International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 8, August - 2013

Up–Stroke Theory

Vassil V. Kojouharov
Pleven, Bulgaria

Abstract

The subject of the present article is a flapping-wing aircraft. The article deals with the behavior of an aircraft with rigid, flat flapping wings in the mode of a vertical flight. The flight of such a craft is leap-like, similar to that of some species of butterflies. An original approach for analysis of the main problem of an aircraft of such type was found - the performance of the up-stroke. By combining of the wing angular speed with the aircraft free fall speed was derived a law of the wing motion which guarantees that the up-stroke will be performed in the fastest possible manner without obstruction of the aerodynamic law. The paper defines the behavior of the craft during the performance of one operation cycle. Mathematical relationship which allows defining of the hover conditions was derived.

1. INTRODUCTION

This article is an attempt to analyze the possibilities for vertical take-off device with flapping wings of the simplest design. In flapping-wing flight all the work required to hold the craft in the air is performed during the down-stroke. In general a part of this performance is used to reach and maintain the horizontal velocity and the remaining part to maintain the flight height. As a result two forces act on the corpus during the down-stroke. They both are components of the aerodynamic wing force and they are a horizontal component / trust/ and a vertical one / lift/. The presence of horizontal velocity allows the up-stroke to be implemented fast enough to generate a lift due to reducing of the horizontal velocity, which value is restored during the next cycle. For this purpose, it is necessary that the wing is twisted so that the angle of attack to be within certain limits throughout its length. The twisting should be changed depending on the speed of the craft and the angular velocity of the wing. This means that the wing must be flexible and its geometry can be operated by changing according to the flight parameters. Such wings can hardly be attained. Flexible wings which shape changes under the action of the air flow are used.

Therefore in the wing operation surface appears an area upon which the air flow acts reversely to the desired direction [1]. As a result, instead of producing a lift, part of the engine is spent to provide a force, which pulls the aircraft downwards, which makes the flight ineffective.The existing aircraft with flapping wings are unable to take off individually and require horizontal acceleration. When the craft is miniature, the horizontal acceleration is done manually. Larger devices are accelerating by means of another vehicle to acquire sufficient horizontal speed. Self-reinforcing on track is possible too, but it is associated with dangerous jumps and hits of the craft to the track [2].

This helplessness in the starting period could be avoided if the craft is able to take off vertically and gradually pass in horizontal flight mode. The vertical take-off unit can not rely on the generation of lift, produced by the wing of wrapping horizontal airflow. This means that the aerodynamic profile which is similar to the profile of an airplane wing is not necessary. In our case, we assume that the wing is flat and has a rectangular shape with a horizontal axis of rotation.

During flight along the vertical / without the horizontal motion / the horizontal component of the aerodynamic force is equal to zero, and the complete performance, accomplished during the down-stroke is used to produce a lift. The value of the aerodynamic force will depend on the area of the wing and its angular velocity .The wing deformation under the action of the air flow is not desirable because it would reduce the effective area of the wing, and hence reduce the aerodynamic force generated by the down-stroke.

Therefore we will assume that the wing is strong enough and is practically rigid. The above speculation lead to the idea of a vertical take-off craft with flat and rigid flapping wings with a horizontal axis of rotation, the behavior of which we will try to analyze.

The flight of such equipment will not resemble the flight of birds, and would be like the jumping flight of butterflies. Within the limits of one operation cycle, the aerodynamic force, acting during the down- stroke, should be able to provide possibility of the craft to reach the end cycle height, equal or more than the height in the beginning of the cycle. A craft complying to these conditions could accomplish a vertical flight without the necessity of prior acceleration along the runway aimed to gain horizontal velocity. The references lack publications about a similar craft. This is probably due to the fact, that the flight of a similar craft would be leap-like, with great amplitude of strokes and considerable accelerations which make it unattractive as transport means. Despite that there are at least two reasons to work in this direction. On the one hand, analyzing such type of flight there could be found regularities which are valid of any flapping-wing flight including the bird flight. On the other hand, such flight type can be considered as the start mode of a more complicated craft, which after gaining height by several strokes, could switch into another mode, suitable for horizontal direction flights.

If we admit, that we have an engine with the required power and strong enough wings, the problem with creating of an aircraft of the above type is reduced to the discovery of a law for wing drive to guarantee the optimal engine energy utilization to get a lift.

During a downward wing stroke with such flight the air cowl is directed to the bottom surface of the wing. It is obvious that by increasing the rotation speed the obtained lift will increase. This means that the wings should be driven by the maximum velocity, defined by the construction strength and the power of the drive unit. There remains the problem how to drive the wings during the up-stroke in order that the aerodynamic requirements are completely observed.

