Unbalanced Radial Distribution System Load Flow and Voltage Profile Enhancement in the presence of Distributed Generators

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***Abstract*— This paper presents the siting and sizing of Distributed Generators (DGs) in a three-phase unbalanced radial distribution system. This method exploits the radial structure of the network and solves the distribution load flow. Mutual coupling between the phases has been included in the mathematical model. The concept of finding the bus with maximum deviation from standard voltage profile is used to site a DG with suitable size in order to maintain the voltage profile of system as close as possible to standard voltages i.e 1p.u. Forward/Backward sweep method is used for the load flow studies. The proposed method is tested on a practical unbalanced three phase distribution feeder emanating from Pathardhi 132/11KV Grid substation in India and the results are presented in agreements with the literature and show that the proposed methodology is valid and reliable.**

***Keywords*— unbalanced distribution load flow, radial network, voltage profile, optimal placement, distributed generation, *Indian bus system*.**

I. INTRODUCTION

with the advent in technology and emerging trends in the area of power generation, renewable sources are gaining more popularity. As we all know that the renewable technology is more eco-friendly and cost effective, hence we are trying to harness these sources locally through wind turbines, PV modules, DG sets etc.[1]. By incorporating DG technology in our distribution system we can achieve reduction in system losses, improvement in voltage profile, system reliability etc.

An Electrical system consists of two parts- i. Power system ii. Distribution system. Power system is considered to be balanced while Distribution system is unbalanced in nature. Power system is highly mesh connected and distribution system is highly radial in nature. Due to radial nature, there is consecutive increase in voltage drop from source bus to end buses. Voltage collapse usually occurs in heavily loaded systems that do not have sufficient local reactive power support and consequently cannot provide secure voltage profile for the system. This reactive power shortage may lead to wide-area blackouts and voltage-stability problems as has occurred in many countries[2]. The shortage can be alleviated by an increased share of DGs in low-voltage (LV) distribution systems to improve voltage stability. For analysis of system we carry out load flow studies. Load flow studies are used to study the electrical power transfer from generators to consumers through the grid system[3]. The main objective of the load flow analysis is to find out the power flow in each line along with the voltage at each bus of the system for the specific loading conditions. Chiang [4] has presented a distribution load flow method by iterative solution of three fundamental equations representing real power, reactive power and voltage magnitude. Das, *et al.* [5]have proposed a load flow method by writing an algebraic equation for bus voltage magnitude. Distribution system with their radial structure and wide ranging resistance and reactance values are inherently ill-conditioned, hence the conventional load flow methods like Gauss-Seidel, Newton-Raphson and fast decoupled techniques are inefficient in solving such networks. For this reason we go for other techniques of distribution load flow. The load flow methods proposed for distribution systems considering the unbalance operation can be grouped into two basic categories. The first category is Forward Backward Sweep (FBS) / Ladder Network based methods, Loop Impedance Method and Implicit Zbus Gauss method or its modified versions[6]. The second category is composed of methods which require information on the derivatives of the network equations. Newton like methods involving formation of Jacobian and computation of power mismatches at the end of the feeder and laterals and other fast decoupled methods. Here in this paper we have opted Forward/Backward approach for the load flow studies.

II. UNBALANCED THREE-PHASE LINE

Fig.1 shows a typical section of the distribution system between bus i and j. The line parameters can be obtained by the method developed by Carson and Lewis[7]. A 4x4 matrix, which takes into account the self and mutual coupling effects of the unbalanced three phase line section, can be expressed as:

$$Zabcn=\begin{bmatrix} Zaa & Zab & Zac & Zan \\ Zba & Zbb & Zbc & Zbn \\ Zca & Zcb & Zcc & Zcn \\ Zna & Znb & Znc & Znn \end{bmatrix}. \quad (1)$$

This equation can be reduced to 3x3 matrix, by applying Kron's reduction[8], still considering effects of neutral and ground wire as shown in (2).

$$Zabc=\begin{bmatrix} Zaa & Zab & Zac \\ Zba & Zbb & Zbc \\ Zca & Zcb & Zcc \end{bmatrix}. \quad (2)$$

