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Validation of the Navigation Receiver Measurements of An Earth Observation Nanosatellite using Intermediate Orbit Theory

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Abstract - It is important to develop and launch more communications satellites and other practical satellites in line with the requirements of the space science and technology era. The satellite is a combination of modern science and technology and has wide application in national economy. For an artificial earth satellite to perform its mission correctly, it is necessary to design well the orbit and attitude control system, and for this, it is important to acquire a deep knowledge of the dynamics that reflect the motion characteristics of the satellite. In order to fully realize the normal operation of the nanosatellite, it is necessary, first of all, to acquire the orbit of the satellite in real time and with high accuracy.

In this paper, we investigate an on-board orbit determination scheme combining SGP4 model, an analytical orbit determination model widely used in low-orbit satellites, and navigation receiver measurements considering the power consumption and orbit characteristics in nanosatellite. We also proposed a methodology to validate the navigation receiver measurements necessary for the determination of the trajectory on-board and verified its effectiveness.

Keywords - Nanosatellite , Intermediate orbit theory, Earth observation, Orbit, Navigation, Receiver measurement

1. INTRODUCTION

The methods for determining the orbit of a satellite include numerical, analytical and quasi-analytical ones.

The numerical method is a method of determining the orbit of a satellite by numerically integrating the equations of motion involving all the disturbances acting on the satellite, which has the advantage of high computational accuracy but the disadvantage of high computational effort. Analytical orbit determination is a method of determining the orbit of a satellite by approximating the equations of motion to reflect the essential characteristics of motion in a limited time interval, which can be analytically integrable [1].

The quasi-analytical method combines the advantages of numerical and analytical methods, which is very effective in computational time and accuracy. The basic idea of the quasi-analytical method is to determine the orbit by separating the short-period components from the long-period components and the permanent component in the orbital element variation [2]. In this method, the orbit is determined by numerically integrating over a large time step by constructing an equation of motion involving the permanent and long-period components using the mean orbit element. Then, for the short-period component, the motion equation is analytically modeled using Fourier series to perform the calculation within the integration step and determined by combining the long-period component and the permanent component.

If we want to obtain the change of the orbital elements for a time shorter than the integration time, we determine using the function interpolation. A typical example of such a quasi-analytical method is the orbit determination method using the SGP4 model. It is reported that the error of 10 km per day occurs in the case of orbit determination for a 635 km altitude sun-synchronous satellite with the SGP4 model using data provided by the North American Aeronautical Defense Command (NORAD) [2].

If, for some reason, TLE data is not available from the ground station, then the orbit determination accuracy of the satellite cannot be achieved. Hence, a method for determining trajectories autonomously is needed [2].

For on-board orbit determination, the navigation receiver is a very reasonable choice. The validation of these navigation receiver measurements is a primary problem for more accurate orbit determination.

In the past, the evaluation of the navigation receiver's measurement data was focused on the estimation of the range of the orbital components based on the visual representation. However, even the data passed in the range estimation may not satisfy the inherent motion characteristics of the satellite.

This is why the method of calculating the invariants characterizing the motion of a satellite using the intermediate -orbit theory

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and applying statistical techniques based on the data to determine the validity of satellite navigation receiver measurements was established and applied to the orbit. Hence, we have derived a detailed analytical expression to evaluate the effectiveness of satellite navigation receiver measurements by applying the intermediate orbit theory and statistical methods, and proposed a decision scheme, and verified the validity of the proposed scheme by simulation on a nanosatellite.

2. A MOTION INTEGRAL OF SATELLITE IN NON-CENTRAL EARTH GRAVITATION FIELD AND A VALIDATION ALGORITHM OF NAVIGATION RECEIVER MEASUREMENTS

2.1. Intermediate orbit theory and the motion integral of satellite

In the central force field, the orbit of the satellite is elliptic. This is only a rough approximation of the true orbit of a satellite, and to calculate the orbit of an artificial satellite with relatively high accuracy, the effect of disturbances on the basis of elliptical orbits must be taken into account.