2. UP-STROKE ANLYSIS

To analyze the wing motion we will accept the following prerequisites:

1. The craft with weight G has a corpus 1 and two flat rectangular non-elastic wings 2 with length R and width B(fig1).
2. The center of gravity of the complete craft lies on the corpus axis at a distance 1/2B of the wing edge, positioned in U point.
3. The craft is driven in a leap-like manner along the vertical line (there is no horizontal motion).
4. The longitudinal axis of the craft keeps horizontal position during the whole cycle.
5. After one cycle performance the craft resumes again initial position, i.e. the craft is held in the air without gaining or losing height.

If the craft corpus is still, at downward wing stroke a lift will be produced. The lift will strive to hold the craft in the air. The lift value will depend on length R and the width B as well as on the angular velocity of the wing ω. The wing efficiency will depend on lift size and the duration of its action.

During the up-stroke, if the angular velocity is not very small, however, there will appear a force, with a direction reverse to that of the lift. In this case the height gained during the down-stroke will be lost during the up-stroke. In order that the effect of the lift is maintained, the up-stroke of the wings should be combined in a certain manner to the motion of the craft center of gravity.

Let us admit that the craft rises upwards under the action of the lift and after termination of this action the rise continues under the action of the inertia to reach the highest point of its trajectory in which the vertical speed of the craft is zero. From this point on the craft will start falling under the action of the gravitational acceleration. This is the moment when the up-stroke should start.

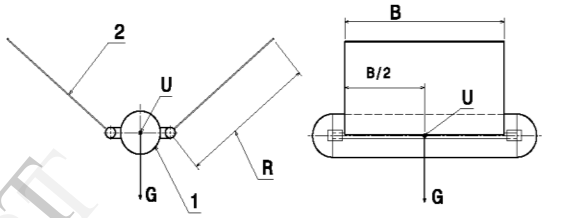

Fig 1: Diagram of a flat rigid flapping wings craft

The resulting force which pulls the craft downwards can be avoided if the up-stroke is performed slowly enough, but this will lead to a great height loss which could be resumed with difficulty for the next stroke. It is necessary to find a way for performance of the up-stroke with the smallest possible height loss, respectively for the shortest time. Two different approaches to solve the above problem are possible:

- Up-stroke with lift.

This version requires performing of rotation in the fastest possible manner in which the air flow is constantly directed against the bottom wing surface.

- Up-stroke with balanced torque.

With this version the up-stroke is performed with a greater velocity. As a result in the opposite end of the rotation axis of the wing appears an area upon which the air flow is directed against its top surface. The stroke should be performed in such a manner that all the time the momentum of the force acting upon the bottom surface of the wing against the rotation axis should be equal in value to the momentum of the force acting upon its top surface. The equalization of both moments allows stroke performance without obtaining a force which pulls the craft downwards. We will discuss both version in a sequence.

2.1 UP-STROKE WITH LIFT

If we admit that in moment t of the free fall of the craft its velocity is $V_t = gt$, and the angle between the wing and the vertical axis is θ_t (fig 2). On the diagram of fig 2 point U shows the gravity center of the craft and point O shows the rotation axis, $OO^/$ shows the wing length. The absolute velocity of any point located on the wing $OO^/$ is equal to the sum of the fall velocity V_t and the angular speed $V_r = \omega_t r$ $(0 < r < R)$.

By increasing the distance from the rotation center O, the angular velocity V_r increases proportionally to the obtained radius, while the vertical velocity V_t is a constant value for all points. The distance $OO^/$ is divided into several parts by the points O_1, O_2, O_3 and O_4 and the absolute velocities V_1, V_2, V_3 and V_4 are defined in each point. Let us admit that the angle φ between the absolute velocity vector and $OO^/$ is positive, when the velocity vector is situated under $OO^/$. It can be seen in the figure that in point O this angle is positive ($\varphi_0 = \theta_t$) and by increasing the distance from this point the angle decreases gradually in value and in point O_4 it is negative. In this case between point 3 and point 4 there exists point N in which the angle φ_N is equal to zero i.e. the direction of the absolute velocity coincides with $OO^/$.

As the velocity of the air flow in each point has a direction which is reverse to the absolute velocity V, in the part ON there will arise a lift and in the part $NO^/$ the force will be negative and its vertical component will pull the craft downwards. This comes to show that the existence of the part $NO^/$ leads to the consumption of energy which creates power, preventing the flight of the craft i.e. to an excessive loss of energy. To discuss the fall velocity V_t we could select such values of the angle velocity ω_t so that point N that coincide with $O^/$. Thus the parasite area will be eliminated for the considered moment of the craft free fall. As the fall velocity V_t constantly changes, in order to observe the above condition in every fall moment it is necessary that the angular velocity to be a variable which is a function of V_t.This condition is the ground on which we will formulate the up- stroke wing motion law. On fig 3 is shown an aircraft with wing length $O_1O_1' = R$ and rotation of axis O_1. To simplify the example it is admitted that the rotation axis coincides with the craft

