# Unsteady Peristaltic Pumping of a Jeffrey Fluid in a Finite Length Tube With Permeable Wall

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***Abstract* - Investigation concerning Peristaltic pumping of a Jeffrey fluid in a finite length tube with permeable wall is made. The walls of the finite length tube are subjected to the contraction waves that do not cross the stationary boundaries. Mathematical analysis is expressed through long wavelength and low Reynolds number approximation in non-dimensional form . Saffman slip boundary condition used in this analysis. Exact analytical expressions of axial velocity, radial velocity, non-dimensional flow rate are obtained. The effect of various parameters on the flow pattern is discussed through graphs. When $\lambda_1 = 0$ the results are in agreement with Ravi Kumar.Y.V.K [2010].**

***Keywords:* Peristaltic Pumping, finite length tube, Jeffrey fluid, Permeable wall.**



## 1. INTRODUCTION

Peristalsis is a mechanism of pumping of viscous fluids in ducts against an adverse pressure gradient by means of a series of moving contractile rings on the wall. Most of the transportations in physiology are due to the peristaltic pumping mechanism. The flow of blood through arteries and veins, the flow of urine through ureter, the passage of bile from the gall bladder to the duodenum, the movement of chime in the gastrointestinal tract and the transportation of the food bolus through the alimentary canal are some examples of peristaltic pumping.

The study of the fluid mechanism of peristaltic transport has been confirmed experimentally by Latham[1] and Weinberg et al.[2]. Several research proposed different models by considering various geometries with relevant boundary conditions [10-20].Shapiro et.al [3] made a detailed investigation of peristaltic pumping of a Newtonian fluid in a flexible channel and a circular tube.

Most of the theoretical investigations have been carried out by assuming blood and the other physiological fluids behave like a Newtonian fluid. Although this approach may provide a satisfactory understanding of the peristaltic mechanism in the ureter ,it fails to provide a satisfactory efferentus of the male reproductive tract and in the transport of spermatozoa in the most of the physiological fluids behave like non-Newtonian fluids. Li and Brasseur [6] considered the non-steady peristaltic transport in a finite length tube by considering Newtonian fluid. Misra and Pandey [8-9] extended the model for peristaltic flow of power-law fluids in a finite length circular cylindrical peristaltic pumping in a finite length tube with permeable wall.

Now, we proposed to study the unsteady peristaltic pumping in a finite length tube with permeable wall with Jeffrey fluid model, Saffman boundary conditions are used at the permeable wall of the tube.

## 2. JEFFREY FLUID MODEL

The equation for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\bar{I} + \bar{S} \tag{1}$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}\left(\frac{\partial \bar{\gamma}}{\partial t} + \lambda_2 \frac{\partial^2 \bar{\gamma}}{\partial t^2}\right) \tag{2}$$

where $\bar{T}$ and $\bar{S}$ are Cauchy stress tensor and extra stress tensor, $\bar{P}$ is the pressure, $\bar{I}$ is the identity tensor, $\lambda_1$ is the ratio of the relaxation to retardation times, $\lambda_2$ is the retardation time and $\gamma$ is the shear rate.

## 3. MATHEMATICAL FORMULATION

We consider the peristaltic pumping of a non-Newtonian fluid namely Jeffrey fluid in a finite length tube with permeable walls. Sinusoidal waves of constant speed propagate along the channel boundaries.

The wall deformation of the peristaltic wave is given by

$$r = h(\bar{z},\bar{t}) = \varepsilon + m\bar{z} + 0.5aA(1 - \cos[\frac{2\Pi}{\lambda}(\bar{Z} - c\bar{t})])$$

(3)

where ε is the minimum tube occlusion, a is the average radius of the bolus, A is the amplitude, $\lambda$ is the wavelength of the peristaltic wave, m is a constant whose magnitude depends on the length of the tube, exist, and inlet dimensions, and c is the Velocity of the peristaltic wave.

Introducing a wave frame ( $\bar{r},\bar{z}$ ) moving with velocity c away from the fixed frame ( $\bar{R},\bar{Z}$ ) by the following transformations

$\bar{z} = \bar{Z} - c\bar{t}$ ,$\bar{r} = \bar{R}$, $\bar{v} = \bar{V} - c$, $\bar{u} = \bar{U}$, where ( $\bar{u},\bar{v}$ ) and ( $\bar{U},\bar{V}$ ) are velocity components in wave and fixed frame ,respectively.

