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Unit Based Fuzzy Expediting In Project Networks

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Abstract

A new algorithm namely, unit based fuzzy expediting algorithm is proposed for finding a fuzzy optimal schedule to fuzzy project networks. An illustrative example is presented to clarify the idea of the proposed approach. The output of the proposed method includes a distribution of the fuzzy project duration, a distribution of the total fuzzy project cost. The unit based fuzzy expediting method can be served as an important tool for the project managers when they are handling various types of project networks having imprecise parameters.

Keywords: *Fuzzy project network, Critical path, Fuzzy cost slope, Expediting.*

1. Introduction

Project management involves project scheduling, planning, controlling, time management, resource management and cost management among which time management is more significant. Critical path method pertains to an algorithm established in the 1950's and is typically used in project management cases for scheduling a set of project activities. It helps us to calculate the minimum length of time in which the project can be completed. When the activity times in the project are deterministic and known, critical path method has been demonstrated to be a useful tool in managing projects in an efficient manner to meet the challenge. Since the critical path determines the completion date of the project, the project can be accelerated by adding the resources required to decrease the time for the activities in the critical path. Such a shortening of the project sometimes is referred as project expediting. It is true that due date for the completion of the project may well affect the cost incurred because more resources are required to perform work in a shorter period of time. This is called time-cost trade-off analysis. Efficient algorithms have been developed for solving the project network when the duration and cost of an activity in the project are certain numbers. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the duration and cost of an activity may be uncertain due to some uncontrollable factors such as weather, productivity level, and human facts and so on. Wang et al. [12] designed a model to project scheduling with fuzzy information. Leu et al. [9] developed a fuzzy optimal model to formulate effects of both certain activity duration and research constraint. Feng et al. [4] introduced a hybrid approach that combines simulation techniques with a genetic algorithm to solve the time cost trade-off problem under uncertainty. Leu et al. [10] gave a genetic algorithm based fuzzy optimal model for the construction of time-cost trade-off. Arikan and Gungor [1] presented fuzzy goal programming model for time-cost trade –off problem. Chao-gunag et al. [8] proposed a fully fuzzy time-cost trade-off based on genetic algorithms. Yang [13] incorporated budget uncertainty into project time-cost trade-off in a chance constraint programming model. Ghazanfari et al. [5] developed a new approach for solving the time-cost trade-off problem with fuzzy decision variable. Yousefli et al. [14] developed a heuristic method to solve a project scheduling problem by using fuzzy decision making in fuzzy environment. Hua Ke et al. [6] provided an algorithm for solving project scheduling problem in hybrid uncertain environments. Shakeela and Ganesan [11] proposed a method for finding an optimal duration by expediting the fuzzy activities of a project network without converting the fuzzy activity times to classical numbers.

In this paper we propose a new algorithm namely, unit based fuzzy expediting algorithm for project networks in fuzzy environment. We have considered project networks in uncertain environment in which normal and expedite durations of each activity are considered imprecise and shown in the form of triangular fuzzy numbers. Optimum durations of activities are calculated in the form of triangular fuzzy numbers. A numerical example has been solved illustrating the validity of the proposed approach.

2. Preliminaries

We need the following mathematical orientated definitions of fuzzy set, fuzzy number and membership function which can be found in Bellman and Zadeh [2].

Definition 2.1 Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0,1]. A fuzzy set $\tilde{A}$ with the membership function $\mu_A(x)$ is defined by

$$\tilde{A} = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1] \}.$$

Definition 2.2 A Fuzzy set $\tilde{A}$ is called positive if its membership function is such that $\mu_{\tilde{A}}(x) = 0$ for all $x \le 0$.

Definition 2.3 A Fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number of its member ship function has the following conditions:

(i) $\mu_{\tilde{A}}(x) : R \to [0,1]$ is continuous.