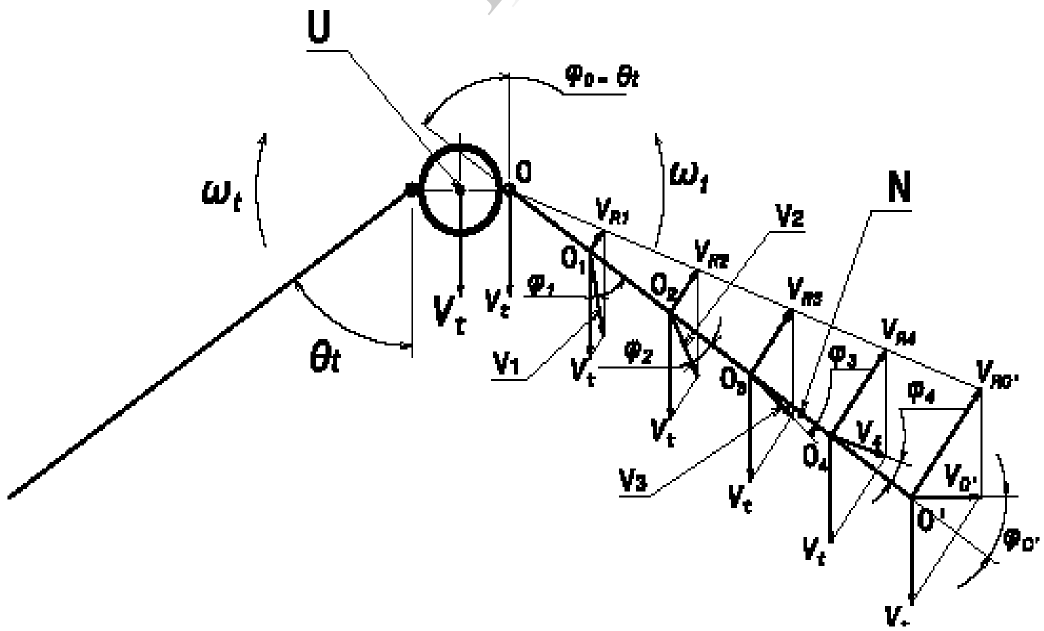

Fig2: Distribution of velocities during up-stroke performance

corpus axis.

During the free fall the gravity center of the craft moves from point O_1 to point O_2. We admit that in top and bottom limit position the wing makes an angle with the line O_1O_2 which is θ_0. During the up-stroke performance the limit point of the wing $O_1^{/}$ moves along the curve $O_1^{/}\,M\,O_2^{/}$, which we will designate as $y = f(x)$. We are going to discuss the wing in intermediate position $O_n O_n^{/}$. In order to cover the condition for wing motion without downward directed force it is necessary in each moment of the motion the direction of the absolute velocity in point $O_n^{/}$ to coincide with the direction of $O_n O_n^{/}$ in the same moment. This means that in each moment the segment $O_n O_n^{/}$ will tangent in point $O_n^{/}$ to the curve $y = f(x)$. In this case the curve equation achieves the type of:

$$-\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{Tan}\theta_t}$$

or : $\mathrm{d}y = -\dfrac{\mathrm{d}x}{\mathrm{Tan}\theta_t}$ or : $y = -\displaystyle\int \frac{\mathrm{d}x}{\mathrm{Tan}\theta_t}$

as $\mathrm{Sin}\theta_t = \dfrac{x}{\mathrm{R}}$ and $\mathrm{Cos}\theta_t = \sqrt{1-\dfrac{x^2}{\mathrm{R}^2}}$, so

$$\frac{1}{\mathrm{Tan}\theta_t} = \frac{\mathrm{Cos}\theta_t}{\mathrm{Sin}\theta_t} = \sqrt{1-\frac{x^2}{\mathrm{R}^2}}.\frac{\mathrm{R}}{x} = \frac{\sqrt{\mathrm{R}^2 - x^2}}{x}$$

After substitution the received value for $\dfrac{1}{\mathrm{Tan}\theta_t}$

we get: $y = -\displaystyle\int \frac{\sqrt{\mathrm{R}^2 - x^2}}{x}\,\mathrm{d}x$

To solve the integral we set : $x = \mathrm{RSin}u$

Then $\mathrm{d}x = \mathrm{RCos}u\,\mathrm{d}u$ and $u = \mathrm{ASin}\dfrac{x}{\mathrm{R}} = \theta_t$

$$y = -\int \frac{\sqrt{\mathrm{R}^2 - x^2}}{x}\mathrm{d}x =$$

$$= -\int \sqrt{\frac{\mathrm{R}^2(1-\mathrm{Sin}^2u)\mathrm{RCos}u}{\mathrm{RSin}u}}\mathrm{d}u = -\mathrm{R}\int \frac{\mathrm{Cos}^2u}{\mathrm{Sin}u}\mathrm{d}u =$$

$$= -\mathrm{R}\int \frac{1-\mathrm{Sin}^2u}{\mathrm{Sin}u}\mathrm{d}u = -\mathrm{R}\left(\int \frac{\mathrm{d}u}{\mathrm{Sin}u} - \int \mathrm{Sin}u\mathrm{d}u\right) =$$

$$= -\mathrm{R}(\mathrm{LnTan}\frac{u}{2} + \mathrm{Cos}u + \mathrm{C}) =$$

$$= -\mathrm{R}(\mathrm{LnTan}\frac{\theta_t}{2} + \mathrm{Cos}\theta_t + \mathrm{C})$$

