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Unconventional Design Approach for Shaft Design

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***Abstract*- The present work proposes an unconventional design approach for designing mechanical components. The component discussed and designed below is any shaft used for transmission of torque / power. Traditional approach or method consists of designing a shaft assuming one support reaction at each bearing. Civil structures like beams, bridges, etc. are designed assuming multiple i.e. redundant supports. This method of using more supports than that are required is known as Indeterminate theory. This approach used to define shaft diameter is discussed in below article.**

Keywords- Shaft design, Indeterminate, Slope-deflection, Bending Moment.

I. INTRODUCTION

Sugar factories have undergone dramatic increase in processing rate over the past few years, which is possible only by increasing the crushing rate. Higher crushing rate has resulted in increase in roller mill size. Rollers are one of the most critical and bulky components thus making roller mills the highest power consuming section.

Increasing the roller mill size or expanding the mills by adding new rollers cannot always be the solution or rather not feasible each time. Also, increase in size means higher power consumption. With these limitations in mind, an attempt has been made to design mill shaft using unconventional design method or technique. The method is based on the concept of Indeterminacy or Indeterminate Structure as compared to conventional determinate theory.

Fig.1 presented in book "Cane Sugar Engineering" by Peter Rein shows critical section to be just adjacent to the journal bearing inner edge (section B-B).Also in "A basic understanding of the mechanics of rolling mill rolls" [1], Dr. Karl Heinrich Schroder and in "Life Prediction for the top roller shafts of the sugar mills [4]", S. A. Rodriguez, J. J. Coronado, N. Arzolahas confirmed that the frequency of shaft failures is maximum at this location.

Our area of interest will be obtaining shaft diameter at this critical section.

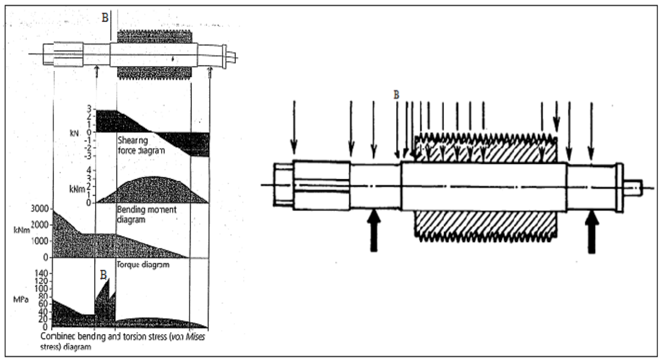

Figure 1: Diagram showing critical section of sugar roller mill shaft.

Fig.2 shows various dimensional parameters of shaft.

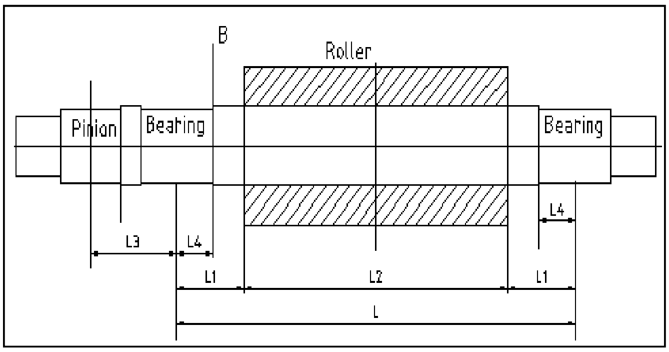

Figure 2: Schematic showing various dimensions of shaft.

Input Data:-
Power (P_o) = 940 HP;
Roller speed= 3.4 rpm;
Roller dia. d = 1270 mm
Baggase load= 560,000 kgf;
Self-weight= 40,000 kgf;
Total load= 5886 KN;
L1= 605 mm;
L2= 2540 mm;
L3= 890 mm;
L4= 385 mm;
L= 3750 mm.
Pressure angle ϕ = 250;
Gear Diameter = 1270 mm;
Modulus of elasticity/Young's Modulus E = 206.01 KN/mm^2.

II. INDETERMINATE STRUCTURE THEORY

Any structure is designed for the stress resultants of bending moment, shear force, deflection, torsional stresses, and axial stresses. If these moments, shears and stresses are evaluated at various critical sections, then based on these, the proportioning can be done. Evaluation of these stresses, moments and forces and plotting them for that structural component is known as analysis. Determination of dimensions for these components of these stresses and proportioning is known as design.

Determinate structures are analyzed just by the use of basic equilibrium equations. By this analysis, the unknown reactions are found for the further determination of stresses. Redundant or indeterminate structures are not capable of being analyzed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like compatibility conditions of deformations etc. to get the unknown reactions for drawing bending moment and shear force diagrams.

Examples of determinate structures are: simply supported beams, cantilever beams, single and double overhanging beams, etc.

Examples of indeterminate structures are: fixed beams, continuous beams, etc.

Special methods like strain energy method, slope deflection method, moment distribution method, column analogy method, virtual work method, matrix methods, etc. are used for the analysis of redundant structures.