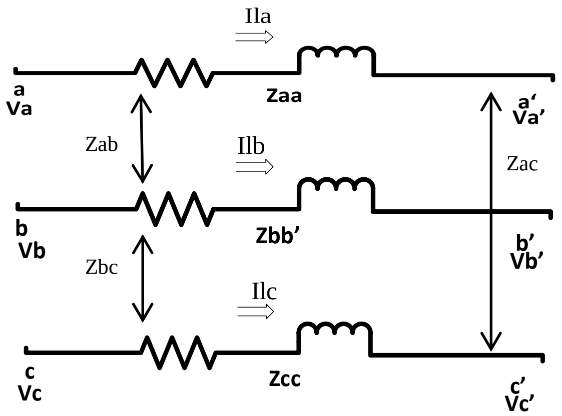

Fig.1. Typical Three Phase Feeder

Now, the receiving end voltages can be linked with the sending end voltages of the feeder shown in Fig.1 as below:

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} = \begin{bmatrix} Va' \\ Vb' \\ Vc' \end{bmatrix} + \begin{bmatrix} Zaa & Zab & Zac \\ Zba & Zbb & Zbc \\ Zca & Zcb & Zcc \end{bmatrix} \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}. \quad (3)$$

The same approach will be used for two phase system as well as single phase system. In a two phase system the phase which is not connected will have the corresponding Impedance matrix's row and column zero.

III. PROBLEM FORMULATION

In this paper we are determining the optimal location and size of the DGs such that voltage profile is improved which further reduces the system losses.

A. *Objective Function:*

The objective function aims at minimizing the Bus Voltage Deviation Index (BVDI).

$$F(i) = \min(BVDI_i) \quad (4)$$

BVDI is the average voltage deviation of all the three phases calculated at each bus.

B. *Constraints:*

For proper system operation certain constraints must be satisfied. These constraints are as follows:

1) *Active and reactive power equality:*

Total generated power must be equal to the power demand given as (5), (6)

$$\sum_{i=1}^{n} P_{Gi} - P_{DG} - \sum_{i=1}^{n} P_{Di} = 0 \quad (5)$$

$$\sum_{i=1}^{n} Q_{Gi} - \sum_{i=1}^{n} Q_{Di} = 0 \quad (6)$$

Where P_{Gi}, Q_{Gi} are active and reactive power generated at bus i, P_{Di}, Q_{Di} are active and reactive loads at bus i, P_{DG} is active power injected by DG.

2) *Distribution Line Capacity Limits:*

Power flow of lines should be less than the allowed flow constrained by its thermal limit.

$$S(i,j) \le S(i,j)_{max} \quad (7)$$

Where S(i,j) is MVA flow in the line connecting bus i and j, $S(i,j)_{max}$ is MVA capacity of line connecting i and j.

3) *Voltage Drop limits:*

Bus voltages should be in the range of minimum and maximum voltage.

$$V_{min} < V < V_{max} \quad (8)$$

Where V_{min} and V_{max} are minimum and maximum allowable voltage at buses.

IV. SOLUTION METHODOLOGY

These days, most DG technologies, such as synchronous machines, power-electronic interface devices (e.g., photovoltaic cells and micro turbines), and even new induction generators [e.g., doubly fed induction generators (DFIGs)], are capable of providing a fast, dynamic reactive power response[9]. This capability can be used by the system operators to enhance system security and stability. Since a generator location affects the system voltage stability, finding the best position for its installation becomes inevitable, as improper placement may further deteriorate the system performance. To identifying the most effective bus for DG installation following procedure is followed:

A. *DG unit positioning algorithm:*

- Calculate the average voltage of all buses in each phase.

$$Vavg(m) = \frac{1}{n}\sum_{i=1}^{n} V(i,m) \quad (9)$$

Where n is number of buses in the system, m is the phase number.

- Calculate the deviation of each bus voltage from Vavg for all phases.

$$VD(m,i) = V(m,i+1) - Vavg(m) \qquad (10)$$

Where m=1 to 3 and i=1, 2, 3…….n-1.

