Unsteady Free Convection MHD Flow past a Vertical Plate with Variable Temperature and Chemical Reaction

*Bhaben Ch. Neog[1], Rudra Kr. Das[2]
[1]Office of the Principal, Jagiroad College, Jagiroad, Morigaon, Assam.

[2]Dept. of Physics, Jagiroad College, Jagiroad, Morigaon, Assam.

Abstract

An analysis is performed to study the effect of magnetic field on transient free convection flow of an electrically conducting fluid over an impulsively started isothermal vertical plate with variable temperature and with chemical reaction. Solutions are obtained by Laplace transform technique and presented graphically for different values of physical parameters. It is observed that chemical reaction parameter and magnetic parameter influence the velocity and concentration profiles significantly.

KeyWords: Free Convection, MHD, Variable Temperature, Chemical Reaction
AMS 2000 subject classification: 76R10, 76W05, 80A20, 80A32.
*Corresponding Author

1. Introduction

Mass transfer with chemical reaction is one of the most commonly encountered circumstances in chemical industry as well as in physical and biological sciences. In some other areas such as food processing industry, paper processing technology, the evaporation or condensation process, solvent extraction, drying humidification, sublimation, oxygenation of blood, food and drug assimilation, respiration mechanism, etc. chemical reaction takes place. There are many situations where convection heat transfer phenomena are accompanied by mass transfer also. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. A number of investigations have already been carried out with combined heat and mass transfer under the assumption of different physical situations. Further due to some important industrial and engineering applications such as liquid metal cooling in nuclear reactors, magnetic control of molten iron flow in steel industry etc., magnetoconvection has also been gaining considerable attention amongst researchers. Hence combined study will surely enhance the already developed areas further for more complex studies.

Exact solutions of free convection flow past a vertical plate in free convective flow was first obtained by Soundalgekar [14] and the same problem with mass transfer effect was considered by Soundalgekar and Akolkar [15]. Das *et. al.* [3] studied the effects of mass transfer on free convection flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. They also studied the transient free convection flow past infinite vertical plate with periodic temperature [5]. Effect of mass transfer on the flow past an infinite vertical oscillating plate with constant heat flux was studied by Soundalgekar *et. al.* [17].

The effects of mass transfer on free convection flow past a semi-infinite vertical isothermal plate was first studied by Gebhart and Pera [10] and the effects of mass transfer on the flow past an impulsively started infinite vertical plate with variable temperature was studied by Soundalgekar *et. al.* [16]. Muthucumaraswamy *et. al.* [12] considered the effects of mass transfer on impulsively started infinite vertical plate with variable temperature and uniform mass flux. All of them considered the fact that free convection current caused by temperature differences is also caused by the differences in concentration or material constitution as suggested by Gebhart [9].

Further, in many cases in the process of free convection, chemical reaction also takes place due to the presence of foreign masses (as impurities) in fluid. It is found that in many chemical engineering processes, chemical reaction takes place between foreign masses (present in the form of ingredients) and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However if the presence of such foreign mass is very low then we can assume the first order chemical reaction so that heat generation due to chemical reaction can be considered to be very negligible. Das *et. al.* [4] considered the effects of mass transfer on flow past an impulsively started vertical plate and Muthucumaraswamy and Meenakshisundaram [13] studied the chemical reaction effects on vertical oscillating plate with variable temperature and chemical reaction. Deka and Neog [6], [7] considered the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium and with MHD respectively.

In a very recent paper Neog [8] studied the effect of MHD with chemical reaction on electrically conducting fluid past oscillating plate. Chaudhary and Jain [2] studied the magnetohydrodynamic transient heat and mass transfer flow by free convection past a vertical plate, when the temperature of the plate oscillates in time about a constant mean temperature and the plate is embedded in a porous medium. They extended the work of Das et.al. [3], which include the effects of mass transfer, magnetic field and porous medium.

Although many authors studied mass transfer with or without chemical reaction in flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer in the presence of transverse magnetic field and chemical reaction with variable temperature has not been found in literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of chemical reaction on hydromagnetic flow past a vertical plate with variable temperature under the assumption of first order chemical reaction.

