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Two -Phase Free Convection Flow and Heat Transfer From A Vertical Flat Plate

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Abstract

Momentum integral method has been employed to study the effect of suspended particulate matter (SPM) on two-phase free convection heat transfer from a vertical plate. The presence of SPM increases the velocity of the carrier fluid where as diffusion of particles and concentration of particles in the fluid has little effect on it. The Grashoff number has a effect to increase the carrier fluid velocity but Prandtl number has a effect to decrease the magnitude of particle velocity and temperature. It is observed that, the heat is transferred from plate to fluid.

Key words: Particulate suspensions; boundary layer characteristics; volume fraction; heat transfer.

Nomenclature:

$(x, y,)$	→	Space co-ordinates i.e. distance along the perpendicular to plate length
$\vec{q}(u, v)$	→	Velocity components for the fluid phase in x and y directions respectively
$\vec{q}_p(u_p, v_p)$	→	Velocity components for the particle phase in x and x and $y$directions respectively
(T, T_p)	→	Temperature of fluid and particle phase respectively
(T_w, T_∞)	→	Temperature at the wall and free-stream respectively
(v, v_p)	→	Kinematic coefficient of viscosity of fluid and particle phase respectively
(ρ, ρ_p)	→	Density of fluid and particle phase respectively
(ρ_{sp}, ρ_m)	→	Material density of particle and mixture respectively
(μ, μ_p)	→	Coefficient of viscosity of fluid and particle phase respectively
(τ_p, τ_T)	→	Velocity and thermal

		equilibrium time
(c_p, c_s)	$\rightarrow$	Specific heat of fluid and SPM respectively
$\vec{g}(0, g)$	$\rightarrow$	Gravitational acceleration vector
Re	$\rightarrow$	Fluid phase Reynolds number
Pr	$\rightarrow$	Prandtl number
Ec	$\rightarrow$	Eckret number
Nu	$\rightarrow$	Nusselt number
Gr	$\rightarrow$	Grassoff number
Fr	$\rightarrow$	Froud number
c_f	$\rightarrow$	Skin friction coefficient
p	$\rightarrow$	Pressure of fluid phase
φ	$\rightarrow$	Volume fraction of Suspended particulate matter (SPM)
d	$\rightarrow$	Diameter of the particle
a	$\rightarrow$	Radius of each particle
δ	$\rightarrow$	Boundary layer thickness
a	$\rightarrow$	Thermal diffusivity
κ	$\rightarrow$	Thermal conductivity
α	$\rightarrow$	Concentration parameter
γ	$\rightarrow$	Loading ratio$\left(= \rho_{sp}/\rho\right)$
β	$\rightarrow$	Coefficient of volume expansion
ϵ	$\rightarrow$	Diffusion parameter
ϵ_p	$\rightarrow$	Particle mass fraction
ρ_0	$\rightarrow$	$M_h \times M_g \times 2.69 \times 10^{19}\ cm^{-3}$,the fluid density at STP
M_h	$\rightarrow$	1.67x 10^{-27} mass of hydrogen at STP
M_g	$\rightarrow$	Molecular weight of fluid
F	$\rightarrow$	Friction parameter between the fluid and the particle $\left(F = 1/\tau_p = 18\mu/\rho_p d^2\right)$
L	$\rightarrow$	Reference / Characteristic length
U	$\rightarrow$	Free stream velocity
A	$\rightarrow$	δ/L

Subscripts:

∞	$\rightarrow$	Values at free stream
w	$\rightarrow$	Values on the plate/wall

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1. Introduction

The study of a flow situation where free and forced convention effects are of comparable order, finds application in several industrial and technical processes such as nuclear reactors cooled during emergency situation, solar central receivers exposed to winds, electronics devices cooled by fans and heat exchanges placed in a low-velocity environment. The simplest physical model convention flow is the two dimensional laminar mixed convection flow along a vertical flat plate and extensive studies have been conducted on this type flow [1- 4].