- Now, calculate bus voltage deviation index BVDI which is the average of Voltage deviation of all the three phases.

$$BVDI(i) = \frac{1}{3}\sum_{m=1}^{3} VD(m,i) \quad \text{i=1, 2, .n-1.} \qquad (11)$$

Now determine the order with the least value of BVDI, 'i'. The bus next to 'i' is considered as the best location for DG placement of proper size given as *DGpos*.

$$DGpos = i+1 \qquad (12)$$

B. Forward/Backward Sweep Load Flow:

The distribution system is very much different from the transmission system as it is more of radial system with low X/R ratio. Thus the conventional load flows like N.R, Gauss Siedel [10][11]etc. fails to converge or are not reliable. Hence in this paper forward backward (FB) sweep load flow is used [12] to compute load flow.

$$[Vabc] = [Vabc'] + [Zabc][Iabc]. \qquad (13)$$

$$Ii = (Pi - Qi)/Vi* \quad \text{i=2, 3…, n} \qquad (14)$$

The FB sweep considers the complex load at each bus to be constant. The load is then converted into equivalent current injection at each bus using equation 14. The FB sweep method takes two steps in each iteration[13]. The first step is called backward sweep wherein starting from the last bus we calculate the branch current till the first bus, given by (14)]. Now the second step is called forward sweep wherein starting from the first bus, voltage is calculated for all the buses using equation (13).

$$Il_j = \text{Current of 'j'th branch} = \sum Il_i \qquad (15)$$

Where i=all subsequent buses to 'j'

C. Algorithm for Load Flow:

Using the above equation (13), (14) and (15) we develop the load flow algorithm for three phase radial distribution system:

- Input the data about the distribution system.
- Assume the initial voltage magnitude at all buses to be 1p.u and voltage phase to be 0,-120 and 120 for phase-a, b and c respectively.
- Construct the Z-bus matrix of the system as per the given data considering mutual inductances[15].
- Set iter=1.
- Calculate load current at each bus using (14).
- Compute the branch currents starting from end nodes using (15).
- Use the (13) to calculate the updated bus voltageV_{iter}+1.
- Compare the present value of the voltage with that of the previous iteration, if converge, go to next step, otherwise iter=iter+1 and go to step-5.
- Calculate losses after the converged voltages are obtained using (16).

$$Sloss = (Vp * Ilpq^*) - (Vq * Ilpq^*) \qquad (16)$$

Where *Vp* is the sending end voltage and *Vq* is the receiving end voltage and *Ilpq* is the current flowing in the respective line.

$$Ploss = real(Sloss) \qquad (17)$$

$$Qloss = imag.(Sloss) \qquad (18)$$

D. DG unit size allocation algorithm:

For DG sizing, first we will fix the maximum limit of DGs based on the number of DGs 'k' we want install.

$$P_{DG\max} = 100/k. \qquad (19)$$

The maximum size of DG considered can be up to P_{DGmax}.
The equivalent aggregated load is calculated as follows:

$$Pload = \sum_{i=1}^{n} Pi \qquad (20)$$

$$Qload = \sum_{i=1}^{n} Qi \qquad (21)$$

Where *Pload* is the equivalent active power component of the aggregated load; *Pi* is the active power component of the load at bus number *i*; *Qload* is the equivalent reactive power component of the aggregated load; *Qi* is the reactive power component of the load at bus number *i*; and *n* is the total number of buses.

To determine the size of DG to be applied at *DGpos* obtained from (12) certain steps are followed:

1. At the obtained DGpos we will apply generation in steps of 1% of Total load to the P_{DGmax} and calculate the voltages at each bus.
2. Check for the violation of any constraint.
3. If YES then STOP at this generation. If No then return to step 1.
4. DG generation, for which the violation occurs, is stored and the generation previous to it is considered as the optimal size.
5. This is the optimal size of DG generation, to be applied at DGpos for improving the voltage profile at each bus.