Indeterminate Structures: A structure is termed as statically indeterminate, if it cannot be analyzed from principles of statics alone, i.e. $\sum H = 0, \ \sum V = 0, \ \ \sum M = 0$

TABLE 1: DIFFERENCE BETWEEN DETERMINATE AND INDETERMINATE STRUCTURES.

S.No.	Determinate Structures	Indeterminate Structures
1	Equilibrium conditions are fully adequate to analyze the structure.	Conditions of equilibrium are not adequate to fully analyze the structure.
2	Bending moment or shear force at any section is independent of the material property of the structure.	Bending moment or shear force at any section depends upon the material property.
3	The bending moment or shear force at any section is independent of the cross-section or moment of inertia.	The bending moment or shear force at any section depends upon the cross-section or moment of inertia.
4	Temperature variations do not cause stresses.	Temperature variations cause stresses.
5	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.
6	Extra conditions like compatibility of displacements are not required to analyze the structure.	Extra conditions like compatibility of displacements are required to analyze the structure along with the equilibrium equations.

III. METHODOLOGY

A Traditional / Conventional Approach:

Here, only one support reaction is assumed at each bearing. Therefore, we have two reactions.

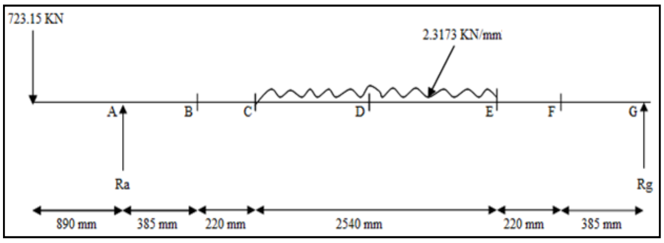

Figure 3: Schematic showing various forces and support reaction on shaft.

Design steps-

1. Determine Vertical & Horizontal components of all the loads acting on shaft.
2. Now considering Vertical Loading case and using the equations of equilibrium $\sum F_y = 0$, & $\sum M_g = 0$ find out all the support reactions namely Ra and Rg. Here, in determinate method, there are only two reactions – one each located at the center length of bearings.
3. Determine bending moments $M_{@A}$, $M_{@B}$ etc. at each points A, B, C, etc. using support reactions.
4. Plot bending moment diagram using these bending moments for vertical loading.
5. Repeat step 2. to step 4. for Horizontal Loading and plot bending moment diagram for horizontal loading as well.
6. A resultant bending moment at critical sections (Section B-B) is calculated.
7. Using equation of "basic shaft design as per *ASME Standard B106.1M"*, solid shaft diameter is determined. Various factors like surface factor, reliability factor, stress concentration factor, press-fitted collar factor, etc. affecting fatigue life are also considered.

Results:

Resultant Bending Moment @ B

$$= \sqrt{(M_{@_{Bh}})^2 + (M_{@_{Bv}})^2}$$

$$= 1291.9412 \times 10^3 \ KNmm$$

Calculation of shaft diameter [2]

$$d_{ob} = \left[\left(\frac{32 \times Fs1 \times B1}{\pi} \right) \times \left[\left(\frac{Mb}{\sigma_{fbf}} \right)^2 + \frac{3}{4} \times \left(\frac{T_d}{\sigma_{ys}} \right)^2 \right]^{1/2} \right]^{1/3}$$

$$d_{ob} = 637.05322 \ mm$$

B Indeterminate Approach:

Here, each bearing is assumed to have two reactions – one at each end. Therefore, we have four reactions as against two in conventional method.

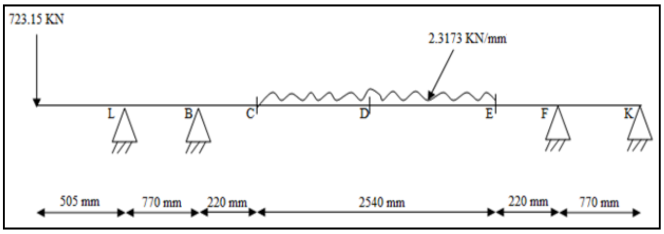

Figure 4: Schematic showing various forces and support reaction on shaft for Indeterminate approach.

Design Steps-

1. Determine Vertical & Horizontal components of all the loads acting on shaft.
2. Now for Vertical Loading case, first convert this indeterminate system into determinate by removing reactions at point B and F i.e. $R_b = 0$ and $R_f = 0$. So, now we have only two support reactions R_l *and* R_k. Using equations of equilibrium determine R_l *and* R_k.
3. Add equal UDL's to length E-F-R such that it still satisfies $\sum F_y = 0$.
4. Using basic equations of slope and deflection, determine deflection at B and F known as y_{BP} and y_{FP}.
5. Now, further removing all the loads and making reaction at B as unity i.e. $R_b = 1KN$, determine deflections at B and F known as y_{BB1} and y_{FB1}.
6. Similarly, find deflections at B y_{BF1} and F y_{FF1} when reaction at F is unity i.e. $R_f = 1KN$.
7. The total deflection at B $y_B = y_{BP} + y_{BB} + y_{BF}$. Where, $y_{BB} = y_{BB1} \times X_B$ and $y_{BF} = y_{BF1} \times X_F$. Therefore, we get $y_B = y_{BP} + (y_{BB1} \times X_B) + (y_{BF1} \times X_F)$. But since point B is supported by journal bearing, $y_B = 0$.
8. Therefore we get $0 = y_{BP} + (y_{BB1} \times X_B) + (y_{BF1} \times X_F)$.
9. Similarly, we get $0 = y_{FP} + (y_{FB1} \times X_B) + (y_{FF1} \times X_F)$.
10. Solving above two equations, we get reactions at B and F i.e. X_B and X_F.
11. Using equations of equilibrium determine bending moment at various points for vertical loading case.
12. Repeat steps 2 to steps 11 for horizontal loading and determine bending moment.
13. Find resultant bending moment at B (i.e. critical section).
14. Using equation of "basic shaft design as per *ASME Standard B106.1M*,.solid shaft diameter is determined. Various factors like surface factor, reliability factor, stress concentration factor, press-fitted collar factor, etc. affecting fatigue life are also considered.