With $x = \mathrm{R}$ $\quad \theta_t = 90^0$ $\quad y = 0$ $\quad \longrightarrow \quad \mathrm{C} = 0$

Then $y = -\mathrm{R}(\mathrm{Cos}\theta_t + \mathrm{LnTan}\dfrac{\theta_t}{2})$ $\quad \ldots(1)$

For the path values $h_1 = \mathrm{O_1O_2}$ and the time of the craft free fall t_1, and its velocity in the limit point O_2 we get: $h_1 = 2(\mathrm{RCos}\theta_0 + y_0)$

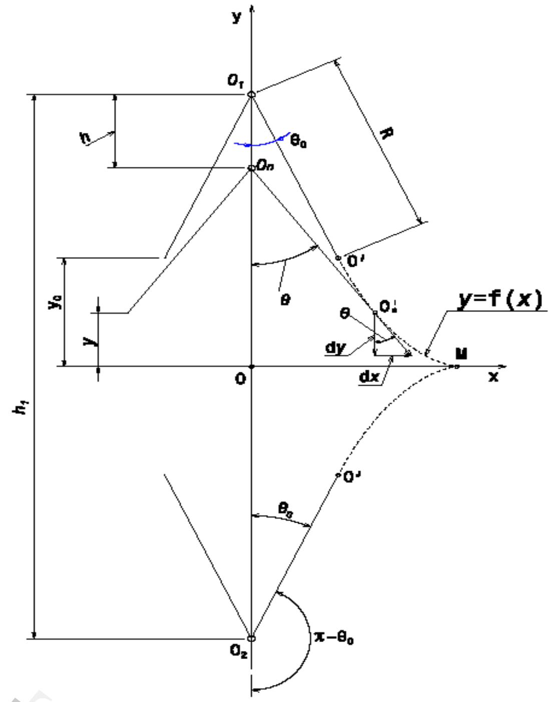

Fig 3: Diagram of wing motion during performance of the up-stroke

As $y_0 = -\mathrm{R}(\mathrm{Cos}\theta_0 + \mathrm{LnTan}\dfrac{\theta_0}{2})$, so:

$h_1 = 2[\mathrm{RCos}\theta_0 - \mathrm{R}(\mathrm{Cos}\theta_0 + \mathrm{LnTan}\dfrac{\theta_0}{2})]$, or

$h_1 = -2\mathrm{RLnTan}\dfrac{\theta_0}{2}$ $\quad \ldots(2)$

As $h_1 = \dfrac{\mathrm{g}t_1^2}{2}$, then $t_1 = \sqrt{\dfrac{2h_1}{\mathrm{g}}}$

So $t_1 = 2\sqrt{-\dfrac{\mathrm{R}}{\mathrm{g}}\mathrm{LnTan}\dfrac{\theta_0}{2}}$ $\quad \ldots..(3)$

The craft velocity in the end of the free fall i. e in point 2 is: $V_2 = \mathrm{g}t_1$ $\quad \ldots.(4)$

Or: $V_2 = 2\sqrt{-\mathrm{RgLnTan}\dfrac{\theta_0}{2}}$ $\quad \ldots(5)$

From fig 3 we could build the equation:

$$h + \mathrm{RCos}\theta_t + y = \mathrm{RCos}\theta_0 + y_0$$

After substitution of y and y_0 with the expression of formula (1) we get:

$$h + \mathrm{RCos}\theta_t - \mathrm{R}(\mathrm{Cos}\theta_t + \mathrm{LnTan}\frac{\theta_t}{2}) =$$

$$= \mathrm{RCos}\theta_0 - \mathrm{R}(\mathrm{Cos}\theta_0 + \mathrm{LnTan}\frac{\theta_0}{2})$$

Of which after reworking we get:

$$\theta_t = 2\mathrm{ATan}(e^{\frac{h}{R}}\mathrm{Tan}\frac{\theta_0}{2}) \quad \ldots(6)$$

To get the value of θ_t related to time t, we substitute $h = \frac{gt^2}{2}$ and get:

$$\theta_t = 2\mathrm{ATan}(e^{\frac{gt^2}{2R}}\mathrm{Tan}\frac{\theta_0}{2}) \quad \ldots.(7)$$

Formulae (6) and (7) represent the wing motion law, providing correct performance of the up-stroke with lift and minimum loss of height.