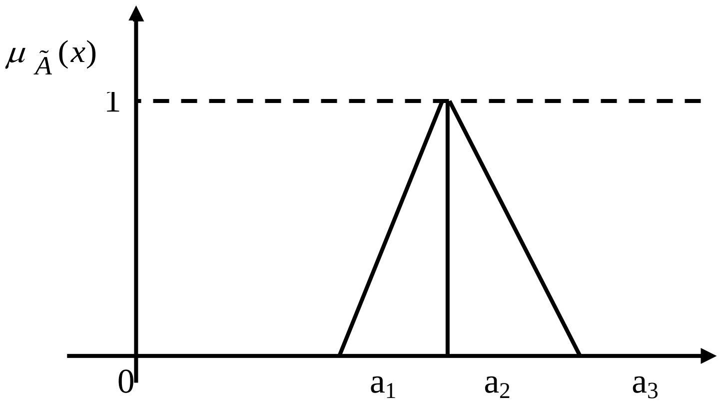
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c].

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in b$ where $a \le b \le c$.

Definition 2.3 A fuzzy number $\tilde{A}$ is denoted as a triangular fuzzy number by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if} \quad a_1 \le x \le a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & \text{if} \quad a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$



We need the following definitions of the basic arithmetic operators on triangular fuzzy numbers based on the function principle which can be found in Chen [3].

Definition 2.4 If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{a} - \tilde{b} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(iii) $\tilde{a} \times \tilde{b} = (c_1, c_2, c_3)$

where, $c_1 = \min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$;

$c_2 = a_2b_2$;

$c_3 = \max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$

(iv) If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are non zero and positive real numbers, then

$\tilde{a} \times \tilde{b} = (a_1b_1, a_2b_2, a_3b_3)$

(v) $\tilde{a} / \tilde{b} = (a_1 / b_3, a_2 / b_2, a_3 / b_1)$

(vi) $k\tilde{a} = (ka_1, ka_2, ka_3)$ for $k \geq 0$.

Definition 2.5 If $\tilde{a} = (a_1, a_2, a_3)$ is a triangular fuzzy number, then the Graded Mean Integration Representation method to defuzzify the number is given by,

$$p(\tilde{a}) = \frac{a_1 + 4a_2 + a_3}{6}.$$

3. Fuzzy Expediting

Fuzzy network expediting developed with the critical path method for planning and controlling of project networks. Fuzzy expediting in critical path method means that by selecting the lowest fuzzy cost slope activity or activities which will shorten the critical path. This procedure is repeated until the project has been shortened sufficiently or the fuzzy cost to shorten the project exceeds the benefits to be derived. The crashing method is focused on reducing the fuzzy duration of the activities on the critical path. The aim is always to strike a balance between the fuzzy cost and fuzzy duration and to obtain a fuzzy optimum project schedule.

3.1. Fuzzy relationship between duration and cost

A simple representation of the possible relationship between the duration of the activity and its direct costs appears in figure 1. Shortening the duration on an activity will normally increase its direct costs. A duration which implies minimum direct cost is called the Normal Duration (ND) and the minimum possible time to complete an activity is called Crash Duration (CD), but at a maximum cost. In this paper the durations and costs are both characterized by triangular fuzzy numbers. The fuzzy relationship between duration and cost is shown in figure 2.