The orbit close to the actual is called the intermediate one [3].

In modeling the intermediate orbit, it is usual to make analytical considerations taking into account the influence of the harmonic disturbances.

From the main conclusions of celestial mechanics, only two-body and some special three-body problems can be solved analytically [4].

The motion of a satellite around the earth is a limiting four-body problem considering the effects of the Sun and the Moon, regarding the Earth as a point of matter, which is also not solved analytically, and the equations of motion are more complicated if the effects of the atmosphere and solar light pressure are taken into account.

If we consider the earth as a rotational symmetry and consider only the gravitational action of the earth, we shall be equivalent to a special restricted three-body problem, in which one corresponds to a central force field and the other is a harmonic term perturbation field, which is similar to a field with two fixed centers.

If we consider the earth as a spheroid, the potential function of the satellite is as follows. [4]

$$V = \frac{\mu}{r}\left[1 - \sum_{n\geq 2}\frac{J_n}{r^n} p_n(\sin\varphi)\right] \tag{1}$$

Where,

μ – gravitational constant

r – position of satellite

φ – geocentric latitude of the satellite

J_n – n^{th} order harmonic coefficient

$p_n(x)$ – Legendre polynomial

The kinetic energy of a satellite is

$$T = \frac{1}{2}\left(\dot{r}^2 + r^2\cos^2\varphi\dot{\lambda}^2 + r^2\dot{\varphi}^2\right) \tag{2}$$

Then the corresponding Hamiltonian is

$$H = \frac{1}{2}\left(p_1^2 + \frac{1}{r^2\cos^2\varphi}p_2^2 + \frac{1}{r^2}p_3^2\right) + V(r,\varphi) \tag{3}$$

Here, γ, λ. ψ is the generalized coordinate q_1, q_2, q_3 and is the generalized momentum p_1, p_2, p_3, which is…

$$p_1 = \frac{\partial T}{\partial \dot{r}} = \dot{r},\ p_2 = \frac{\partial T}{\partial \dot{\lambda}} = r^2\cos^2\varphi\dot{\lambda},\ p_3 = \frac{\partial T}{\partial \dot{\varphi}} = r^2\dot{\varphi} \tag{4}$$

Hence, the canonical equation of satellite motion can be written as;

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$$\left.\begin{aligned} \dot{r} &= \frac{\partial H}{\partial p_1}, \dot{p}_1 = -\frac{\partial H}{\partial r} \\ \dot{\lambda} &= \frac{\partial H}{\partial p_2}, \dot{p}_2 = -\frac{\partial H}{\partial \lambda} \\ \dot{\varphi} &= \frac{\partial H}{\partial p_3}, \dot{p}_3 = -\frac{\partial H}{\partial \varphi} \end{aligned}\right\} \quad (5)$$

Since the Hamiltonian H is independent of time 't' and coordinates 'γ', there exist two integrals.

$$H = \alpha_1, p_2 = \alpha_2 \quad (6)$$

α_1, α_2 are the integral constants, which mean the Hamiltonian and angular momentum of the satellite.

From $p_2 = \alpha_2$, we can obtain;

$$\dot{\lambda} = \frac{\partial H}{\partial p_2} = \frac{\alpha_2}{r^2 \cos^2 \varphi}$$

Hence, we have

$$\lambda = \int \frac{\alpha_2}{r^2 \cos^2 \varphi} dt + \alpha_3 \quad (7)$$

α_3 is also an integral constant.

