mathbb{C}$.

Theorem C [3] Suppose that $f(z)$ is a transcendental meromorphic function of order less than one with maximal deficiency sum. Then

$$K(\Delta_c f) \leq \frac{2(1 - \delta(\infty, f))}{2 - \delta(\infty, f)}$$

where

$$K(\Delta_c f) = \limsup_{r\to\infty} \frac{N(r, \Delta_c f) + N\left(r, \frac{1}{\Delta_c f}\right)}{T(r, \Delta_c f)}$$

Theorem D [3] Let $f(z)$ be a transcendental meromorphic function of order less than one and assume $\delta(\infty, f) = 1$. Then

$$\sum_{a\in\mathbb{C}} \delta(a, f) \leq \delta(0, \Delta_c f)$$

Later on, Zhaojun Wu[10] investigated the relationship between the Characteristic function of a meromorphic function $f(z)$ with maximal deficiency sum using difference operator and also established an inequality on the zeros and poles for difference operator by giving an example to show that the upperbound of the inequality is accurate.
Recently, Subhas. S. Bhoosnurmath, R.S.Dyavanal et. al[11] obtained the relationship between

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the Characteristic function of meromorphic function having maximal deficiency sum and its higher order exact difference $\Delta_c^n(f(z))$ and generalized the results of Zhaojun Wu to a great extent.
Inspiring by these results,we investigate the value distribution for q-shift difference polynomial $\Delta_q^n(f(z))P(f(qz+c))$ of meromorphic function with maximal deficiency sum.

2 Lemmas

Lemma 2.1. *[14]. Let q and c be two non-zero finite complex numbers and $f(z) \in F$ with zero order*

$$m\left(r, \frac{f(qz+c)}{f(z)}\right) = S(r,f).$$

Lemma 2.2. *[13]. Let $f(z) \in F$ of finite order, $c \in \mathbb{C}$ and $\delta < 1$, then*

$$m\left(r, \frac{\Delta_c^n f(z)}{f(z)}\right) = o\left(\frac{T(r,f)}{r^\delta}\right) = S(r,f).$$

Lemma 2.3. *[16]. Let $f(z) \in F$ of finite order and $q, c \in c^*$, then*

$$T(r, f(z+c)) \le T(r, f(z)) + S(r,f),$$
$$N(r, f(qz+c)) \le N(r, f(z)) + S(r,f),$$
$$N\left(r, \frac{1}{f(qz+c)}\right) \le N\left(r, \frac{1}{f(z)}\right) + S(r,f).$$

Lemma 2.4. *[13]. Let $f(z) \in F$ of finite order and $c \in \mathbb{C}$, then $N(r, \Delta_c^n f(z)) = (n+1)N(r,f) + S(r,f)$.*

Lemma 2.5. *[15]. Let $f(z) \in F$ of finite order and $P(f(qz+c))$ be q-shift polynomial of order s and $c \in \mathbb{C}$. Then*

$$N(r, P(f(qz+c))) \le sN(r,f) + S(r,f).$$

3 Results:

Theorem 1.1 Suppose that $f(z)$ is a transcendental meromorphic function of finite order with maximal deficiency sum. Then

$$\sum_{a\in\mathbb{C}} \delta(a,f) \le \liminf_{r\to\infty} \frac{T\left(r, \frac{\Delta_q^n(f(z))}{f(z)} P(f(qz+c))\right)}{T(r,f)}$$

$$\le \limsup_{r\to\infty} \frac{T\left(r, \frac{\Delta_q^n(f(z))}{f(z)} P(f(qz+c))\right)}{T(r,f)} \le (n+s+1) - n\delta(\infty, f)$$

and

(i) For $n = 1$ and $s = 0$ with maximal deficiency and $\sum_{a\in C} \delta(a,f) = \beta. \quad (1 \le \beta \le 2)$

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$$\text{We obtain } T(r,\Delta_q(f(z))) \sim \beta T(r,f) \text{ as } r\to\infty$$

(ii) For $n \geqslant 1$ and $s = 0$ with maximal deficiency and $\sum_{a\in C}\delta(a,f) = 1 and \delta(\infty,f) = 1$

$$\text{We obtain } \lim_{r\to\infty}\frac{T\left(r,\Delta_q^n(f(z))P(f(qz+c))\right)}{T(r,f)} = 1$$