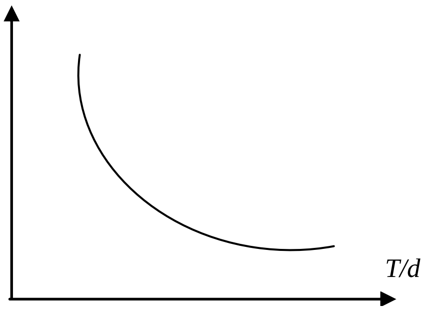
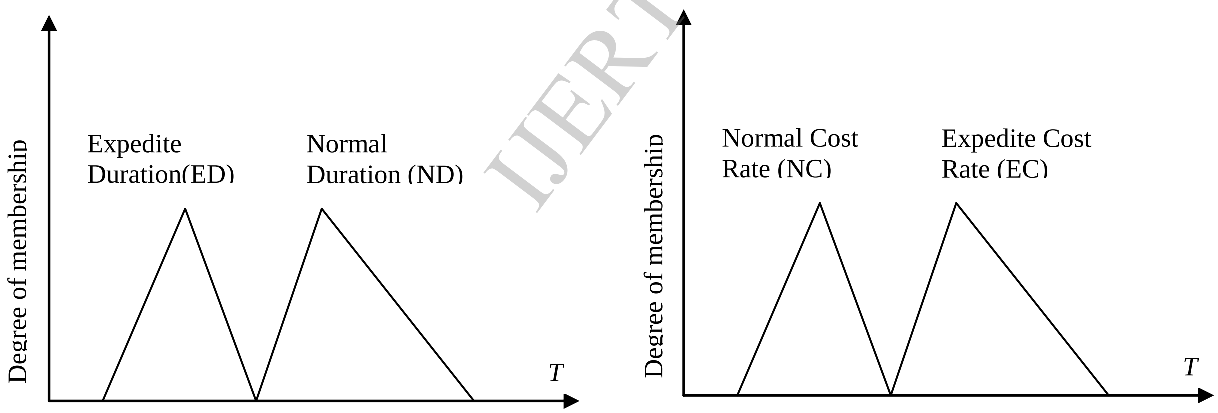


Fig 1 Deterministic relationship between duration and cost of an activity



(a) Expedite & Normal Activity Fuzzy Duration (FD) (b) Expedite & Normal Activity Fuzzy Cost (FC)

Fig 2 Fuzzy relationship between duration and cost of an activity

4. Unit based Fuzzy Expediting Algorithm

We now introduce the new method namely, unit based fuzzy expediting algorithm for finding the fuzzy optimum project schedule.

The proposed algorithm proceeds as follows:

Step1: Draw the project network and determine the critical path.

Step2: Compute the fuzzy cost slope for each activity by the formula.

Fuzzy cost slope= (Expedite FC – Normal FC)/ (Normal FD-Expedite FD)

Step3: Identify the activity with the minimum fuzzy cost slope and expedite the activity by one day.

Step4: Identify the new critical path and find the fuzzy cost of the project by the formula
Fuzzy project cost= (Direct FC + Expedite FC of crashed activity) + (Indirect cost x Fuzzy project duration)

Step5: Now in the new critical path select the activity with the next minimum cost slope and expedite by one day.

Step6: Repeat the process until all the activities in the critical path have been expedited by one day upto the desired level.

Step7: At this point all the activities are expedited and further expedition is not possible. The expedition of non critical activity does not alter the fuzzy project duration and is of no use.

5. Illustration

The proposed method namely, unit based fuzzy expediting method for solving a fuzzy expediting in project management is illustrated by the following:

A precedence relationships network with 12 activities is depicted in the figure 5.1. The expedite duration and normal duration of an activity are triangular fuzzy numbers and expedite cost rate and normal cost rate are also triangular fuzzy numbers. The indirect cost is considered as 0.10 x 10^4 Yuan per day. The complete data is given in table 5.1.