It can be seen that there are three integration constants for the intermediate -orbit satellite. For this potential function, Аксенов approximate the complex potential field with two fixed centers as follows. [5-7]

$$V = \frac{\mu}{2}\left(\frac{1+id}{r_1} - \frac{1-id}{r_2}\right) \quad (8)$$

Here,

$$\left.\begin{aligned} x &= \sqrt{(\xi^2 + c^2)(1-\eta^2)} \cdot \cos \omega \\ y &= \sqrt{(\xi^2 + c^2)(1-\eta^2)} \cdot \sin \omega \\ z &= c\sigma + \xi\eta \end{aligned}\right\} \quad (9)$$

Then the attractive potential function is;

$$V = \frac{\xi - c\sigma\eta}{\xi^2 + c^2\mu^2} \quad (10)$$

Then, by adjusting the constants c and σ, we can make the attractive and radiative potential fields coincide for rotational symmetry. Taking into account the J_3 perturbation at the attractive potential, we can derive the constants c, σ follows :

$$c = R_e \sqrt{J_2 - \left(\frac{J_3}{2J_2}\right)^2} = 209.7290630022\ 334 \quad (11)$$

$$\sigma = \frac{J_3}{2J_2 \sqrt{J_2 - \left(\frac{J_3}{2J_2}\right)^2}} = -3.5571502674\ 694 \quad (12)$$

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Based on the above intermediate orbit theory and intuition, the validation of satellite navigation receiver measurements can be determined based on the range and invariance of the values.

Let us assume that the satellite navigation receiver data received at any time (in the form of years, months, days, hours, minutes and seconds) are given, and the position and velocity in the ECEF are given. The range of the available satellite navigation receiver raw data is defined as Tables 1, 2 and 3 and the validity can be determined.

Table 1. Validation range for calendar-type time

Time	Validation range
Month	1 ~ 12
day	1 ~ 31 (Consider year, mont h)
hour	0 ~ 23
minute	0 ~ 59
second	0 ~ 59

Table 2. Value range of the long radius, eccentricity and orbital tilt angle

partitioning	Effective range
Long radius(km)	6678 ~ 7078
altitude(km)	300~ 700
mean movement (time/day)	14.5 ~15.9
eccentric rate	0 ~ 0.5
orbital tilt angle (°)	87 ~ 107

Table 3. Value range of velocity vector

velocity vector magnitude	Effective range
V(km/s)	6.5 ~ 9.0

Next, we calculate invariants for the measured data that are evaluated to be valid in the range estimation.

2.2. Calculation of invariants of satellite navigation receiver measurements

First, we calculate the satellite system time from the time t_i in the hourly-second form of the year and then the stellar time ST_i from the time of the satellite system.

Calculate the position and velocity of the satellite in the geocentric inertial coordinate system.

$$\begin{aligned} x_i &= x_{gi}\cos(ST_i) - y_{gi}\sin(ST_i) \\ y_i &= x_{gi}\sin(ST_i) + y_{gi}\cos(ST_i) \\ z_i &= z_{gi} \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{x}_i &= \dot{x}_{gi}\cos(ST_i) - \dot{y}_{gi}\sin(ST_i) \\ \dot{y}_i &= \dot{x}_{gi}\sin(ST_i) + \dot{y}_{gi}\cos(ST_i) \\ \dot{z}_i &= \dot{z}_{gi} \end{aligned} \tag{14}$$

For the obtained velocities, we consider the earth rotation.

$$\begin{aligned} \dot{x}_i &\leftarrow \dot{x}_i - \omega_e y_i \\ \dot{y}_i &\leftarrow \dot{y}_i + \omega_e x_i \end{aligned} \tag{15}$$

Here, $\omega_e = 7.2921151467 \cdot 10^{-5}$ rad/s

Work out generalized coordinates ξ_i, η_i for all data.

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We find preliminary quantities to determine ξ, η.

$$\bar{z} = z - c\sigma$$
$$\bar{r}^2 = x^2 + y^2 + \bar{z}^2$$

We find that $\xi' = \xi^2$ as follows;

$$A' = c^2 - \bar{r}^2$$
$$B' = c^2 \cdot \bar{z}^2$$
$$\xi' = \frac{-A' + \sqrt{A'^2 + 4B'}}{2}$$

When $|\bar{z}| < 10$ km , we calculate $\xi' = \xi^2$ as follows;

$$\xi' \approx |A'|$$

Get ξ, η.