Proof of theorem 1.1:

Proof. We define, $F_1 = \Delta_q^n(f(z))P(f(qz+c))$. Using the first main theorem of Nevanlinna and Lemmas 2.2 and 2.4, we have

$$T(r,F_1) = m(r,F_1) + N(r,F_1)$$

$$= m(r,\Delta_q^n f(z)p(f(qz+c))) + N(r,\Delta_q^n f(z)P(f(qz+c)))$$

$$\leq m\left(r,\frac{\Delta_q^n(f(z))f(z)}{f(z)}P(f(qz+c))\right) + N(r,\Delta_q^n f(z)) + N(r,P(f(qz+c)))$$

$$\leq m\left(r,\frac{\Delta_q^n f(z)}{f(z)}\right) + m(r,f(z)) + m(r,P(qz+c)) + N(r,\Delta_q^n f(z)) + N(r,P(qz+c))$$

$$\leq S(r,f) + m(r,f) + Sm(r,f) + (n+1)N(r,f)$$

$$T(r,f) \leq T(r,f)(S+1) + nN(r,f) + S(r,f)$$

$$\lim_{r\to\infty} Sup\frac{T(r,F_1)}{T(r,f)} \leq (S+1) + n\lim_{r\to\infty} Sup\frac{N(r,f)}{T(r,f)} + n\lim_{r\to\infty} Sup\frac{S(r,f)}{T(r,f)}$$

$$(S+1) + n(1-\delta(\infty,f))$$

$$\lim_{r\to\infty} Sup\frac{T(r,F_1}{T(r,f)} \leq (n+S+1) - n\delta(\infty,f) \tag{3.1}$$

Let $\{a_i\}(i = 1,2,...,q)$ be distinct complex numbers continuing all the finite deficient values of $f(z)$.
Consider

$$\Phi(z) = \sum_{i=1}^{q}\frac{1}{f-a_i}$$

We know that $T(r,f(z)-a_i) = T(r,f) + O(1)$
Since Δ_q^n is a linear operator and Δ_q^n of a constant is zero, it follows that

$$\Delta_q^n\left[f(z)-a_i\right] = \Delta_q^n f(z)$$

Using Lemma 2.2, we deduce

$$m(r,\phi(z)F_1(z)) \leq \sum_{i=1}^{q} m\left(r,\frac{\Delta_q^n(f(z)-a_i)P(f(qz+c))}{f(z)-a_i}\right) + logq$$

$$= S(r,f)$$

Now the above results yields,

$$m(r,\phi(z)) = m\left(r,\phi(z)F_1(z)\frac{1}{F_1(z)}\right)$$

$$\leq m(r,\phi(z)F_!(z)) + m\left(r,\frac{1}{F_1(z)}\right) + c$$

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therefore

$$m(r,\phi(z)) \le m\left(r,\frac{1}{F_1(z)}\right) + S(r,f) \tag{3.2}$$

In view of **Valirmon Mahonko** identity (1.1), we have

$$qT(r,f) = T(r,\phi(z)) + O(1)$$

$$= m(r,\phi(z)) + N(r,\phi(z)) + O(1)$$

$$\le m\left(r,\frac{1}{F_1(z)}\right) + \sum_{i=1}^{q} N(r,a_i) + S(r,f)$$

$$\le T(r,F_1(z)) = \sum_{i=1}^{q} N(r,a_i) + S(r,f)$$

Hence

$$q \le \lim_{r\to\infty} inf \frac{T(r,F_1(z))}{T(r,f)} + \sum_{i=1}^{q} \lim_{r\to\infty} Sup \frac{N(r,a_i)}{T(r,f)}$$

$$q = \lim_{r\to\infty} inf \frac{T(r,F_1(z))}{T(r,f)} + \sum_{i=1}^{q} (1-\delta(a_i,f))$$

$$q \le \lim_{r\to\infty} inf \frac{T(r,F_1(z))}{T(r,f)} + q - \sum_{i=1}^{q} \delta(a_i,f)$$