Activity	Normal Fuzzy		Expedite Fuzzy	
	Duration (NFD)	Cost x 10^4 Yuan (NFC)	Duration (EFD)	Cost x 10^4 Yuan (EFC)
1--2	(7, 8, 10)	(0.65 , 0.7 , 0.75)	(2, 3, 4)	(0.90, 1.00, 1.10)
1--3	(4, 4, 5)	(0.55, 0.6, 0.65)	(1,2,3)	(0.70, 0.80, 0.90)
2--3	(0, 0, 0)	(0,0,0)	(0, 0, 0)	(0, 0, 0)
2--5	(3, 6, 9)	(0.80, 0.90, 1.00)	(1, 1, 1)	(1.10, 1.15, 1.20)
3--4	(7, 7, 7)	(0.25, 0.25, 0.25)	(4, 5, 6)	(0.30, 0.30, 0.30)
4--6	(11, 12, 13)	(0.90, 1.00, 1.10)	(7, 8, 10)	(1.50, 1.60, 1.70)
5--6	(12, 15, 18)	(1.10, 1.20, 1.30)	(8, 10, 10)	(1.55, 1.60, 1.65)
5--7	(7, 7, 8)	(1.15, 1.20, 1.25)	(6, 6, 6)	(1.40, 1.40, 1.45)
6--8	(4, 5, 6)	(0.90, 1.00, 1.10)	(4,5,6)	(0.90, 1.00, 1.10)
7--8	(10, 14, 17)	(0.50, 0.60, 0.70)	(5, 7, 8)	(0.72, 0.74, 0.76)
7--9	(7, 8, 10)	(0.55, 0.60, 0.65)	(4, 5, 6)	(1.10, 1.20, 1.30)
8--9	(6, 6, 9)	(0.50, 0.60, 0.70)	(3,4,5)	(0. 76, 0.78, 0.80)

Table 5.1

Stage 1: According to the step 1, we construct a project network for the given data in figure 5.1.

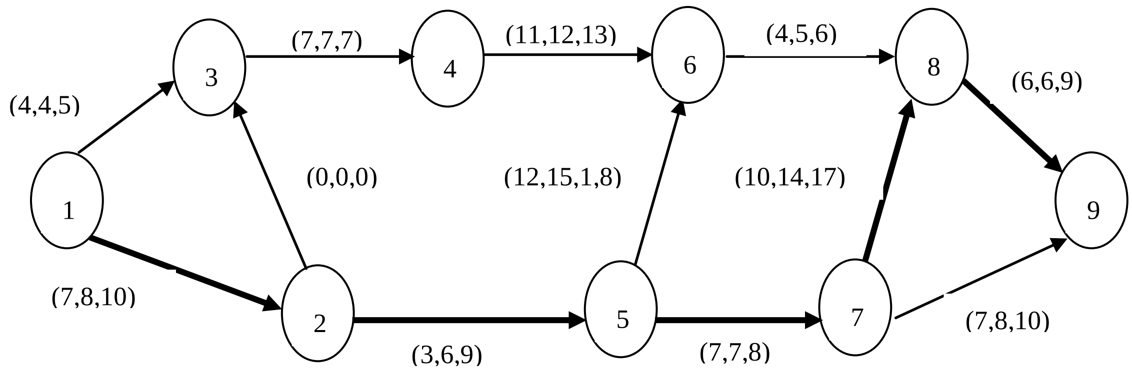


Figure 5.1

Now the Critical Path in this network is highlighted with dark lines. The fuzzy project duration of the critical path in figure 5.1. is (33,41,53) and the defuzzification value is 41.67, whereas the corresponding total fuzzy cost is (11.15, 12.75, 14.75) x10^4 Yuan and its defuzzification value is 12.81 x10^4 Yuan.

Now as in step 2, we compute the fuzzy cost slope for each activity.

Activity	Fuzzy cost slope x 10^4 Yuan	Activity	Fuzzy cost slope x 10^4 Yuan
1--2	(0.02, 0.06, 0.15)	5--6	(0.03, 0.08, 0.28)
1--3	(0.03, 0.10, 0.35)	5--7	(0.08, 0.20, 0.30)
2--3	(0, 0, 0)	6--8	(0, 0, 0)
2--5	(0.01, 0.05, 0.20)	7--8	(0.0, 0.02, 0.13)
3--4	(0.02, 0.03, 0.05)	7--9	(0.08, 0.20, 0.75)
4--6	(0.07, 0.15, 0.80)	8--9	(0.01, 0.09, 0.30)

Stage 2: Now as in step 3, activity 7-8 has minimum cost slope and it is expedited by one day.