$$\xi = \sqrt{\xi'}, \quad \eta = \bar{z} / \xi \tag{16}$$

Calculate invariants $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}$ for all data.

$$\left.\begin{aligned} \bar{r}^2 &= x^2 + y^2 + (z - c\sigma)^2 \\ \dot{\bar{r}} &= x\dot{x} + y\dot{y} + (z - c\sigma)\dot{z} \\ Q &= \frac{2GM_e\xi\eta(c^2\eta + c\sigma\xi)}{J} \end{aligned}\right\} \tag{17}$$

$$J = \xi^2 + c^2\eta^2, \quad V^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2, \quad U' = \frac{GM_e(\xi - c\sigma\eta)}{J}$$

$$\alpha_1 = -U' + V^2/2, \quad \alpha_2 = \bar{r}^2V^2 - \dot{\bar{r}}^2 - c^2\dot{z}^2 + Q, \quad \alpha_3 = x\dot{y} - y\dot{x}$$

Since $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}$ are about 29, 10^9, and 7000, respectively, we use the following multipliers to handle the quantity of order 1.

$$\left.\begin{aligned} \alpha_1 &= \alpha_1 \cdot 0.1 \\ \alpha_2 &= \alpha_2 \cdot 10^{-9} \\ \alpha_3 &= \alpha_3 \cdot 10^{-3} \end{aligned}\right\} \tag{18}$$

2.3 Efficiency evaluation algorithm

Based on the calculation of these invariances, following validation algorithms can be accepted.

First compute the mean invariant.

$$\bar{\alpha}_1 = \frac{\sum_{i=1}^{n} \alpha_{1i}}{n}, \quad \bar{\alpha}_2 = \frac{\sum_{i=1}^{n} \alpha_{2i}}{n}, \quad \bar{\alpha}_3 = \frac{\sum_{i=1}^{n} \alpha_{3i}}{n}$$

Then, find the relative deviations of the invariants.

$$\alpha'_{1i} = \alpha_{1i} - \bar{\alpha}_1, \qquad i = \overline{1, n}$$
$$\alpha'_{2i} = \alpha_{2i} - \bar{\alpha}_2, \qquad i = \overline{1, n}$$
$$\alpha'_{3i} = \alpha_{3i} - \bar{\alpha}_3, \qquad i = \overline{1, n}$$

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Find the standard deviation from the relative deviations.

$$RMS_1 = \sqrt{\frac{1}{n-1}\sum_i (\alpha'_{1i})^2}, \qquad i = \overline{1,n}$$

$$RMS_2 = \sqrt{\frac{1}{n-1}\sum_i (\alpha'_{2i})^2}, \qquad i = \overline{1,n}$$

$$RMS_3 = \sqrt{\frac{1}{n-1}\sum_i (\alpha'_{3i})^2}, \qquad i = \overline{1,n}$$

If the relative deviations of the first, second, and third invariants $\alpha'_{1n}, \alpha'_{2n}, \alpha'_{3n}$ satisfy the following conditions, we judge by valid data, otherwise we judge by not valid data.

$$|\alpha'_{1n}| < 3 \cdot RMS_1, \quad |\alpha'_{2n}| < 3 \cdot RMS_2, \quad |\alpha'_{3n}| < 3 \cdot RMS_3$$

3. ANALYSIS AND RESULTS

The simulation was performed on earth observation nanosatellite. To simulate satellite navigation receiver data, the orbital elements obtained by high-precision orbit propagation (HPOP) model on a sun synchronous orbit with repeating period of 5 days were set as a reference and considered position and velocity deviation of receiver as $\sigma_r = 25m, \sigma_v = 0.12m/s$ respectively.

And for 1% of the data, we added singular noise. The calculations were performed with 60 samples in the expected and variance calculations of the invariants. The results of the simulation analysis based on the aforementioned decision algorithm are shown in Figures 1, 2, 3 and 4.