2. Mathematical Analysis

An unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate is considered here. To visualize the flow pattern a Cartesian co-ordinate system is considered where x'-axis is taken along the infinite vertical plate, the y'-axis is normal to the plate and fluid fills the region $y' \geq 0$. Initially, the fluid and the plate are kept at the same constant temperature T'_∞ and species concentration C'_∞. At time $t' > 0$, the plate is assumed to be moving continuously in its own plane with a uniform velocity U_0 and at the same time the plate temperature is raised linearly with time and the level of species concentration is raised to C'_w . A magnetic field of uniform strength B_0 is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only. Thus the motion is one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \quad (3)$$

Along with the following initial and boundary conditions:

$$\left.\begin{array}{ll} u'=0, \quad T'=T'_\infty, \; C'=C'_\infty \;\; \text{for all } y' \text{ and } t' \leq 0 & \\ \left.\begin{array}{l} u'=U_0, \; T'=T'_\infty+(T'_w-T'_\infty)At', \; C'=C'_w \text{ at } y'=0 \\ u' \to 0, \; T' \to T'_\infty, \; C' \to C'_\infty \quad \text{as } y' \to \infty \end{array}\right\} , t>0 & \end{array}\right\} \quad (4)$$

where $A = \frac{u_0^2}{\nu}$.

Now to reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\left.\begin{array}{l} u = \frac{u'}{U_0}, \; t = \frac{t'U_0^2}{\nu}, y = \frac{y'U_0}{\nu}, \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, \\ \phi = \frac{C'-C'_\infty}{C'_w-C'_\infty}, \quad Gr = \frac{g\beta\nu(T'_w-T'_\infty)}{U_0^3}, \\ Gm = \frac{g\beta^*\nu(C'_w-C'_\infty)}{U_0^3}, \; \Pr = \frac{\mu C_p}{\kappa}, \\ R = \frac{\nu K_1}{U_0^2}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2} \end{array}\right\} \quad (5)$$

Thus with the help of above non dimensional quantities (5), the equations (1), (2) and (3) then reduce to the following forms.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi-Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-R\phi \tag{8}$$

And the initial and boundary conditions (4) are as follows:

$$\left.\begin{aligned} &u=0, && \theta=0, && \phi=0, \text{ for all } y \text{ and } t\le 0\\ &\left.\begin{aligned} &u=1, && \theta=t, && \phi=1 \quad at\ y=0\\ &u\to 0, && \theta\to 0, && \phi\to 0 \quad as\ y\to\infty \end{aligned}\right\}, \ t>0 \end{aligned}\right\} \tag{9}$$

Solutions of the equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained with the help of Abramowtz and Stegum [1] and Hetnarski's [11] algorithm. They are obtained as follows:

$$\begin{aligned}\theta(y,t)=&\left(t+\frac{y^2\,\mathrm{Pr}}{2}\right)erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)\\ &-\frac{y\sqrt{t}\,\mathrm{Pr}}{\sqrt{\pi}}e^{-\frac{y^2\,\mathrm{Pr}}{4t}}\end{aligned} \tag{10}$$

$$\phi(y,t)=\frac{1}{2}\left\{\begin{aligned}&e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)\\ &+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\end{aligned}\right\} \tag{11}$$

$$\begin{aligned}u(y,t)=&\frac{G_4}{2}\left\{e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\\ &+\frac{yG_1}{4b\sqrt{M}}\left\{e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)-e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right\}\\ &+\frac{G_1}{2b^2}\left[e^{bt}\left(\left\{e^{-y\sqrt{c}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ct}\right)+e^{y\sqrt{c}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ct}\right)\right\}\right.\right.\\ &\left.\left.-\left\{e^{-y\sqrt{b\,\mathrm{Pr}}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}-\sqrt{bt}\right)+e^{y\sqrt{b\,\mathrm{Pr}}}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}+\sqrt{bt}\right)\right\}\right)\right]\\ &+\frac{G_2}{2d}\left[e^{dt}\left(\left\{e^{-y\sqrt{k}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{kt}\right)+e^{y\sqrt{k}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{kt}\right)\right\}\right.\right.\\ &\left.-\left\{e^{-y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{ht}\right)+e^{y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{ht}\right)\right\}\right)\\ &\left.+\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\}\right]\\ &+\frac{G_1}{b}\left[\left(t+\frac{y^2\,\mathrm{Pr}}{2}\right)erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-\frac{y\sqrt{t}\,\mathrm{Pr}}{\sqrt{\pi}}e^{-\frac{y^2\,\mathrm{Pr}}{4t}}\right]+\frac{G_3}{2}erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)\end{aligned} \tag{12}$$

Here, the following symbols are used in the above solutions:

$$\left.\begin{aligned}&b=\frac{M}{\mathrm{Pr}-1}, c=M+b, d=\frac{RSc-M}{1-Sc}, h=R+d,\\ &k=M+d,\quad G_1=\frac{Gr}{\mathrm{Pr}-1}, G_2=\frac{Gm}{Sc-1},\\ &G_3=1-\frac{G_2}{d}-\frac{G_1}{b^2},\quad G_4=G_3-\frac{tG_1}{b}\end{aligned}\right\} \tag{13}$$

3. Results and Discussion

In order to know the influence of different physical parameters viz., chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number, Hartmann number, Prandtl number and time on the physical flow situation, computations are carrier out for vertical velocity, temperature and concentration and they are presented graphically.