Results:

Resultant Bending Moment @ B

$$= \sqrt{(M_{@Bh})^2 + (M_{@Bv})^2}$$

$$= 531.2487 \times 10^3 \; KNmm$$

Calculation of shaft diameter [2]

$$d_{ob} = \left[\left(\frac{32 \times Fs1 \times B1}{\pi} \right) \times \left[\left(\frac{Mb}{\sigma_{fbf}} \right)^2 + \frac{3}{4} \times \left(\frac{T_d}{\sigma_{ys}} \right)^2 \right]^{1/2} \right]^{1/3}$$

$$\boxed{Dia.\ at\ B\ calculated\ as = d_{ob} = 605.946\ mm}$$

IV. VALIDATION USING FEA

To validate the result, Static Structural Analysis was performed using ANSYS.

Resultant bending moment similar to above theoretical calculation was derived using Vertical & Horizontal components of all the loads acting on shaft.

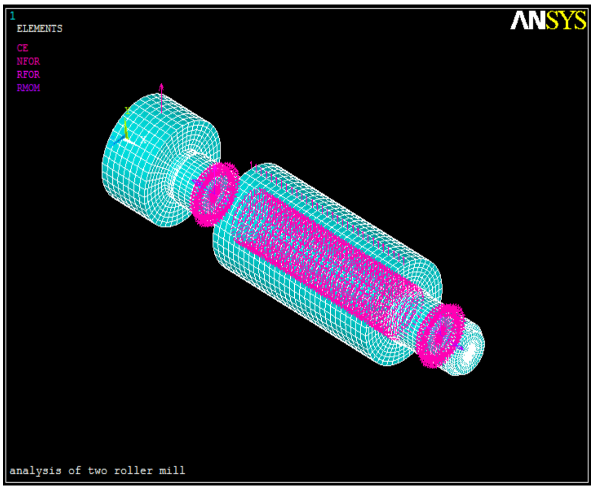

Figure 5 : Constraint at bearing center for vertical direction.

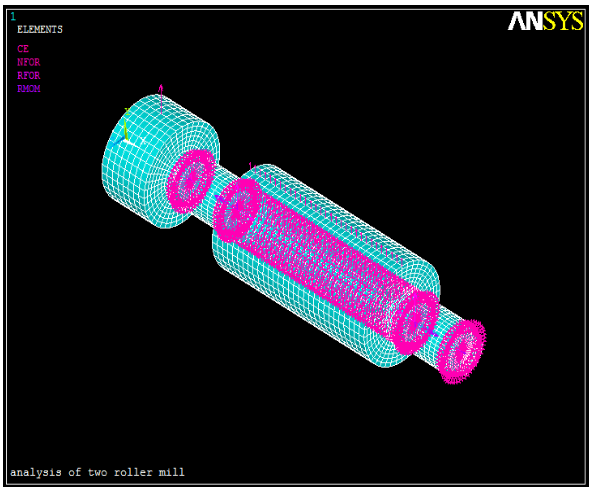

Figure 6 : Constraint at end of bearing for vertical direction.

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A resultant bending moment at B using ANSYS was $415.4 \times 10^3\ KNmm$. This value is close by to that obtained theoretically. Thus we can say that an Indeterminate Approach is fairly optimistic in designing shaft.

V. CONCLUSION

Based on above results following conclusion can be arrived:

1. With determinate method, the shaft diameter is 637.053 mm at the critical section B-B.
2. By using an approach of indeterminate system, the shaft diameter at same section B-B reduces to 605.943 mm.
3. Bending moment reduces to $531.2487 \times 10^3\ KNmm$ as against $1291.9412 \times 10^3\ KNmm$.
4. Simulation using ANSYS predicts the resultant bending moment to be $415.4 \times 10^3\ KNmm$. This value is close-by to the theoretical calculations.
5. In indeterminate approach, by taking into account the considerable bearing width and multiple reactions at its ends a significant reduction in bending moment at bearing is observed as compared to single reaction at center of bearing.
6. Even though bending moment appears to reduce significantly, the change in shaft diameter is small. This is because, bending moment is not the only factor controlling shaft design, other factors like torque, axial force, fatigue, stress concentration also affect shaft design and needs to be considered in shaft design.

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