The effect of buoyancy forces in flow and heat transfer is usually ignored when a forced convection flow over a cooled or heated surface is studied. But it is not justified as the buoyancy forces influences the flow and temperature functions despite the presence of forced convention flow.

The steady, laminar, free convective two phase boundary layer flow of a viscous incompressible fluid with suspended particulate matter along a semi-infinite vertical flat plate is considered.

It has shown Rudinger [5], that the particle contribution towards the pressure can be neglected even at extremes of the density ratio and mass fraction as long as the particle diameter "d" is larger than a few hundredths of a micron i.e. $d \geq \left[\left(\frac{1-\epsilon}{\epsilon_p}\right)\left(\frac{\rho_{sp}}{\rho_0}\right)\right]^{-1/3} \times 1.92198 \times 10^{-6}$ Further the particle volume fraction $\varphi = \left(\frac{\epsilon_p}{1-\epsilon_p}\right)\left(\frac{\rho}{\rho_{sp}}\right)$ may become significant, if the density ratio or the loading ratio or both become sufficiently large. If we consider the gravitational force as a body force, we should also include buoyancy force as a part of the interaction force between the pseudo-fluid and gas phases. Buoyancy force on pseudo-fluid is $-\varphi\rho g$. If we combine buoyancy force with the corresponding body force due to gravitational acceleration, we get for the pseudo-fluid, the total force due to gravitational acceleration as $\varphi(\rho_{sp}-\rho)g$.

Soo[6] has clarified the condition under which the gravity effect is negligible. the conditions are the gravity effect on the fluid phase must be small, compared with the effect of friction. In addition to the above conditions one of the following conditions:

1. The density of the material of the fluid phase and the particle phase are equal
2. In case of Turbulent flow, the Intensity of the random particle motion due to fluid drag is much greater than that due to the gravity effect.
3. In case of Turbulent flow of electrically charged particles, the intensity of the particle motion due to electrostatic force is greater than g/F .

2. Mathematical Formulation

In the present problem, to show the free convections effects, we have considered a very long and wide plane perpendicular to the floor, which is either heated or cooled. In either case the mathematical solution is same. The x-coordinate and y-coordinate are chosen along and perpendicular to the plate. Further the temperature of the heated or cooled surface be taken as T_w and the temperature decreases asymptotically to the value of ambient fluid T_∞ far away. The fluid viscosity on the plate increases from zero at the plate surface to a maximum value in the immediate vicinity of the surface and there after decreases asymptotically to a zero value far away. The velocity and temperature of the particle phase differs from fluid velocity on the plate and approaches asymptotically to the fluid velocity and temperature.

Assuming the boundary layer flow laminar, and since the flow is two dimensional in the present consideration and steady the boundary layer simplification gives the following governing equations for the present flow problems. Here in free convection the buoyancy force sustains the fluid motion, and hence the gravity term $-(1-\varphi)\rho g$ must be included in momentum integral equations.

The boundary layer equations for two dimensional two phase flow after simplifications are given by

$$\frac{\partial}{\partial x}[(1-\varphi)u]+\frac{\partial}{\partial y}[(1-\varphi)v]=0 \qquad (1)$$

$$\frac{\partial}{\partial x}(\varphi\rho_{sp}u_p)+\frac{\partial}{\partial y}(\varphi\rho_{sp}v_p)=0 \qquad (2)$$

$$(1-\varphi)\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-(1-\varphi)\frac{\partial p}{\partial x}+\mu(1-\varphi)\frac{\partial^2 u}{\partial y^2}+\frac{\varphi\rho_{sp}}{\tau_p}(u_p-u)-(1-\varphi)\rho g \qquad (3)$$