In figure-1 concentration profiles are presented for different values of *Sc* and *R*.

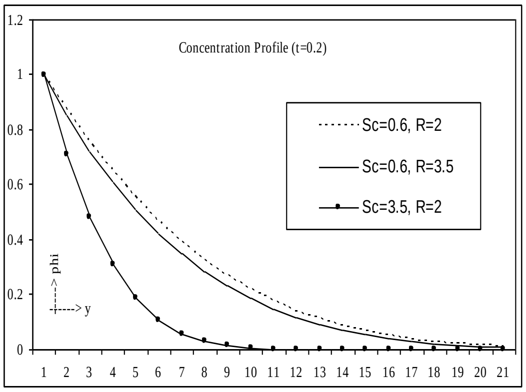

Figure-1 : Concentration Profile (Effect of Chemical reaction parameter and Schmidt number).

It is observed that increase of Schmidt number and chemical reaction parameter lead to the decrease in concentration of the species.

Figure-2a represents the temperature profiles for different values of *Pr* and figure-2b represents temperature profiles at different times *t*.

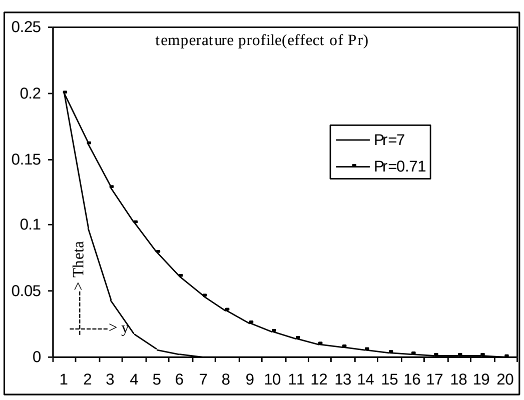

Figure-2a:Temperature Profile (Effect of Pr)

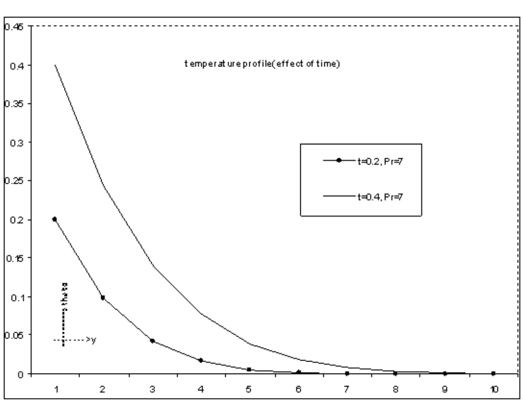

Figure-2b: Temperature Profile (Effect of time)

Since temperature is considered as time dependent, therefore this figure clearly reflects the situation. It is further observed that temperature decreases with the increase of *Pr*.

Velocity Profiles for different values of parameters are shown in figures-3a and 3b. Influence of *Gr*, *Gm* and *R* are shown in figure 3a for fixed values of *M* , *Sc* and *Pr* .

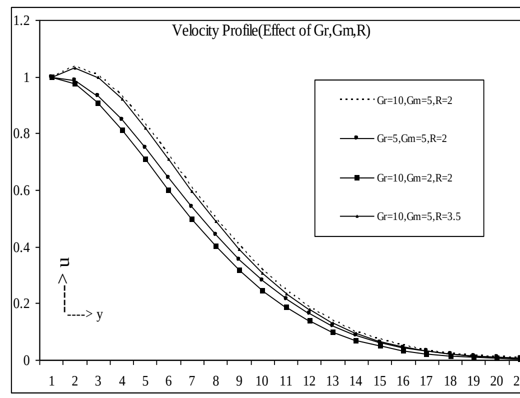

Figure-3a: Velocity Profile (Effect of Gr, Gm and R)

It is clear from the figure that velocity increases with the increase of *Gr* and *Gm* but decreases with the increases of *R*. In figure 3b effects of *M* and *Sc* are presented for some fixed values of *t* , *Gr* , *Gm*, *Pr* and *R*.