TABLE I
BASE CASE LOAD FLOW RESULTS

Bus No.	Phase A		Phase B		Phase C	
	Magn.	Angle	Magn.	Angle	Magn.	Angle
1	1.0000	0.0000	1.0000	2.0944	1.0000	-2.0944
2	0.9860	0.0009	0.9879	2.0951	0.9865	-2.0943
3	0.9849	0.0012	0.9876	2.0951	0.9856	-2.0945
4	0.9803	0.0011	0.9821	2.0954	0.9810	-2.0940
5	0.9802	0.0011	0.9819	2.0954	0.9809	-2.0940
6	0.9769	0.0013	0.9786	2.0956	0.9777	-2.0938
7	0.9765	0.0013	0.9783	2.0956	0.9774	-2.0938
8	0.9697	0.0015	0.9708	2.0959	0.9703	-2.0934
9	0.9620	0.0018	0.9620	2.0960	0.9617	-2.0927
10	0.9512	0.0024	0.9503	2.0962	0.9497	-2.0919
11	0.9498	0.0026	0.9490	2.0962	0.9478	-2.0918
12	0.9495	0.0024	0.9484	2.0964	0.9482	-2.0918
13	0.9495	0.0027	0.9485	2.0961	0.9472	-2.0918
14	0.9492	0.0026	0.9485	2.0962	0.9473	-2.0918
15	0.9473	0.0024	0.9455	2.0964	0.9456	-2.0915
16	0.9488	0.0023	0.9471	2.0964	0.9474	-2.0916
17	0.9483	0.0027	0.9479	2.0963	0.9467	-2.0919
18	0.9489	0.0027	0.9481	2.0962	0.9469	-2.0918
19	0.9466	0.0023	0.9447	2.0965	0.9451	-2.0914

TABLE II
OPTIMIZATION RESULTS

Phase	W/o DG		With One DG		With Two DG	
	Ploss (KW)	Qloss (KVar)	Ploss (KW)	Qloss (KVar)	Ploss (KW)	Qloss (KVar)
A	16.596	7.033	7.222	2.738	6.202	2.324
B	16.479	7.173	6.185	2.878	5.326	2.513
C	16.972	7.312	7.211	3.247	6.216	2.791