2.2 UP STROKE WITH BALANSED TORQUE

The performance of the up-stroke also begins with reaching the highest point in the flight trajectory. The craft starts falling. In moment of the free fall the craft has velocity $V_t = gt$. The wing has angular velocity ω_t and angle to the vertical θ_t. The velocity distribution along the wing is shown on the fig 4. The air flow velocities at both sides of the wing have opposite directions. The forces which these flows exercise on the wing are proportional to the squares of the respective velocities. The force by which the air flow acts on the bottom surface of the wing will be designated by F_t and the force on the top surface by F_r. For the forces F_t and F_r we can write $F_t = cBR\mathrm{Sin}\theta_t g^2 t^2$ and

$$F_{tN} = F_t \mathrm{Sin}\theta_t = cBR\mathrm{Sin}^2\theta_t g^2 t^2$$

$$dF_r = cB\omega_t^2 r^2 dr$$

$$F_r = cB\omega_t^2 \int_0^R r^2 dr = \frac{cB\omega_t^2 R^3}{3}$$

or: $$F_r = \frac{cB\omega_t^2 R^3}{3} \quad \ldots\ldots\ldots (8)$$

Where c is a proportionality coefficient reading the air resistance. To get zero torque the following condition should be observed:

$$F_{tN} \cdot \frac{R}{2} = F_r \cdot \frac{3R}{4}$$,or:

$$cBR\mathrm{Sin}^2\theta_t . g^2 . t^2 . \frac{R}{2} = \frac{cB\omega_t^2 R^3}{3} . \frac{3R}{4}$$

Or: $\omega_t^2 R^2 = 2g^2 t^2 \mathrm{Sin}^2\theta_t$ and

$\omega_t R = gt\sqrt{2}\mathrm{Sin}\theta_t$ As

$\omega_t = \frac{d\theta_t}{dt}$, then $\frac{d\theta_t}{dt} R = gt\sqrt{2}\mathrm{Sin}\theta_t$,or:

$$\frac{d\theta_t}{\mathrm{Sin}\theta_t} = \frac{g\sqrt{2}}{R} t dt$$

Upon integration, we get:

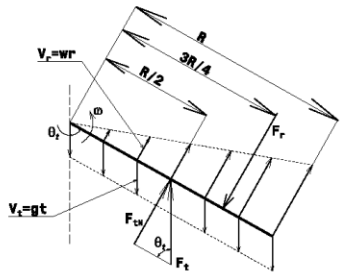

Fig 4: Diagram of the equilibrium of aerodynamic forces at up-stroke with balanced torque

$$\mathrm{LnTan}\frac{\theta_t}{2} = \frac{g\sqrt{2}}{R} . \frac{t^2}{2} + C$$

At $t = 0$ $\theta_t = \theta_0$ we get:

$C = \mathrm{LnTan}\frac{\theta_0}{2}$, or:

$$\mathrm{LnTan}\frac{\theta_t}{2} = \frac{g\sqrt{2}}{R}\frac{t^2}{2} + \mathrm{LnTan}\frac{\theta_0}{2}$$ and

$$\mathrm{LnTan}\frac{\theta_t}{2} - \mathrm{LnTan}\frac{\theta_0}{2} = \frac{g\sqrt{2}}{R} . \frac{t^2}{2}$$

Or: $$\mathrm{Ln}\frac{\mathrm{Tan}\frac{\theta_t}{2}}{\mathrm{Tan}\frac{\theta_0}{2}} = \frac{gt^2}{R\sqrt{2}} \quad \ldots (9)$$

Upon rearrangement we get:

$$\theta_t = 2\mathrm{ATan}(e^{\frac{gt^2}{R\sqrt{2}}}\mathrm{Tan}\frac{\theta_0}{2}) \quad \ldots.(10)$$

As $h = \frac{gt^2}{2}$ or $t^2 = \frac{2h}{g}$

after substitution in (9) we get:

$$\theta_t = 2\mathrm{ATan}(e^{\frac{h\sqrt{2}}{R}}\mathrm{Tan}\frac{\theta_0}{2}) \quad \ldots\ldots\ldots(11)$$

where θ_t is a function of the path.

For the angular velocity ω_t we get: $\omega_t = \frac{d\theta_t}{dt} =$

$$= 2\frac{1}{1 + (e^{\frac{gt^2}{R\sqrt{2}}}\mathrm{Tan}\frac{\theta_0}{2})^2}\mathrm{Tan}\frac{\theta_0}{2} e^{\frac{gt^2}{R\sqrt{2}}} \frac{g2t}{R\sqrt{2}}$$

$$\omega_t = \frac{2\sqrt{2}gt\mathrm{Tan}\dfrac{\theta_0}{2}\, e^{\frac{gt^2}{R\sqrt{2}}}}{R[1+(e^{\frac{gt^2}{R\sqrt{2}}}\mathrm{Tan}\dfrac{\theta_0}{2})^2]} \qquad \text{….(12)}$$