(OR)

$$\lim_{r\to\infty} inf \frac{T(r,F_1(z))}{T(r,f)} \ge \sum_{i=1}^{q} \delta(a_i,f)$$

Since q is arbitrary, we have

$$\sum_{a\in\phi} \delta(a,f) \le \lim_{r\to\infty} inf \frac{T(r,F_1(z))}{T(r,f)} \tag{3.3}$$

Thus from 3.2 and 3.3 we get (1.2)

$$\sum_{a\in\phi} \delta(a,f) \le \lim_{r\to\infty} inf \frac{T(r,F_1)}{T(r,f)} \le \lim_{r\to\infty} Sup \frac{T(r,F_1)}{T(r,f)}$$

$$\le (n+s+1) - n\delta(\infty,f)$$

Now for $n=1$ and $s=0$ with maximal deficiency and $\sum_{a\in\phi}\delta(a,f) = \beta. \quad (1\le\beta\le 2)$

We obtain $T(r,F_1) \sim \beta T(r,f)$ as $r\to\infty$

Again for $n\ge 1$ and $s=0$ with $\sum_{a\in\phi}\delta(a,f) = 1$ and $\delta(\infty,f)=1$

We get $\lim_{r\to\infty} \frac{T(r,F_1)}{T(r,f)} = 1$

□

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Theorem 1.2 Suppose that $f(z)$ is a transcendental meromorphic function of finite order. Then

$$\limsup_{r\to\infty} \frac{N\left(r, \dfrac{1}{\Delta_q^n(f(z))P(f(qz+c))}\right)}{T\left(r, \dfrac{1}{\Delta_q^n(f(z))P(f(qz+c))}\right)}$$

$$\leq 1 - \frac{\sum\limits_{a\in C} \delta(a,f)}{(n+s+1) - n\delta(\infty, f)}$$

and

For $n \geqslant 1$ and $s = 0$ with maximal deficiency and $\sum\limits_{a\in C} \delta(a,f) = 1 and \delta(\infty, f) = 1$

$$\text{We obtain } \lim_{r\to\infty} \frac{N\left(r, \dfrac{1}{\Delta_q^n(f(z))P(f(qz+c))}\right)}{T\left(r, \dfrac{1}{\Delta_q^n(f(z))P(f(qz+c))}\right)} = 0$$

consequently $\delta(0, \Delta_q^n(f(z))P(f(qz+c))) = 1$

Proof of theorem 1.2:

Proof. Consider $\sum\limits_{i=1}^{q} \dfrac{1}{f-a_i}$

As in the proof of Theorem (2.1) in Hayman's[1], we have

$$m(r, \phi(z)) \geq \sum_{i=1}^{q} m(r, a_i) - q log^+ \frac{3q}{\delta} - log 2$$

$$\sum_{i=1}^{q} m(r, a_i) + N\left(r, \frac{1}{F_1(z)}\right) \leq m(r, \phi(z)) + N\left(r, \frac{1}{F_1(z)}\right) + O(1)$$

By using the first main theorem and eq (3.2), we get

$$\sum_{i=1}^{q} m(r, a_i) + N\left(r, \frac{1}{F_1(z)}\right) \leq +(r, F_1(z)) + S(r, f)$$

$$\text{Thus, } \frac{\sum\limits_{i=1}^{q} m(r, a_i)}{T(r, F_1(z))} + \frac{N\left(r, \dfrac{1}{F_1(z)}\right)}{T(r, F_1(z)} \leq 1 + \frac{S(r,f)}{T(r, F_1(z))}$$

We derive from (1.2) that

$$\sum_{i=1}^{q} \lim_{r\to\infty} inf \frac{m(r, a_i)}{T(r, F_1(z)} + \lim_{r\to\infty} Sup \frac{N\left(r, \dfrac{1}{F_1(z)}\right)}{T(r, F_1(z)}$$

$$\leq 1 + \lim_{r\to\infty} Sup \frac{S(r,f)}{T(r, F_1(z)}$$

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$$\leq 1 + \lim_{r\to\infty} Sup \frac{S(r,f)}{T(r,f)} \frac{T(r,f)}{T(r,F_1(z)}$$
$$= 1$$