After using these transformations, the equations of motion are

$$\frac{\partial \bar{V}}{\partial \bar{r}} + \frac{\partial \bar{U}}{\partial \bar{z}} + \frac{\bar{V}}{\bar{r}} = 0 \qquad (4)$$

$$\rho[\bar{V}\frac{\partial \bar{V}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{z}}] = -\frac{\partial \bar{p}}{\partial \bar{r}} + \mu[\frac{\partial^2 \bar{V}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{V}}{\partial \bar{r}} + \frac{\partial^2 \bar{V}}{\partial \bar{z}^2} - \frac{\bar{V}}{\bar{r}^2}] \qquad (5)$$

$$\rho[\bar{V}\frac{\partial \bar{U}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{z}}] = -\frac{\partial \bar{p}}{\partial \bar{z}} + \mu[\frac{\partial^2 \bar{U}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{U}}{\partial \bar{r}} + \frac{\partial^2 \bar{V}}{\partial \bar{z}^2}] \qquad (6)$$

where $\bar{U}$ and $\bar{V}$ are the velocity components in the $\bar{r}$ and $\bar{z}$ directions respectively.

Now we introduce the following non-dimensional variables

$$r = \frac{\bar{r}}{a}, Z = \frac{\bar{z}}{\lambda}, t = \frac{c\bar{t}}{\lambda}, H = \frac{h}{a}, \delta = \frac{a}{\lambda}, U = \frac{\bar{U}}{c}$$

where

$$V = \frac{\bar{V}}{\delta c}, p = \frac{\bar{p}a^2}{\mu c\lambda}, \phi = \frac{b}{a}, \mathrm{Re} = (\frac{\rho ca}{\mu})\delta, \alpha = (\frac{am}{\sqrt{k}})^{-1}$$

U and V are the axial and radial velocity components in the laboratory frame and k is the permeability of the wall.

After nondimensionalization, equations (4) – (6) become

$$\frac{1}{r}\frac{\partial}{\partial r}\frac{(rV)}{(1+\lambda_1)} + \frac{\partial U}{\partial z} = 0 \qquad (7)$$

$$\mathrm{Re}\,\delta^2[V\frac{\partial V}{\partial r} + U\frac{\partial V}{\partial z}] = -\frac{\partial p}{\partial r} + \delta^2\frac{\partial}{\partial r}[\frac{1}{r}\frac{\partial}{\partial r}\frac{(rV)}{(1+\lambda_1)}] + \delta^4\frac{\partial^2 V}{\partial z^2} \qquad (8)$$

$$\mathrm{Re}[V\frac{\partial U}{\partial r} + U\frac{\partial U}{\partial z}] = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(\frac{r}{(1+\lambda_1)}\frac{\partial U}{\partial r}) + \delta^2\frac{\partial^2 U}{\partial z^2} \qquad (9)$$

By considering long wavelength and low Reynolds number approximations and dropping terms of order $\delta$ and higher, it follows from equations (7) and (8) that the appropriate equations describing the flow in the frame are

$$\frac{1}{r}\frac{\partial}{\partial r}\frac{(rV)}{(1+\lambda_1)} + \frac{\partial U}{\partial z} = 0 \qquad (10)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(\frac{r}{(1+\lambda_1)}\frac{\partial U}{\partial r}) \qquad (11) \qquad \frac{\partial p}{\partial r} = 0$$

(12)

The non dimensional boundary conditions are discussed bellow

The wall itself undergoes radial vibrations with the velocity

i.e. $V = \frac{\partial H}{\partial t}$ at r = H (13)

The Saffman condition is imposed on permeable wall of the tube

i.e $U = -\alpha\frac{\partial U}{\partial r}$ at r = H (14)

The centre line velocity and velocity gradient in the radial direction are taken as zero

i.e V=0 at r = 0 (15)

$\frac{\partial U}{\partial r} = 0$ at r = 0 (16)

The finite tube length requires the pressures at the two ends of the tube

i.e $P = p_0$ at z = 0 (17)