From equation (9) we can get: $t = \sqrt{\dfrac{R\sqrt{2}}{g}\mathrm{Ln}\dfrac{\mathrm{Tan}\dfrac{\theta_t}{2}}{\mathrm{Tan}\dfrac{\theta_0}{2}}}$

At $\theta_t = \pi - \theta_0$, $t = t_1$, we get:

$$t_1 = \sqrt{\frac{R\sqrt{2}}{g}\mathrm{Ln}\frac{\mathrm{Tan}\dfrac{\pi-\theta_0}{2}}{\mathrm{Tan}\dfrac{\theta_0}{2}}}$$

and after rearrangement:

$$t_1 = \sqrt{\frac{R\sqrt{2}}{g}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\dfrac{\theta_0}{2}}} \qquad \text{…(13)}$$

where t_1 is the time for performing the up-stroke. For the path h_1 and the velocity in the limit point O we get:

$$h_1 = \frac{gt_1^2}{2} = \frac{g}{2}.\frac{R\sqrt{2}}{g}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\dfrac{\theta_0}{2}} \qquad \text{or:}$$

$$h_1 = \frac{R}{\sqrt{2}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\dfrac{\theta_0}{2}} \qquad \text{……(14)}$$

$$V_2 = gt_1 = g\sqrt{\frac{R\sqrt{2}}{g}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\dfrac{\theta_0}{2}}} \qquad \text{or}$$

$$V_2 = \sqrt{gR\sqrt{2}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\dfrac{\theta_0}{2}}} \qquad \text{…..(15)}$$

To obtain acceptable values of t_1 it is necessary to select a wing with a comparatively less length R at which the provision of the necessary for the craft lift could be accomplished on behalf of the greater wing width B. Equations (7) and (10) can be used only for prior programming of the wing motion because they do not account the air resistance at craft fall. During tests the required corrections have to be made in order that the up-stroke performance could be accomplished in accordance to equations (6) or (11). This programming will be done once as the craft fall at every cycle will be performed in the same manner regardless of the height gained. Due to the fact that balanced torque version offers better conditions for the up-stroke performance, it will serve as basis of the further analysis of the flight operation cycle.

3. ANALYSIS OF CRAFT OPERATION CYCLE

After completing of the up-stroke starts the downward wing stroke. During this stroke under the action of the lift $P > G$ the craft will initially continue to fall until its vertical velocity equals zero, and then it will start to rise. After completing of the stroke the craft will continue to move upwards by the inertia. The vertical velocity which is achieved during the stroke should be enough so that the end of the cycle the craft could reach initial position.
The operation cycle could be divided into four stages (fig 5):

Stage I – free fall from point 1 to point 3 (up-stroke) During the fist stage the craft travels path h_1 and its velocity from zero in point 1 reaches V_2 in point 2. The path, time t_1 and velocity V_2 are defined in formulae (13),(14) and (15).
Stage II – motion from point 2 to point 3 at the action of the lift *P*, produced during the down- stroke. Two forces act on the craft – the weight G and the lift *P*.

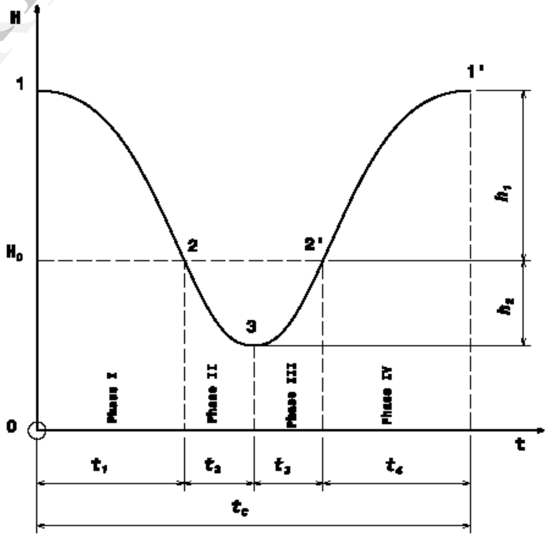

Fig 5: Craft operation cycle diagram

We admit that the lift value is a constant during the down-stroke. Then:

$$P - G = m.a$$

where a is acceleration and m- the craft mass.

Or: $$a = g(\frac{P}{G} - 1) \qquad \text{….(16)}$$

For time t_2 the craft velocity from V_2 in point 2 should become equal to zero in point 3. For the velocity in the segment from point 2 to pint 3 we have:

$$V_t = \mathrm{g}t_1 - at \ \text{, or } V_t = \mathrm{g}t_1 - \mathrm{g}(\frac{P}{\mathrm{G}} - 1)t$$

At $t = t_2$, $V_t = 0$. Then: $\mathrm{g}t_1 - \mathrm{g}(\frac{P}{\mathrm{G}} - 1)t_2 = 0$,

or: $t_2 = \dfrac{t_1}{\frac{P}{\mathrm{G}} - 1}$, or:

$$t_2 = \frac{1}{\frac{P}{\mathrm{G}} - 1}\sqrt{\frac{\mathrm{R}\sqrt{2}}{\mathrm{g}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \qquad \text{…(17)}$$