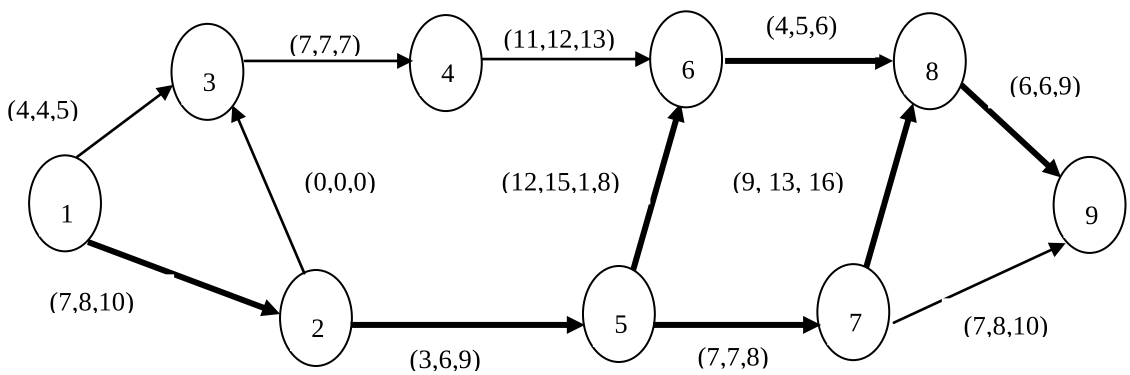


Figure 5.2

According to step 4, we identify there are two critical paths in figure 5.2. They are 1→2→5→7→8→9 and 1→2→5→6→8→9. The fuzzy project duration of the two critical paths are (32, 40, 52) and the defuzzification value is 40.67, whereas the corresponding total fuzzy cost is (11.05, 12.67, 14.78) x10^4 Yuan and its defuzzification value is 12.75 x10^4 Yuan.

Repeating step 3 to step7 by seven times, we obtain the following project network.

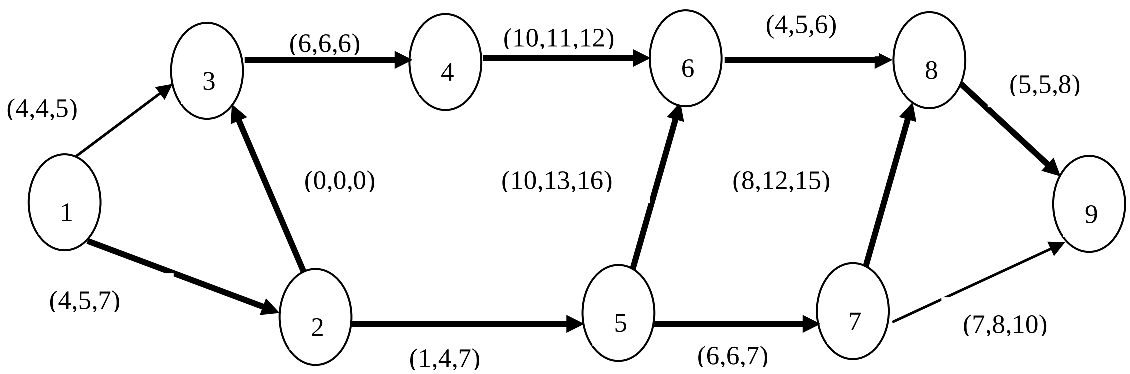


Figure 5.3

From figure 5.3, it is identified that there are three critical paths. They are
1→2→5→7→8→9, 1→2→5→6→8→9 and 1→2→3→4→6→8→9.

The following table shows the results for the given project network.