$$\frac{\partial p}{\partial y}=o(\delta) \qquad (4)$$

$$\varphi\rho_{sp}\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\varphi\mu_{sp}\frac{\partial^2 u_p}{\partial y^2}+\frac{\varphi\rho_{sp}}{\tau_p}(u-u_p)+\varphi(\rho_{sp}-\rho)g \qquad (5)$$

$$(1-\varphi)\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=(1-\varphi)k\frac{\partial^2 T}{\partial y^2}+\frac{\varphi\rho_{sp}c_s}{\tau_T}(T_p-T)+(1-\varphi)\mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6)$$

$$\varphi\rho_{sp}c_s\left(u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=\frac{\varphi\rho_{sp}c_s}{\tau_T}(T-T_p) \qquad (7)$$

In free convection, outside the boundary layer $y\to\delta\ as\ \rho\to\rho_\infty$, where as u and v approaches to zero, equation (3) becomes

$$0=-(1-\varphi)\frac{\partial p}{\partial x}-(1-\varphi)\rho_\infty g$$

$$\text{Or, } \frac{\partial p}{\partial x}=-\rho_\infty g \qquad (8)$$

Putting (8) in (3), we get

$$(1-\varphi)\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu(1-\varphi)\frac{\partial^2 u}{\partial x^2}-(1-\varphi)(\rho-\rho_\infty)g+\frac{\varphi\rho_{sp}}{\tau_p}(u_p-u) \qquad (9)$$

The coefficient of thermal expansion β is given by $\beta=-\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$

$$\text{Or, } \beta=-\frac{1}{v}\frac{v-v_\infty}{T-T_\infty}=\frac{\rho_\infty-\rho}{\rho(T-T_\infty)}$$

$$\text{Or, } \rho_\infty-\rho=\beta\rho(T-T_\infty) \qquad (10)$$

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Substituting (10) in (9), we get

$$(1-\varphi)\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu(1-\varphi)\frac{\partial^2 u}{\partial x^2}$$

$$+(1-\varphi)\rho g\beta(T-T_\infty)+\frac{\varphi\rho_{sp}}{\tau_p}(u_p-u)$$

$$\text{Or, }\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)$$

$$+\left(\frac{\varphi}{1-\varphi}\right)\frac{\rho_{sp}}{\rho}\frac{(u_p-u)}{\tau_p} \qquad (11)$$

Hence the governing equations for the present problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (12)$$

$$\frac{\partial(\rho_p u_p)}{\partial x}+\frac{\partial(\rho_p v_p)}{\partial y}=0 \qquad (13)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+\gamma\frac{\varphi}{1-\varphi}\frac{1}{\tau_p}(u_p-u)$$

$$+g\beta(T-T_\infty) \qquad (14)$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=v_{sp}\frac{\partial^2 u_p}{\partial y^2}+\frac{1}{\tau_p}(u-u_p)$$

$$+\left(1-\frac{\rho}{\rho_{sp}}\right)g \qquad (15)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

$$+\gamma\frac{\varphi}{1-\varphi}\frac{c_s}{c_p}\frac{1}{\tau_T}(T_p-T) \qquad (16)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=\frac{1}{\tau_T}(T-T_p) \qquad (17)$$

We need one more auxiliary condition to make the system consistent. Here we use the condition that flux of the particulate mass across any control volume is zero

$$\text{i.e} \qquad \rho_{p\infty}U\delta=\int_0^\delta \rho_p u_p\,dy \qquad (18)$$

Boundary condition for dimensional quantities

$$At\ \ y=0: u=0, u_p=u_{pw},\ \rho_p=\rho_{pw},$$

$$T=T_w, T_p=T_{pw}$$

$$At\ \ y=\delta: u=0,\ u_p=0,\ \rho_p=\rho_{p\infty},$$

$$T=T_\infty,\quad T_p=T_\infty \qquad (19)$$

Also, for smooth velocity and temperature profiles we assume at $y=\delta, \frac{\partial T}{\partial y}=0,\ \frac{\partial u}{\partial y}=0.$