$P = p_L$ at z = L (18)

## 4. SOLUTION OF THE PROBLEM

Solving equation (10) subject to the boundary conditions given by (14) and (16), we get axial velocity as

$$U = \frac{(1+\lambda_1)}{4}\frac{\partial p}{\partial z}(r^2 - 2\alpha H - H^2) \qquad (19)$$

where $H = \frac{\varepsilon}{a} + Mz + 0.5A(1 - \cos 2\Pi(z - ct))$,

$$M = m\frac{\lambda}{a}$$

Using equation (19) in equation (10) and solving together with the boundary condition given by (15), we get the radial velocity as

$$V = -(1+\lambda_1)^2(\frac{r^3}{16} - \frac{Hr\alpha}{4} - \frac{Hr^2}{8})\frac{\partial^2 p}{\partial z^2} + (1+\lambda_1)^2(\alpha + H)\frac{\partial p}{\partial z}\frac{\partial H}{\partial t}\frac{r}{4}$$

(20) Substituting equation (11) in equation (8) and integrating with respect to z yields

$$\frac{\partial p}{\partial z}=\frac{1}{H(H+4\alpha)^3}[G_0(t)+\frac{16}{(1+\lambda_1)}\int_0^z\frac{(H+4\alpha)^2}{H}\frac{\partial H(s,t)}{\partial t}ds]$$

(21) where $G_0(t)$ at most depends on time.Integrating again yields a relationship between intraluminal pressure, wall geometry, and wall velocity as

$$p(z,t)=p_0(t)+\int_0^z\frac{\partial}{\partial t}p(s,t)dz$$

$$p(z,t)=p_0(t)+\int_0^z\frac{16}{H(1+\lambda_1)(H+4\alpha)^3}[\frac{G_0(t)}{H(H+4\alpha)^3}+\int_0^{s_1}\frac{1}{H}(H+4\alpha)^2\frac{\partial H}{\partial t}ds_2]ds_1$$

(22)

$G_0(t)$ is determined by evaluating equation (22) together with the equation (18) as

$$G_0(t)=\frac{\Delta p(t)16\int_0^L\frac{16}{H(1+\lambda_1)(H+4\alpha)^3}(\int_0^{s_1}\frac{1}{H}(H+4\alpha)^2\frac{\partial H(s_2,t)}{\partial t}ds_2)ds_1}{\int_0^L\frac{1}{H(H+4\alpha)^3}ds}$$

(23)

where $\Delta p(t)=p_L(t)-p_0(t)$

*4.1 Volume Flow Rate*

The non dimensional flow rate is given as

$$Q(z,t)=2C_Q\int_0^H Urdr=2C_Q\frac{\partial p}{\partial z}[\frac{H^4}{16}+\frac{\alpha H^3}{4}]$$

(24)

where $C_Q$ = 1 for train wave and $C_Q=\frac{L}{\lambda}$ for single wave transport.

Pumping performance is characterized by the relationship between a time averaged the volume flow rate is $\bar{Q}=\frac{1}{T}\int_0^T Q(z,t)\,dt$ and the pressure difference between the end of the tube $\Delta p$ , where $\Delta p$ is fixed in time. Then we get

$$\bar{Q}=\bar{Q}_0[1-\frac{\Delta p}{\Delta p_0}] \qquad (25)$$

where $\bar{Q}_0$ is the maximal flow rate at z,

$$\Delta p=\Delta p_0[1-\frac{\bar{Q}}{\bar{Q}_0}] \qquad (26)$$

$$\bar{Q}=\bar{Q}_0-(\frac{2C_Q(1+\lambda_1)}{T})\int_0^T[\frac{\Delta p}{16(4\alpha+H)^2}\frac{H^2}{\int_0^L\frac{1}{H(H+4\alpha)^3}ds}]dt$$

$$\Delta p_0=\frac{8\bar{Q}_0}{C_Q(1+\lambda_1)}[\frac{1}{T}\int_0^T\frac{H^2}{(4\alpha+H)^2}\frac{1}{\int_0^L\frac{1}{H(H+4\alpha)^3}ds}]dt]^{-1}$$

where $\Delta p_0$ is the pressure difference required to maintain zero net flow rate at z.