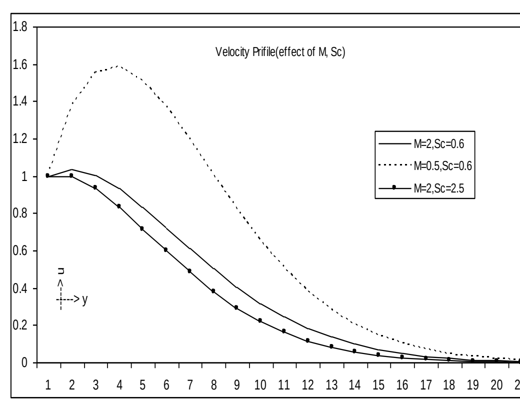

Figure-3b: Velocity Profile (Effect of M and Sc)

It is clear from the figure that velocity decreases when *M* and *Sc* increase.

4. Conclusions

An exact analysis is performed to study the influence of chemically reacting hydro-magnetic flow past a vertical plate with variable temperature and chemical reaction. Exact solutions of equations are obtained by Laplace transform technique. Some of the important conclusions of the study are as follows:

- Concentration decreases as *Sc* and *R* increase.

- Velocity increases with increasing *Gr*, *Gm* and decreasing *M* and *R*.
- Also increase in *Sc and R* lead to decrease in velocity.

5. References

[1] Abramowitz B. M. and Stegum I. A.(1965) **:** Handbook of Mathematical Functional function, Dover Publications, NewYork.

[2] Chaudhary R. C. and Jain A. (2009)**:** MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium, Theoret. Appl. Mech., **36**(1),1-27

[3] Das U. N., Deka R. K. and Soundalgekar V. M. (1994) **:** Effects of mass transfer on flow past an impulsively started vertical infinite plate with constant heat flux and chemical reaction, Forschung in Ingenieurwesen, **60**, 284-287.

[4] Das U. N., Deka R. K. and Soundalgekar V. M.(1998) **:** Effect of Mass Transfer on Flow Past an Impulsively Started Infinite Vertical Plate With Chemical Reaction, GUMA Bulletin, **5**, 13-20

[5] Das U.N., Deka R.K. and Soundalgekar V.M. (1999)**:** Transient free convection flow past an infinite vertical plate with periodic temperature variation, Journal of Heat Transfer, Trans. ASME, **121**, 1091-1094.

[6] Deka R.K. and Neog B C.(2009)**:** Combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium, Adv. Appl. Fluid Mech, **6(2)**, 181-195

[7] Deka R. K. and Neog B. C. (2009)**:** Unsteady MHD Flow past a vertical Oscillating Plate with Thermal Radiation and Variable Mass Diffusion, Cham. J. Math, **1**, 79-92.

[8] Neog B. C.(2010) **:** Unsteady MHD Flow past a vertical Oscillating Plate with Variable Temperature and Chemical Reaction, J. AAM, **1**, 97-109.

[9] Gebhart B.(1971) **:** Heat Transfer, TMcGraw Hill.

[10] Gebhart B. and Pera L. (1971)**:** The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, Int. J. Heat and Mass Transfer, **14**, 2025-2050

[11] Hetnarski R. B.(1975) **:** An algorithm for generating some inverse Laplace transforms of exponential form, ZAMP, **26**, 249-253.

[12] Muthucumaraswamy R., Ganesan P., Soundalgekar V. M. (1999)**:** Impulsively Started Vertical Plate with Variable Temperature and Uniform Mass Flux, The Bulletin, GUMA, **6**, 37-49.

[13] Muthucumaraswamy R., Meenakshisundaram S.(2006) **:** Theoretical Study of Chemical Reaction Effects on Vertical Oscillating Plate With Variable Temperature, Theoretical Applied Mechanics, **33**(3), 245-257.

[14] Soundalgekar V. M.(1979) **:** Free convection effects on the Flow Past a Vertical Oscillating Plate, Astrophysics Space Science, **66**, 165-172.

[15] Soundalgekar V. M. and Akolkar S. P.(1983) **:** Effects of free convection and mass transfer on flow past a vertical oscillating plate, Astrophysics and Space Science, **89**, 241-254.

[16] Soundalgekar V. M., Birajdar N. S., and Darwhekar V. K.(1984) **:** Mass-Transfer Effects on the Flow Past an Impulsively Started Vertical Plate with Variable Temperature or C. H. F., Astrophy. and Sp. Sc., **100**, 159-164

[17] Soundalgekar V. M., Lahurikar R. M., Pohanerkar S. G. and Birajdar N. S. (1994)**:** Effects of Mass Transfer on the Flow Past an Oscillating Infinite Vertical Plate with Constant Heat Flux, Thermophy and AeroMech., **1**, 119-124.