For path h_2 from point 2 to point 3 we have:

$h_2 = \dfrac{at_2^2}{2}$, or after substitution of t_2 and a from (16) and (17) and reworking:

$$h_2 = \frac{\mathrm{R}}{\sqrt{2}(\frac{P}{\mathrm{G}} - 1)}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}} \qquad \text{….(18)}$$

Stage III- rising of the craft from point 3 to point $2'$ at the action of the lift *P*. The craft rises for time t_3 with acceleration $a = \mathrm{g}(\frac{P}{\mathrm{G}} - 1)$.

To be able at the end of the cycle to reach $1'$, in the end of the third stage the craft should be in point $2'$, and the value of its velocity should be equal to its velocity in point 2: $V_{2'} = V_2$ ...(19)

Then the time and the path of the third stage will be equal to the time and the path in the second stage.

$$t_3 = t_2 = \frac{1}{\frac{P}{\mathrm{G}} - 1}\sqrt{\frac{\mathrm{R}\sqrt{2}}{\mathrm{g}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \qquad \text{..(20)}$$

$$h_3 = h_2 = \frac{\mathrm{R}}{\sqrt{2}(\frac{P}{\mathrm{G}} - 1)}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}} \qquad \text{….(21)}$$

For craft velocity in point $2'$ we have:

$$V_{2'} = at_3 = \mathrm{g}(\frac{P}{\mathrm{G}} - 1)t_3 = \mathrm{g}(\frac{P}{\mathrm{G}} - 1)t_2 \qquad \text{...(22)}$$

Stage IV- equally delayed motion from point $2'$ to point $1'$ under the action of the gravity acceleration. The time and the path of the fourth stage should be equal to those in the first stage or:

$$t_4 = t_1 = \sqrt{\frac{\mathrm{R}\sqrt{2}}{\mathrm{g}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \qquad \text{….(23)}$$

and $$h_4 = h_1 = \frac{\mathrm{R}}{\sqrt{2}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}} \qquad \text{…(24)}$$

The resistance exercised by the air on the craft during

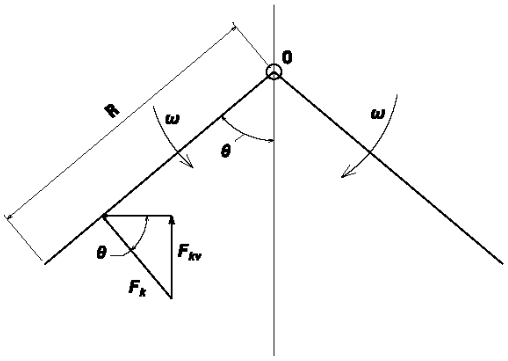

Fig 6: Relationship between aerodynamic force and the lifting force

the complete cycle of the free fall is ignored. The necessary corrections to account this factor could be made after craft testing because it is very difficult to be theoretically defined. According to the diagram on fig 6 and formula (19) the condition for hovering of the craft in the air without height loss after every cycle is during the third stage its velocity to reach a value equal to V_2.The observing of this condition depends on the lift value as well on the time of its action. If we substitute expressions (4) and (22) in equation (19) we get: $\mathrm{g}t_1 = \mathrm{g}(\frac{P}{\mathrm{G}} - 1)t_2$,or $t_1 = (\frac{P}{\mathrm{G}} - 1)t_2$ As $t_1 = t_4$ and $t_2 = t_3$, for the performance time of the complete cycle t_C we can write $t_C = t_1 + t_2 + t_3 + t_4 =$

$$= (\frac{P}{\mathrm{G}} - 1)t_2 + t_2 + t_3 + (\frac{P}{\mathrm{G}} - 1)t_3 =$$

$$= \frac{P}{\mathrm{G}}t_2 + \frac{P}{\mathrm{G}}t_3 = \frac{P}{\mathrm{G}}(t_2 + t_3) = \frac{P}{\mathrm{G}}t_P$$

Or: $P.t_P = \mathrm{G}.t_C$ …(25)

Or: $I_P = \mathrm{G}.t_C$ ……(26)

Where t_P is time of action of lifting force P, t_C - complete cycle time and I_P -impulse of the lifting force. Equation (26) shows that during flight in an established mode the impulse I_P does not depend on the wing geometry and the manner of wing motion. It is defined just by the craft weight and the operation cycle time. At $I_P > \mathrm{G}t_C$ the craft rises gaining height after every cycle At deriving formulae (17),(18),(20) and (21) we admitted that the lift is a constant value. Actually at wing driving with a constant torque the lift is a variable and it is defined by the vertical component F_{kv} of force F_k ,by which the air flow acts on the wing (fig 6) Or:
$P_t = 2F_{kv} = 2F_k \mathrm{Sin}\theta_t$

For approximate defining of the impulse I_p of the lift we will ignore the angular speed deviations during the stroke and the acceleration of the wing in the beginning and the end of the down-stroke i.e. we will admit that the angular velocity ω_d is a constant equal to $\frac{\theta_p}{t_2+t_3}$. We admit that the stroke is symmetrical, i.e. angle θ_p of wing rotation during the down-stroke is defined by the formula $\theta_p = \pi - 2\theta_0$.