Thus,

$$1 \geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} + \sum_{i=1}^{q} \lim_{r\to\infty} inf \frac{m(r,a_i)}{T(r,F_1)}$$

$$1 \geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} + \sum_{i=1}^{q} \lim_{r\to\infty} inf \frac{m(r,a_i)}{T(r,F_1)} \lim_{r\to\infty} \frac{T(r,f)}{T(r,F_1(z)}$$

It follows from (1.2) that

$$1 \geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} + \frac{\sum_{i=1}^{q} \delta(a_i,f)}{(n+s+1) - nn\delta(\infty,f)}$$

Thus, since q is arbitrary, we have

$$\lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} \leq 1 - \frac{\sum_{a\in\phi} \delta(a_i,f)}{(n+s+1) - nn\delta(\infty,f)}$$

For $n = 1$ and $s = 0$ with maximal deficiency sum

$$\lim_{r\to\infty} \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} = 0 \tag{3.4}$$

Now for $n \geq 0$ and $s = 0$ with $\sum_{a\in\phi} \delta(a,f) = 1$ and $\delta(\infty,f) = 1$

$$\lim_{r\to\infty} \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r,F_1(z))} = 0 \tag{3.5}$$

Consequently, we have that the deficiency of $F_1(z)$ with respect to 0 is 1. i.e.,

$$\delta(0, F_1(z)) = 1$$

□

Theorem 1.3 Suppose that $f(z)$ is a transcendental meromorphic function of finite order with maximal deficiency sum. Then

$$\frac{1-\delta(\infty,f)}{[(n+s+1) - n\delta(\infty,f)} \leq \lim_{r\to\infty} Sup \frac{N(r,F_1(z))}{T(r,F_1(z))} \leq \frac{(n+s+1)[1-\delta(\infty,f)]}{\sum_{a\in\phi} \delta(a,f)}$$

(i) For $n = 1$ with maximal deficiency sum, we get

$$\frac{1-\delta(\infty,f)}{(2+s) - \delta(\infty,f)} \leq \lim_{r\to\infty} Sup \frac{N(r,\Delta_q^n(f(z))P(f(qz+c))}{T(r,\Delta_q^n(f(z))P(f(qz+c))} \leq \frac{(2+s)[1-\delta(\infty,f)]}{(2+s) - \delta(\infty,f)}$$

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(ii)For $n \geq 1$ with $\sum\limits_{a\in C} \delta(a, f) = 1$ and $\delta(\infty, f) = 1$,

$$\lim_{r\to\infty} Sup \frac{N(r, \Delta_q^n(f(z))P(f(qz + c))}{T(r, \Delta_q^n(f(z))P(f(qz + c)))} = 0$$

Proof of theorem 1.3

Proof. By using Lemma(2.4), we have

$$N(r, F_1(z)) \leq (n + s + 1)N(r, f) + S(r, f)$$

Which implies that,

$$\frac{N(r, F_1(z))}{T(r, F_1(z))}\frac{T(r, F_1(z))}{T(r, f)} \leq \frac{(n + s + 1)N(r, f)}{T(r, f)} + \frac{S(r, f)}{T(r, f)}$$

Using Theorem (1.1), we have

$$\sum_{a\in\phi} \delta(a, f) \lim_{r\to\infty} Sup \frac{N(r, F_1)}{T(r, F_1)} \leq (n + s + 1)[1 - \delta(\infty, 1)]$$

Hence,

$$\lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1(z))} \leq \frac{(n + s + 1)[1 - \delta(\infty, 1)]}{\sum\limits_{a\in\phi} \delta(a, f)} \tag{3.6}$$

On the other hand, from lemma 2.5, we have

$$N(r, f) \leq N(r, F_1(z))$$

which implies that,

$$\frac{N(r, f)}{T(r, f)} \leq \frac{N(r, F_1(z))}{T(r, F_1(z))}\frac{T(r, F_1(z))}{T(r, f)}$$