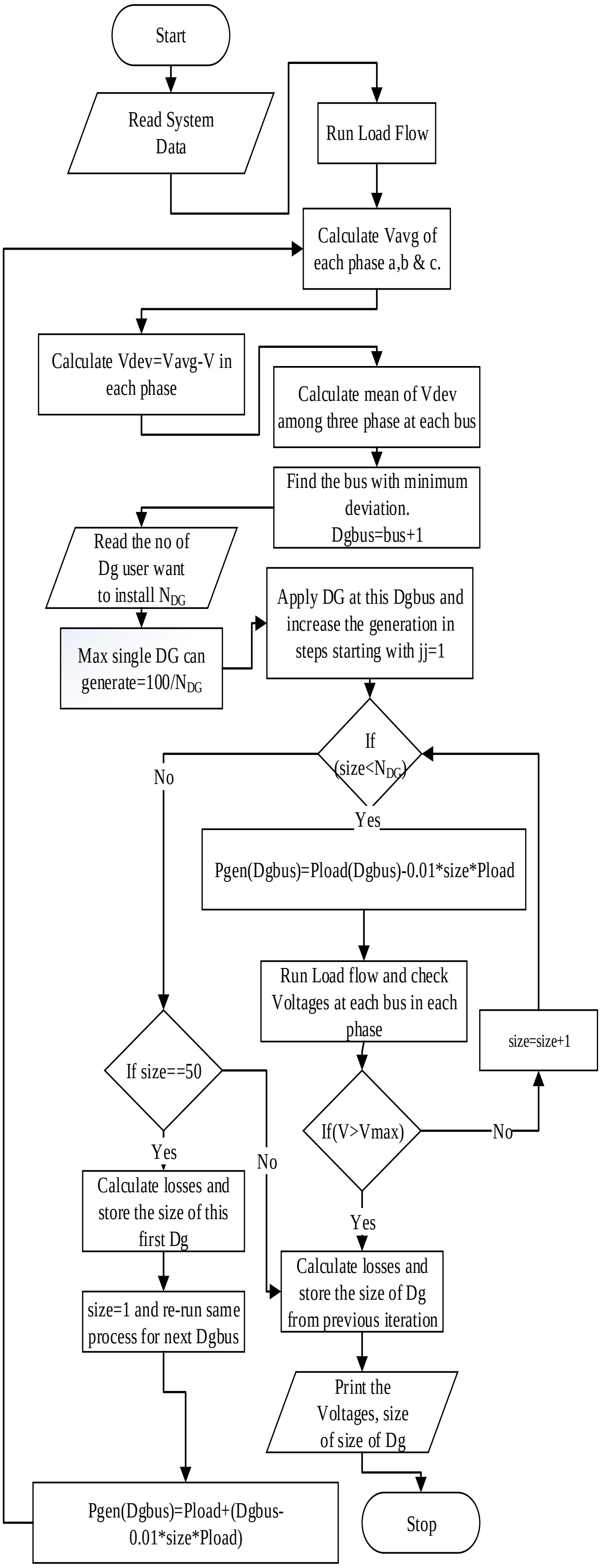

Fig.2. Flowchart for optimal DG placement.