## 5. NUMERICAL RESULTS AND DISCUSSION

Figs 1-4 are drawn for the time varying pressure distribution for a integral number of peristaltic waves in the tube for $L$□ $\lambda$=2. From fig.1 , at t = 0 ,it is observed that the pressure rises very sharply at the initial end reaches the cusp , then decreases to a lower rate at the midpoint of the bolus and it finally rises sharply to meet the leading end of the bolus to transport it under huge control. Similar phenomenon is observed for different values of . It is noticed that there is a large decrease in the peak pressures (maximum and minimum pressures) with increasing permeability parameter.

From figs1-4, it is observed that the increase in time shifts the bolus to the right of the axis for increasing " t ". The pressure distribution exhibits how the two boluses are carried along the length by changing the pressure distribution.

Figs 5-7, are drawn by considering the propagation of a non – integral number of waves in the train. It can be observed that $\Delta p$ decreases with an increase $\alpha$. From both the cases, it is observed that the peaks of the pressures are identical in the integral case while different in the non-integral case. fig 8 is drawn to study the effect of Jeffrey parameter for non-integral case when $\lambda_1=0$ the results are in agreement with that of Ravi Kumar [2010].

From figs. 1–8 and fig. 9 we can observe that the effect of $\alpha$ and $\lambda_1$ on $\Delta p$ as $\Delta p$ decreases with an increase in $\alpha$ and $\lambda_1$.

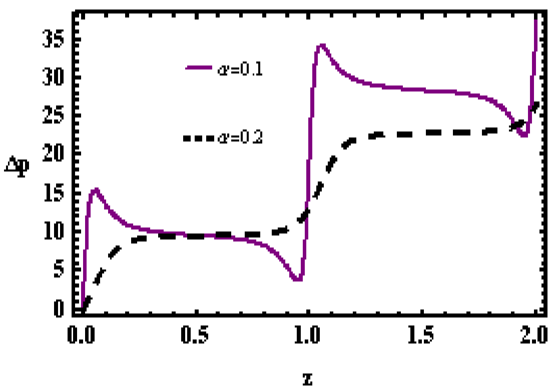


Fig 1 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha=0.1$, $\alpha$ = 0.07, with $\frac{\varepsilon}{a}$ = 0.5 , A = 1.609, $\frac{L}{\lambda}$ = 2 , when t = 0 and $\lambda_1=1$.

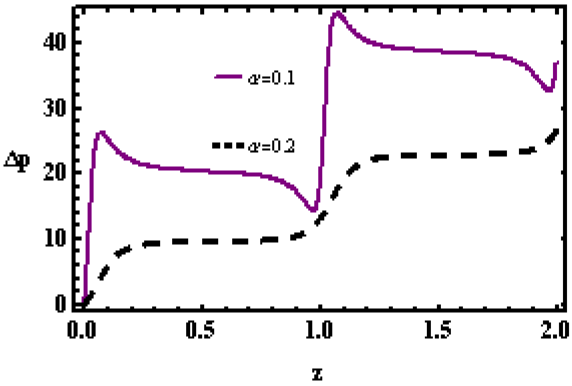


Fig 2 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 2 , when t = 0.02 and $\lambda_1 = 1$.

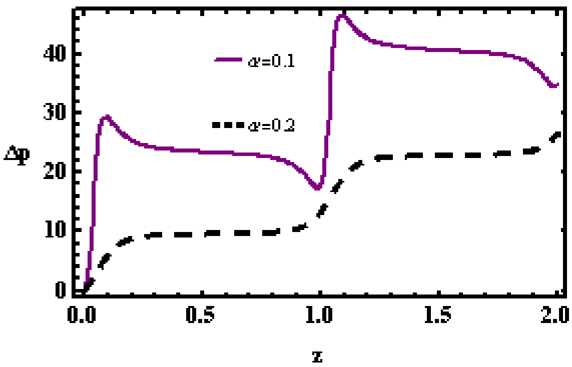


Fig 3 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 2 , when t = 0.04 and $\lambda_1 = 1$.