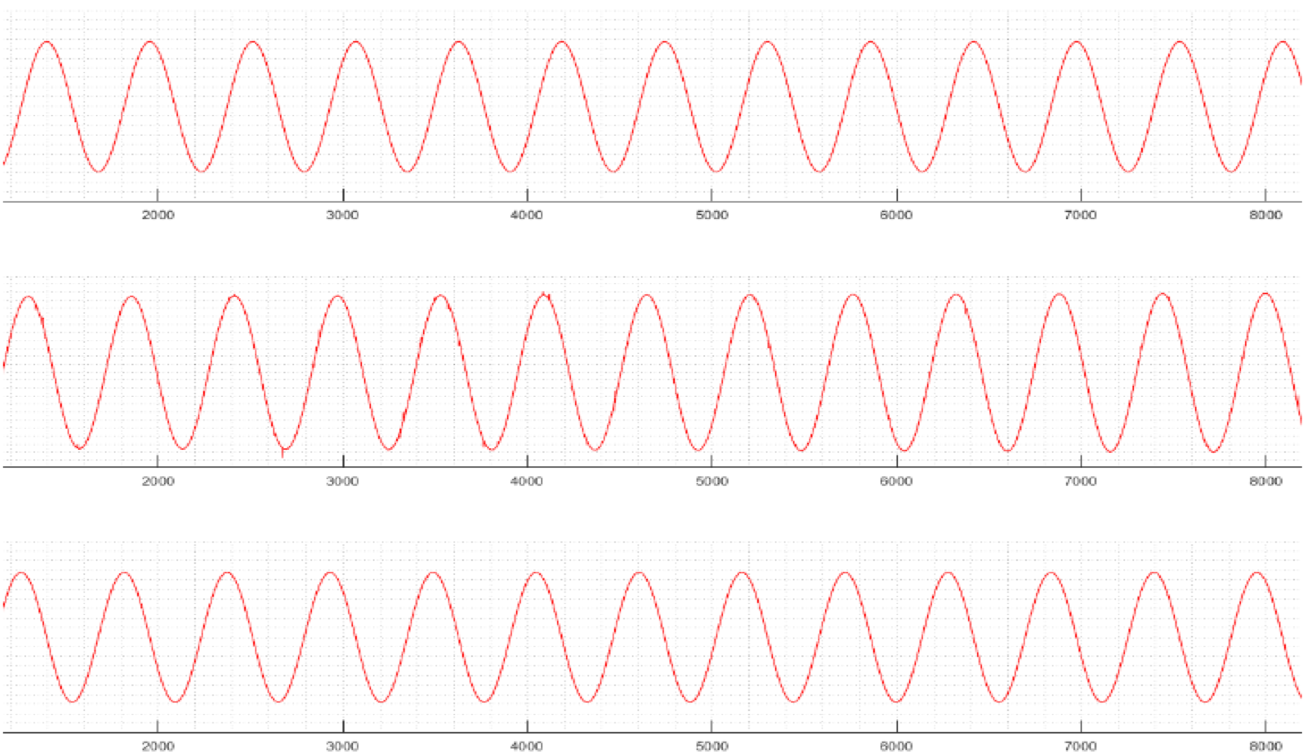

Figure 1. Satellite position measured by satellite navigation receiver

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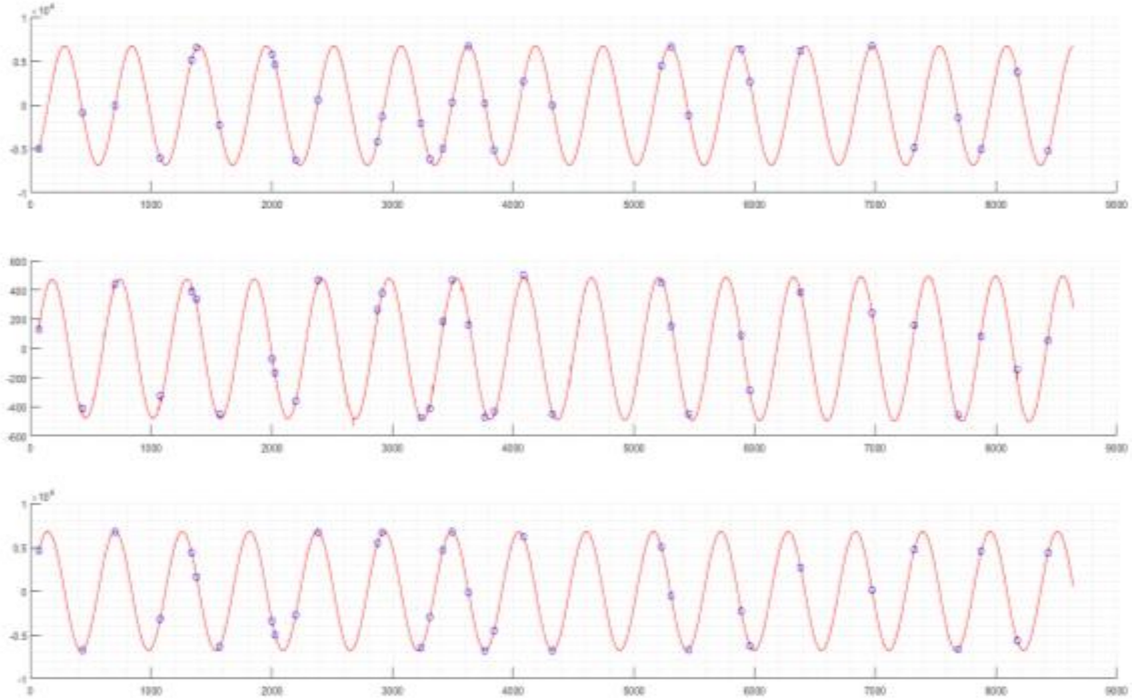