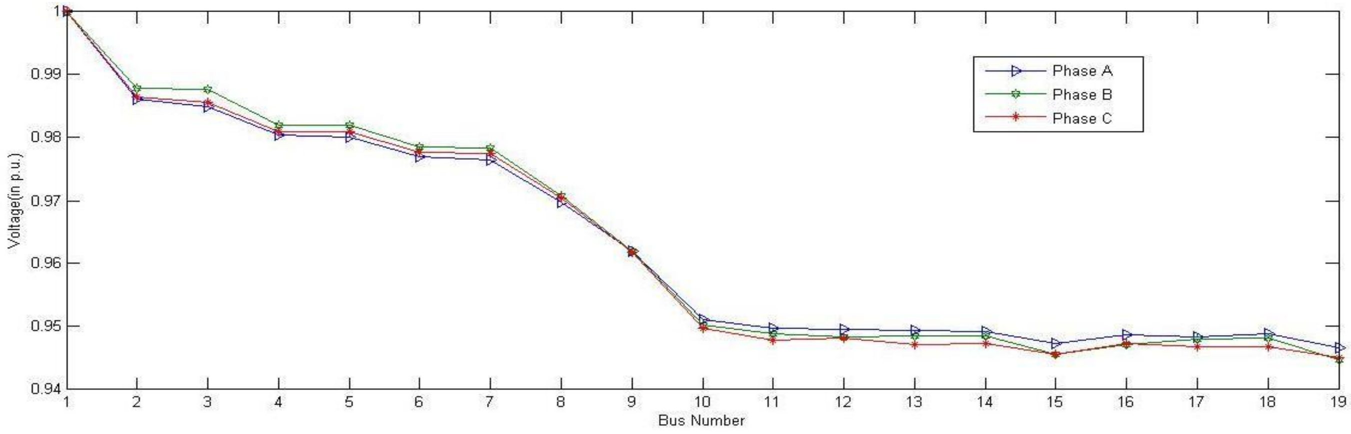

Fig.3 Voltage profile before DG placement

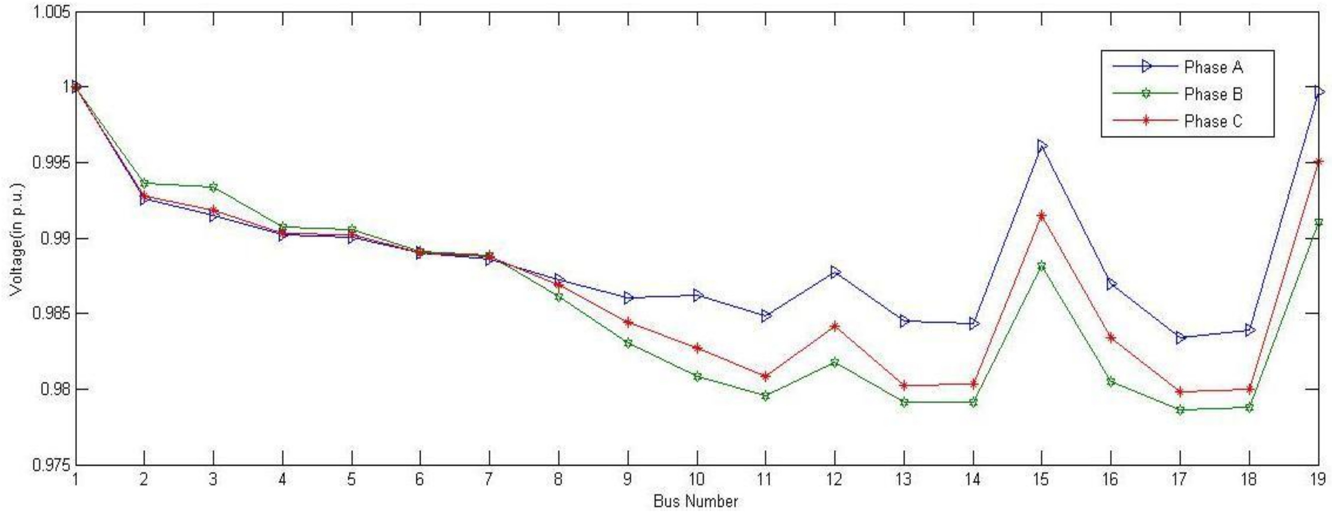

Fig.4 Voltage profile with single DG placed.

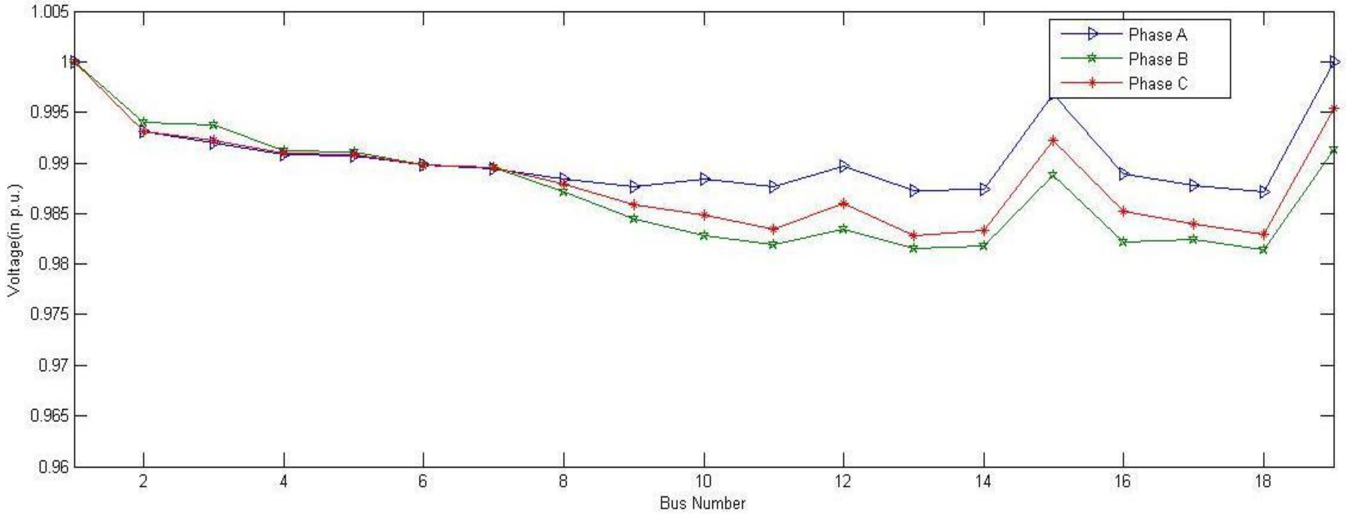

Fig.5 Voltage profile with two DGs placed.