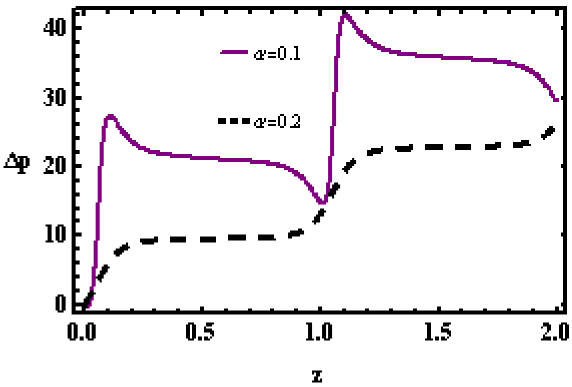


Fig 4 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 2 , when t = 0.06 and $\lambda_1 = 1$.

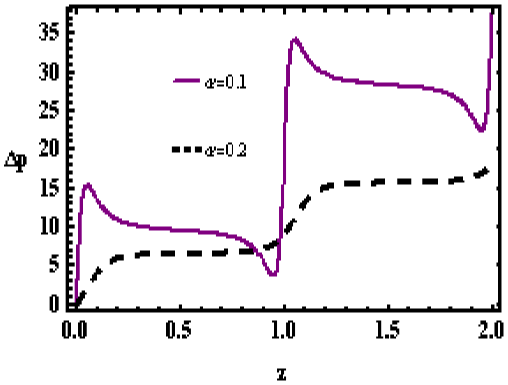


Fig 5 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 1.69, when t = 0 and $\lambda_1 = 1$.



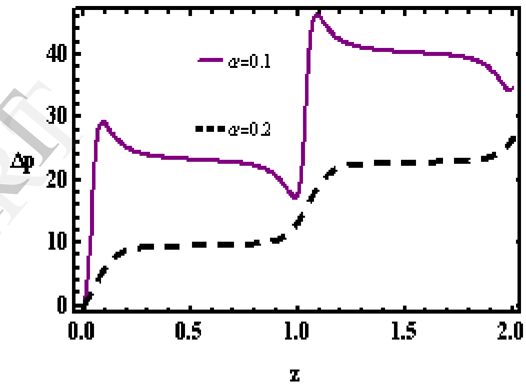


Fig 6 Pressure distribution with z along the tube for $\alpha$ = 0.0, $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 1.69, when t = 0.04 and $\lambda_1 = 1$.

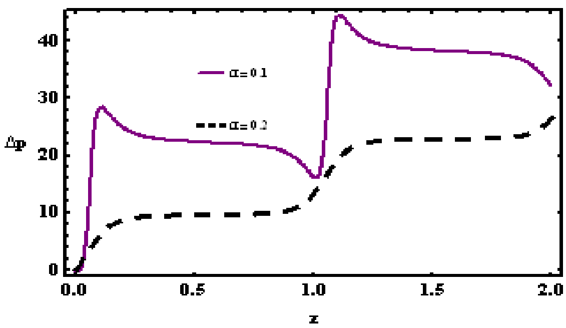


Fig 7 Pressure distribution with z along the tube for $\alpha$ = 0.05, $\alpha = 0.06$, $\alpha$ = 0.07, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda}$ = 1.69 , when t = 0.3 and $\lambda_1 = 1$.

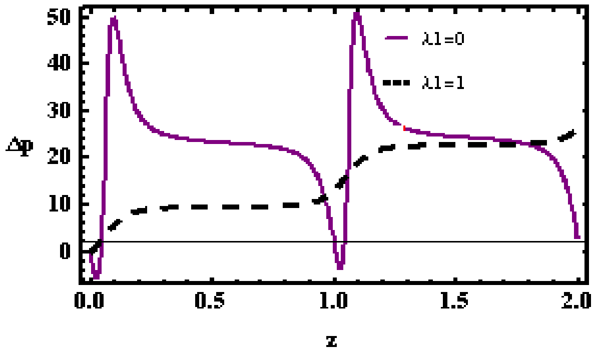


Fig 8 Pressure distribution with z along the tube for $\alpha = 0.1$, with $\frac{\varepsilon}{a} = 0.5$ , A = 1.609, $\frac{L}{\lambda} = 1.89$, when t = 0.0 and $\lambda_1 = 0,\ \lambda_1 = 1.$

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