Therefore,

$$\lim_{r\to\infty} Sup \frac{N(r, f)}{T(r, f)} \leq \lim_{r\to\infty} Sup \frac{N(r, F_1)}{T(r, F_1)} \lim_{r\to\infty} Sup \frac{T(r, F_1)}{T(r, f)}$$

using Theorem (1.1), we have

$$1 - \delta(\infty, f) \leq \lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1(z))}[(n + s + 1) - n\delta(\infty, f)]$$

$$\frac{1 - \delta(\infty, f)}{[(n + s + 1) - n\delta(\infty, f)} \leq \lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1(z))} \tag{3.7}$$

Thus, from equations 3.6 and 3.7 we get (1.3)

$$\frac{1 - \delta(\infty, f)}{[(n + s + 1) - n\delta(\infty, f)} \leq \lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1(z))} \leq \frac{(n + s + 1)[1 - \delta(\infty, f)]}{\sum\limits_{a\in\phi} \delta(a, f)} \tag{3.8}$$

Now for $n = 1$ with maximal deficiency sum, using 3.4 and 3.8, we get

$$\frac{1 - \delta(\infty, f)}{(2 + s) - \delta(\infty, f)} \leq \lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1(z))} \leq \frac{(2 + s)[1 - \delta(\infty, f)]}{(2 + s) - \delta(\infty, f)}$$

Also for $n \geq 1$ with $\sum\limits_{a\in\phi} \delta(a, f) = 1$ and $\delta(\infty, f) = 1$, using 3.5 in 3.8, we get

$$\lim_{r\to\infty} Sup \frac{N(r, F_1(z))}{T(r, F_1)} = 0$$

□

Theorem 1.4 Suppose that $f(z)$ is a transcendental meromorphic function of finite order with maximal deficiency sum $\delta(\infty, f) = 1$. Then

$$\sum_{a\in C} \delta(a, f) \leq (s+1)\delta(0, \Delta_q^n(f(z))P(f(qz+c)))$$

Proof of Theorem 1.4

Proof. Suppose $\sum_{a\in\phi} \delta(a, f) = 0$, then the result is trivially true.
Next let $\sum_{a\in\phi} \delta(a, f) > 0$
Let $\{a_i\}(i = 1, 2, ..., q)$ be a sequence of distinct complex numbers in $\mathbb{C}$ containing all the finite deficient values of $f(z)$. Proceeding as i the proof of theorem 1.2, we have

$$\sum_{i=1}^{q} m(r, a_i) + N\left(r, \frac{1}{F_1(z)}\right) \leq T(r, F_1(z)) + S(r, f)$$

holds for any q finite complex numbers in $\{a_i\}$.
Therefore, we have

$$\frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r, F_1(z)} + \frac{T(r, f)}{T(r, F_1(z))}\left[\frac{\sum_{i=1}^{q} m(r, a_i)}{T(r, f)} - O(1)\right] \leq 1$$

Hence from (1.2), we can get

$$\lim_{r\to\infty} Sup\left[\frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r, F_1(z))} + \frac{T(r, f)}{T(r, F_1(z))}\left[\frac{\sum_{i=1}^{q} m(r, a_i)}{T(r, f)} - O(1)\right]\right]$$

$$1 \geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r, F_1(z))} + \lim_{r\to\infty} inf \frac{T(r, f)}{T(r, F_1(z))}\left[\frac{\sum_{i=1}^{q} m(r, a_i)}{T(r, f)} - O(1)\right]$$

$$\geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r, F_1(z))} + \lim_{r\to\infty} inf \frac{T(r, f)}{T(r, F_1(z))} \lim_{r\to\infty} inf \frac{\sum_{i=1}^{q} m(r, a_i)}{T(r, f)}$$

$$\geq \lim_{r\to\infty} Sup \frac{N\left(r, \frac{1}{F_1(z)}\right)}{T(r, F_1(z))} + \frac{\sum_{i=1}^{q} \delta(a_i, f)}{(n+s+1) - n\delta(\infty, f)}$$

Since q is arbitrary and $\delta(\infty, f) = 1$, we have

$$\sum_{a\in C} \delta(a, f) \leq (s+1)\delta(0, F_1(z))$$

□

3.1 Conclusion:

Using Valiron Mohonko identity, we have obtained the value distribution of q-shift difference polynomial of meromorphic function with maximum deficiency sum.

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