V. RESULTS AND DISCUSSION

The proposed algorithm is tested on practical unbalanced three phase distribution feeder emanating from Pathardhi 132/11KV Grid substation in India. Using the proposed algorithm, following optimum DG locations and sizes have been calculated. Fig.5 shows the effectiveness of the proposed algorithm on system performance in comparison with base case (without DG) in Fig.3. The optimum DG size and position when only one DG is placed are found to be 56% of total load with DG placed at 19th bus. When two DGs are placed first DG size is inferred to be 50% of total load placed at 19th bus and the second DG size is inferred to be 19% of remaining load placed at 17th bus. The base case voltages of each phase for the given loading are shown in Table I. From the figures we can see that DGs had improved the voltage profile to great extent. Thus reducing the system losses without exceeding the line limits. From Table I we see that the minimum voltage in the absence of any DG was 0.9447 p.u. at 19th bus for phase B while as given in Table V, we can see with two DGs optimally placed the minimum system voltage has risen to 0.9815 p.u at 13th bus for phase B.

TABLE III
LOAD DATA

	Pload(KW)	Qload(KVar)
Phase A	421.100	204.100
Phase B	387.800	187.800
Phase C	410.900	199.000

VI FUTURE SCOPE

This method is presently tested on practical unbalanced three phase distribution feeder emanating from Pathardhi 132/11KV Grid substation in India, which can further be extended to even larger bus system present in literature. Here, DGs are considered to generate only active power however it can be extended to DGs operating at different power factors. The loads are considered to be constant which can be extended to time varying loads. We have considered BVDI as our objective for siting the location which can be extended to multi-objective optimization.

TABLE IV
DISTRIBUTED GENERATION INPUT

	One DG	Two DG	
Location	19th	19th	17th
Phase A	235.82	210.550KW	40.005KW
Phase B	217.17	193.900KW	36.841KW
Phase C	230.10	205.450KW	39.036KW

VII. CONCLUSION

In this paper new Bus voltage deviation algorithm has been proposed for optimal placement A systematic simple approach to allocate multiple DG units in radial unbalanced distribution network is proposed, based on voltage deviation index a straightforward algorithm for sizing and locating multiple DG units is developed[16]. The proposed technique is applied to radial test systems. Results show that installing the decided DG units achieves great reduction in power loss and keep node voltages between Vmin and Vmax. This method is easy to implement and the results obtained are found to be good, as the optimal locations and size of DGs have improved the voltage profile as can be seen by graphs and the system losses are also.

TABLE V
FINAL VOLTAGE PROFILE AFTER PLACEMENT OF TWO DG

Bus No.	Phase A		Phase B		Phase C	
	Magn.	Angle	Magn.	Angle	Magn.	Angle
1	1.0000	0.0000	1.0000	2.0944	1.0000	-2.0944
2	0.9931	0.0036	0.9940	2.0975	0.9931	-2.0913
3	0.9919	0.0039	0.9937	2.0975	0.9922	-2.0915
4	0.9909	0.0052	0.9912	2.0990	0.9909	-2.0895
5	0.9907	0.0052	0.9911	2.0990	0.9908	-2.0895
6	0.9898	0.0062	0.9898	2.1000	0.9898	-2.0884
7	0.9894	0.0062	0.9895	2.1000	0.9895	-2.0884
8	0.9884	0.0087	0.9871	2.1024	0.9879	-2.0854
9	0.9876	0.0118	0.9844	2.1050	0.9859	-2.0817
10	0.9884	0.0170	0.9828	2.1094	0.9848	-2.0759
11	0.9876	0.0174	0.9820	2.1095	0.9835	-2.0756
12	0.9897	0.0181	0.9834	2.1105	0.9859	-2.0745
13	0.9873	0.0174	0.9815	2.1094	0.9829	-2.0755
14	0.9875	0.0176	0.9819	2.1097	0.9833	-2.0755
15	0.9969	0.0220	0.9888	2.1140	0.9923	-2.0701
16	0.9889	0.0180	0.9822	2.1105	0.9852	-2.0744
17	0.9878	0.0181	0.9824	2.1102	0.9840	-2.0750
18	0.9871	0.0176	0.9816	2.1097	0.9829	-2.0754
19	1.0000	0.0234	0.9913	2.1155	0.9954	-2.0683

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