Integrating (14) to (17) between the limit $y=0$ to $y=\delta$, we get

$$\frac{d}{dx}\int_0^\delta u^2dy=v\left(\frac{\partial u}{\partial y}\right)_{y=0}dy+g\beta\int_0^\delta(T-T_\infty)dy$$

$$+\frac{\varphi}{(1-\varphi)}\frac{1}{\tau_p}\gamma\int_0^\delta(u_p-u)dy \qquad (20)$$

$$\frac{d}{dx}\int_0^\delta \rho_p u_p^2dy=\varphi\mu_{sp}\left[\frac{\partial u_p}{\partial y}\right]_0^\delta+\frac{1}{\tau_p}$$

$$\int_0^\delta \rho_p(u-u_p)dy+g\int_0^\delta \rho_p\left(1-\frac{\rho}{\rho_{sp}}\right)dy \qquad (21)$$

$$\frac{d}{dx}\int_0^\delta u(T-T_\infty)d\eta=-\frac{k}{\rho c_p}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$+\frac{\varphi}{1-\varphi}\frac{c_s}{c_p}\frac{1}{\tau_T}\gamma\int_0^\delta(T_p-T)dy+\int_0^\delta\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2dy \qquad (22)$$

$$\frac{d}{dx}\int_0^\delta(\rho_p u_p)(T_p-T_\infty)dy=-\frac{1}{\tau_T}\int_0^\delta \rho_p(T_p-T)\,dy \qquad (23)$$

By introducing non-dimensional quantities

$$\overline{x}=\frac{x}{L},\overline{y}=\frac{y}{\delta},\overline{u}=\frac{u}{U},\ \overline{v}=\frac{v}{U},\overline{u}_p=\frac{u_p}{U},\overline{v}_p=\frac{v_p}{U},$$

$$\overline{\rho}_p=\frac{\rho_p}{\rho_{p\infty}},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_p=\frac{T_p-T_\infty}{T_w-T_\infty} \qquad (24)$$

In equations (20) to (23), we get after dropping bars

$$\frac{d}{dx}\left[\int_0^1 Au^2dy\right]=\frac{1}{A}\frac{1}{Re}\left[\frac{\partial u}{dy}\right]_{y=0}+\frac{GrA}{Re^2}\int_0^1\theta\,dy$$

$$+\frac{\varphi}{1-\varphi}\frac{FL}{U}A\gamma\int_0^1(u_p-u)dy \qquad (25)$$

$$\frac{d}{dx}\left[A\int_0^1\rho_p u_p^2dy\right]=\frac{1}{Rep}\frac{1}{A}\left[\frac{\partial u_p}{dy}\right]_{y=0}$$

$$+A\frac{FL}{U}\int_0^1\rho_p(u-u_p)\,dy+\frac{A}{Fr}\left(1-\frac{\rho}{\rho_{sp}}\right)\int_0^1\rho_p\,dy \quad (26)$$

$$\frac{d}{dx}\left[\int_0^1 Au\theta dy\right]=-\frac{1}{Pr.Re}\frac{1}{A}\left[\frac{\partial\theta}{dy}\right]_{y=0}$$

$$+\frac{2}{3}\frac{\varphi}{1-\varphi}\gamma\frac{FL}{U}\frac{1}{Pr}\int_0^1(\theta_p-\theta)\,dy+\frac{Ec}{Re}\frac{1}{A}\int_0^1\left(\frac{\partial u}{dy}\right)^2dy \quad (27)$$

$$\frac{d}{dx}\left[A\int_0^1\rho_p u_p\theta_p dy\right]=-A\frac{FL}{U}\int_0^1\rho_p\,(\theta_p-\theta)dy \qquad (28)$$