Figure 2. Value range estimation of navigation receiver measurement data

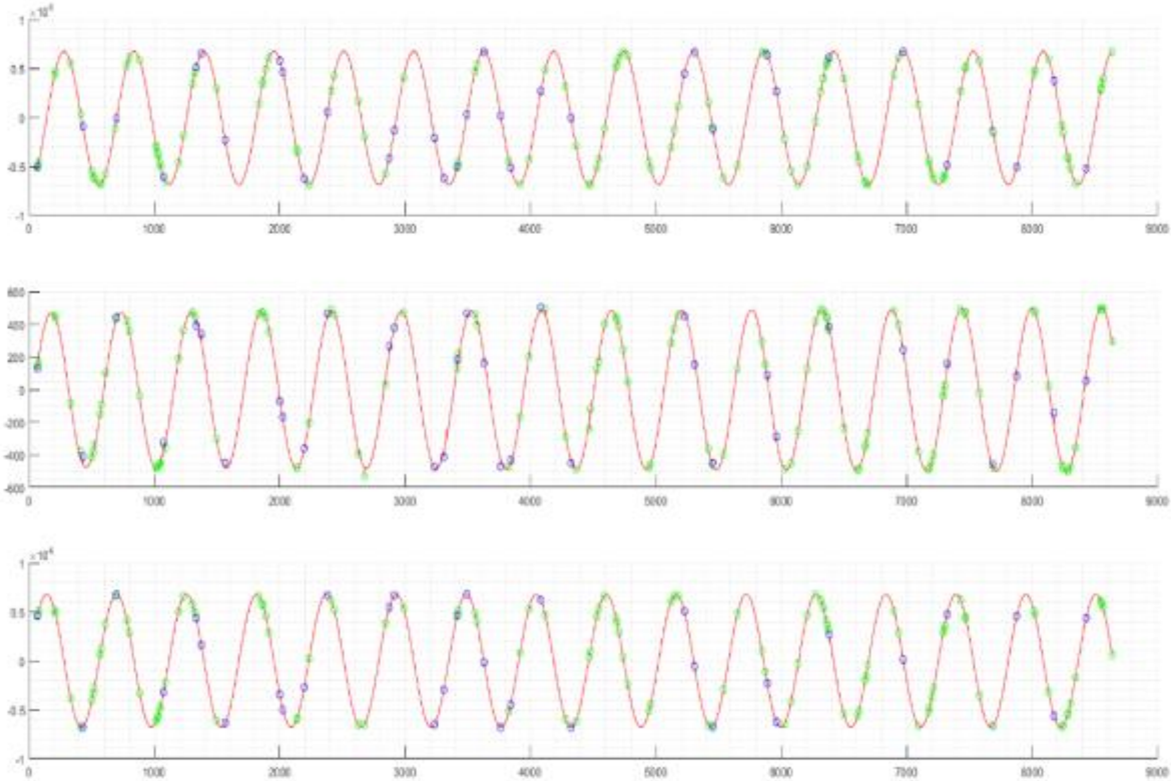

Figure 3. Results of simultaneous evaluation of range and invariants

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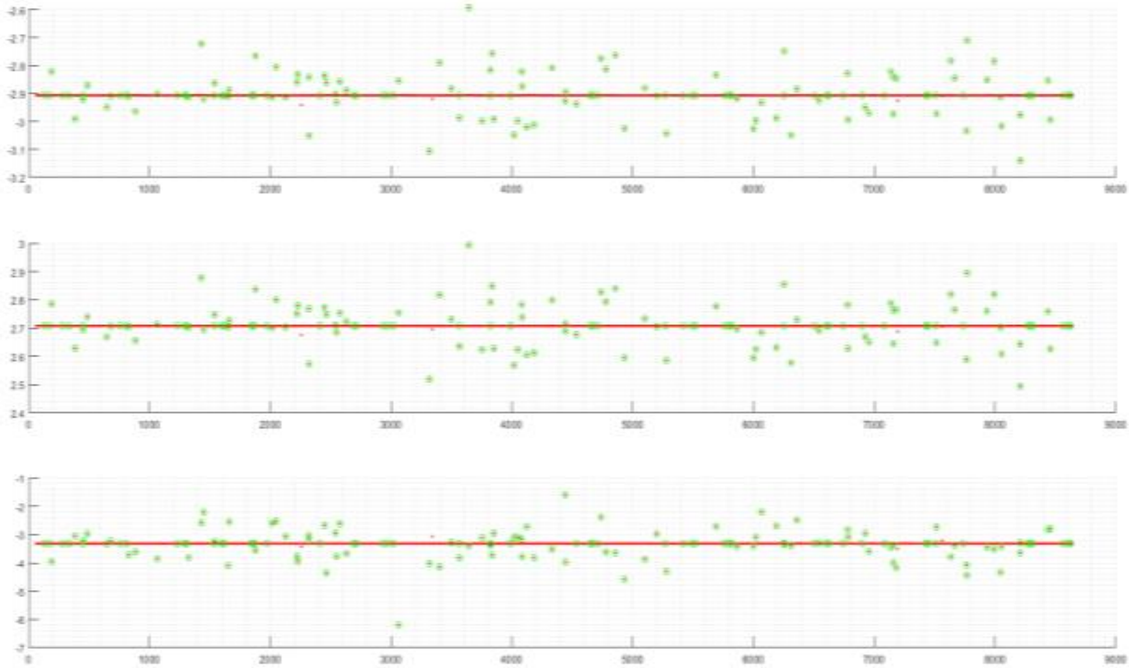

Figure 4. Criteria for invariants

As can be seen from the simulation results, the proposed method can only output valid data by evaluating all the singular data.

4. CONCLUSION

In this paper, we propose a method for validation of satellite navigation receiver measurements by applying the intermediate - orbit theory and statistical methods on an earth observation nanosatellite. The method of verification, which was limited to the estimation of the range of values in the past, was applied to the determination of three invariants in the medium gravity field, so as to further improve the validity of the data, thus increasing the accuracy of the orbit determination system.

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CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

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