According to these conditions for the impulse I_p we get $\frac{dI_p}{dt} = 2F_k \mathrm{Sin}\theta_t$ as $\theta_t = \theta_0 + \omega_d t$, then: $\mathrm{d}I_p = 2F_k \mathrm{Sin}(\theta_0 + \omega_d t)\mathrm{d}t$, or:

$$I_P = \int_0^{t_p} 2F_k Sin(\theta_0 + \omega_d t)dt =$$

$$= 2F \frac{-Cos(\theta_0 + \omega_d t)}{\omega_d}\Big|_0^{t_p} + C$$

Upon rearrangement we get: $I_p = \frac{4F_{kN}\mathrm{Cos}\theta_0 t_p}{\theta_p} + \mathrm{C}$

At $t_p = 0$, $I_p = 0$, or: $I_p = \frac{4F_{kN}\mathrm{Cos}\theta_0 t_p}{\theta_p}$ …(27)

After substitution equation (26) we have:

$\frac{4F_k \mathrm{Cos}\theta_0 t_p}{\theta_p} = \mathrm{G}t_C$, or $\frac{4F_k \mathrm{Cos}\theta_0}{\pi - 2\theta_0} t_p = \mathrm{G}t_C$

or $F_k.t_p = \frac{\mathrm{G}t_C(\pi - 2\theta_0)}{4\mathrm{Cos}\theta_0}$ …(28)

Formula (28) gives the relation between the required for craft holding in the air values of t_p and F_k, at defined prior to that G, t_C and θ_0. Both equations (26) and (28) do not depend on the geometry of the wing and the manner of wing motion. If the suitable values of t_p and F_k have already been defined experimentally we can define the craft path in second stage, the complete cycle time t_C, and the complete height loss H during the first and second stage:

$$h_2 = \frac{V_2}{2}\frac{t_p}{2} = \frac{1}{4}\sqrt{\mathrm{gR}\sqrt{2}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}}.t_p \quad \text{Or:}$$

$$h_2 = \frac{t_p}{4}\sqrt{\mathrm{gR}\sqrt{2}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \quad \ldots (29)$$

$$t_C = t_1 + t_p + t_4 = t_p + 2\sqrt{\frac{\mathrm{R}\sqrt{2}}{\mathrm{g}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \quad ...(30)$$

$$H = h_1 + h_2 =$$

$$= \frac{\mathrm{R}}{\sqrt{2}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}} + \frac{t_p}{4}\sqrt{\mathrm{Rg}\sqrt{2}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}} \quad ..(31)$$

By equation (28) we can define the maximum weight $\mathrm{G}_{\max}$ of the craft which can be held in the air by wings with pre- defined F_k:

$$\mathrm{G}_{\max} = \frac{4F_k \mathrm{Cos}\theta_0 t_p}{(\pi - 2\theta_0)t_C} \quad \ldots(32)$$

For a craft with flat and rigid flapping wings during flight along the vertical after substitution of t_C of (30) in equation (32), for maximum weight of the craft we get:

$$\mathrm{G}_{\max} = \frac{4F_k \mathrm{Cos}\theta_0 t_p}{(\pi - 2\theta_0)(t_p + 2\sqrt{\frac{\mathrm{R}\sqrt{2}}{\mathrm{g}}\mathrm{Ln}\frac{1}{\mathrm{Tan}^2\frac{\theta_0}{2}}}} \quad ...(33)$$

CONCLUSION

1. The present article develops a law for wing motion which allows the up- stroke performance in the fastest possible manner, without producing a force that pulls the craft downwards.
2. The author formulates the relationship between the craft weight, the operation cycle time and the impulse of the lifting force.
3. On the ground of the operation cycle analysis are defined the conditions at which the craft can be held in the air or to perform a vertical take-off.
4. During elaboration of the above type aircraft it is necessary to be relied on a wing with relatively short length and greater width (unlike the plane wing). This allows performance of the up-stroke for a shorter period of time, and for the down-stroke applying a weaker motor torque
5. The vertical leap-like take-off, analyzed in the present article can be considered as a start mode of a more complex craft.

REFERENCES

[1]. Lilienthal Otto, "Bird flight as the basis of aviation", Longmans, Green,1911

[2]. De Laurier, James D. "The Development and Testing of a Full-Scale Piloted Ornithopter." *Canadian Aeronautics and Space Journal*. 45. 2 (1999), 72–82. (accessed November 30, 2010).