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Further the equation (18) reduces to

$$\frac{d}{dx}\left[\int_0^1 \rho_p u_p dy\right] = 0 \tag{29}$$

and the boundary condition are

$$At \quad y = 0: u = 0, u_p = a_2(x), \rho_p = a_3(x),$$

$$\theta = 1\ ,\ \theta_p = a_4(x) \tag{30}$$

$$At \quad y = 1: u = 0,\ u_p = 0,\ \rho_p = 1,\ \theta = 0\ , \theta_p = 0,$$

$$\frac{\partial \theta}{\partial y} = 0,\ \ \frac{\partial u}{\partial y} = 0 \tag{31}$$

Selecting the velocity profiles subjected to the boundary conditions (30) and (31) as

$$\left.\begin{aligned} u &= u_1 y(1-y)^2 \\ u_p &= a_2(1-y)^3 \\ \rho_p &= 1-(1-a_3)(1-y)^3 \\ \theta &= (1-y)^2 \\ \theta_p &= a_4(1-y)^2 \end{aligned}\right\} \tag{32}$$

and substituting (32) in (25) to (29), we get

$$\frac{1}{105}\frac{d}{dx}\left[Au_1{}^2\right] = \frac{u_1}{A\,Re} + \frac{1}{3}\frac{Gr\,A}{Re^2} + \frac{\varphi}{1-\varphi}\frac{FL}{U}A\gamma\left[\frac{a_2}{4} - \frac{u_1}{12}\right] \tag{33}$$

$$\frac{d}{dx}\left[A\left\{\frac{a_2{}^2}{7} - \frac{a_2{}^2(1-a_3)}{10}\right\}\right] = \frac{-3a_2}{A\,Rep} + A\frac{FL}{U}\left[\frac{u_1}{12} - \frac{a_2}{4} - \frac{u_1(1-a_3)}{42} + \frac{a_2(1-a_3)}{7}\right] + \frac{A}{Fr}\left(1 - \frac{\rho}{\rho_{sp}}\right)\left(\frac{3+a_3}{4}\right) \tag{34}$$

$$\frac{1}{30}\frac{d}{dx}[Au_1] = \frac{2}{\Pr Re}\frac{1}{A} + \frac{2}{3}\frac{\varphi}{1-\varphi}\gamma\frac{FL}{U}\frac{1}{pr}A\left[\frac{a_4}{3} - \frac{1}{3}\right] + \frac{4}{3}\frac{Ec}{Re}\frac{1}{A}u_1{}^2 \tag{35}$$

$$\frac{d}{dx}\left[A\left\{\frac{a_2 a_4}{6} - \frac{a_2 a_4(1-a_3)}{9}\right\}\right] = -A\frac{FL}{U}\left[\frac{a_4}{3} - \frac{1}{3} - \frac{a_4(1-a_3)}{6} + \frac{(1-a_3)}{6}\right] \tag{36}$$

$$\frac{d}{dx}\left[\frac{a_2}{4} - \frac{(1-a_3)a_2}{7}\right] = 0 \tag{37}$$

Simplifying equations (33) to (37), we get

$$\frac{du_1}{dx} = \frac{1}{2Au_1}\left[\frac{105u_1}{A\,Re} + \frac{35\,GrA}{Re^2} + 105\frac{\varphi}{1-\varphi}\frac{FL}{U}A\gamma\left(\frac{a_2}{4} - \frac{u_1}{12}\right) - u_1{}^2\frac{dA}{dx}\right] \tag{38}$$

$$\frac{da_2}{dx} = \frac{1}{2Aa_2(3+7a_3)}\left[AP + CP + +\frac{70\,A}{Fr}\left(1 - \frac{\rho}{\rho_{sp}}\right)\left(\frac{3+a_3}{4}\right)\right] \tag{39}$$