Stage	Activity	Expedite Day	Fuzzy Project Duration	Direct Fuzzy Cost x 10^4 Yuan	Indirect Fuzzy Cost x 10^4 Yuan	Total Fuzzy Cost x 10^4 Yuan
1	Nil	0	(33, 41, 53) ≈41.67	(7.85, 8.65, 9.45)	(3.3, 4.1, 5.3)	(11.15, 12.75, 14.75) ≈12.81
2	7--8	1	(32, 40, 52) ≈40.67	(7.85, 8.67, 9.58)	(3.2, 4.0, 5.2)	(11.05, 12.67, 14.78) ≈12.75
3	2--5	1	(31, 39, 51) ≈39.67	(7.86, 8.72, 9.78)	(3.1, 3.9, 5.1)	(10.96, 12.62, 14.88) ≈12.72
4	1--2	1	(30, 38, 50) ≈38.67	(7.88, 8.78, 9.93)	(3.0, 3.8, 5.0)	(10.88, 12.58, 14.93) ≈12.68
5	1--2	1	(29, 37, 49) ≈37.67	(7.90, 8.84, 10.08)	(2.9, 3.7, 4.9)	(10.80, 12.54, 14.98) ≈12.65

6	1--2	1	(28, 36, 48) ≈36.67	(7.92, 8.90, 10.23)	(2.8, 3.6, 4.8)	(10.72, 12.50, 15.03) ≈12.625
7	8--9	1	(27, 35, 47) ≈35.67	(7.93, 8.99, 10.53)	(2.7, 3.5, 4.7)	(10.63, 12.49, 15.23) ≈12.63
8	2--5	1	(26, 34, 46) ≈34.67	(7.94, 9.04, 10.73)	(2.6, 3.4, 4.6)	(10.54, 12.44, 15.33) ≈12.60
9	(7-8), (5-6) & (3-4)	1	(25, 33, 45) ≈33.67	(7.99, 9.17, 11.19)	(2.5, 3.3, 4.5)	(10.49, 12.47, 15.69) ≈12.67
10	(5-7), (5-6) & (4-6)	1	(24, 32, 44) ≈32.67	(8.17, 9.60, 12.57)	(2.4, 3.2, 4.4)	(10.57, 12.8, 16.97) ≈13.12

Graphical Representation

The figure 5.3 portrays the defuzzified results obtained by the proposed algorithm of a given project network.

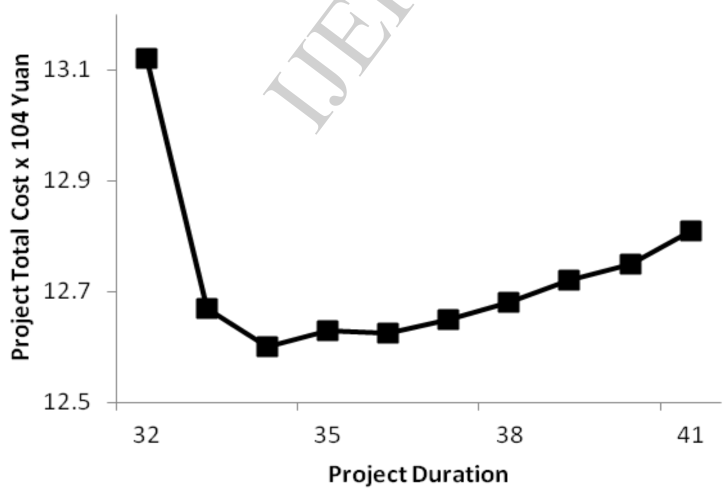


Figure 5.3

6. Conclusions

This paper investigates the unit based fuzzy expediting algorithm in the project networks. The validity of the proposed algorithm is examined with the illustration. By using the proposed approach, the project manager will be able to determine how much to crash each activity in order to minimize the total fuzzy cost of meeting any specified deadline for the project. This method can be served as an important tool for the project managers when they are handling various types of project networks in fuzzy environment.

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