Where $AP = 70\,A\frac{FL}{U}\left[\frac{u_1}{12} - \frac{a_2}{4} - \frac{u_1(1-a_3)}{42} + \frac{a_2(1-a_3)}{7}\right]$

and $CP = \frac{-210a_2}{A\,Rep} - 7Aa_2{}^2\frac{da_3}{dx} - a_2^2(3+7a_3)\frac{dA}{dx}$

$$\frac{dA}{dx} = \frac{1}{u_1}\left[\frac{60}{\Pr Re}\frac{1}{A} + 20\frac{\varphi}{1-\varphi}\gamma\frac{FL}{U}\frac{1}{pr}A\left(\frac{a_4}{3} - \frac{1}{3}\right) + \frac{4}{3}\frac{Ec}{Re}\frac{1}{A}u_1{}^2 - A\frac{du_1}{dx}\right] \tag{40}$$

$$\frac{da_4}{dx} = \frac{1}{(1+2a_3)Aa_2}\left[(1+2a_3)\left(a_2a_4\frac{dA}{dx} + Aa_4\frac{da_2}{dx}\right) + 2Aa_2a_4\frac{da_3}{dx} + \frac{18AFL}{U}\left(\frac{a_4}{3} - \frac{1}{3} - \frac{a_4(1-a_3)}{6} + \frac{(1-a_3)}{6}\right)\right] \tag{41}$$

$$\frac{da_3}{dx} = -\frac{1}{a_2}\left(a_3 + \frac{3}{4}\right)\frac{da_2}{dx} \tag{42}$$

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3. Discussion of Results and conclusion

The 1st order differential equation (38) to (42) are solved by using 4th order Runge-Kutta method for various values of the non-dimensional parameters, Reynolds number(Re), Grassoff number(Gr), Volume fraction (φ), Species density of the particle (ρ_{sp}), density of the particle(ρ), Prandtl number(Pr), Froud number(Fr), Eckert number(Ec).

Fig.1 depicts the pattern of the carrier fluid phase u and temperature T. From Table-1, it can be observed that the presence of SPM results in increase in magnitude of the carrier fluid velocity. The diffusion and concentration parameter has a little effect on velocity, temperature of the carrier fluid and as well as particle phase.

From Figs.2, 3, 4; it is observed that magnitude of the carrier fluid velocity u increase with the increase of Gr where as particle phase velocity and temperature decreases with the increase of Pr. From Table-2, it is observed that the Nusselt number(Nu)decreases with the increase of Pr. Further heat is transferred from plate to fluid in case of air $(Pr = 0.71)$ and from fluid to plate in case of electrolyte solution $(Pr = 1.0)$ and water$(Pr = 7.0)$.

Table-3 depicts the presence of SPM increase the skin friction and decrease the Nusselt number.

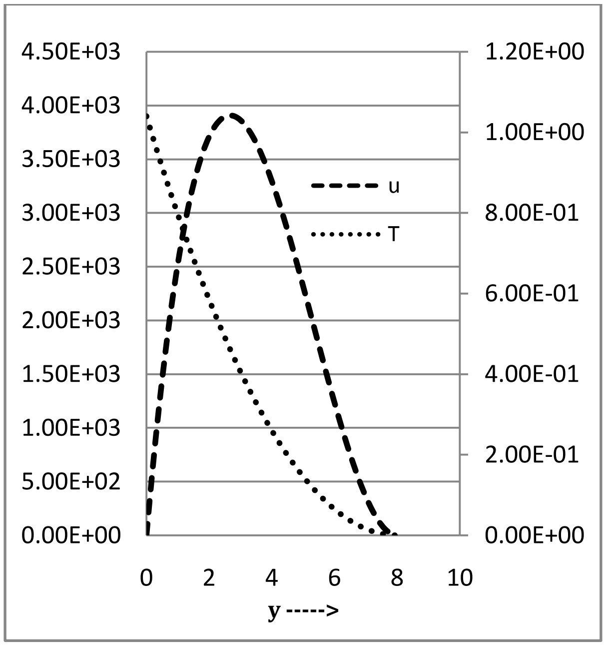

Figure 1: Variation of u & T with y with SPM

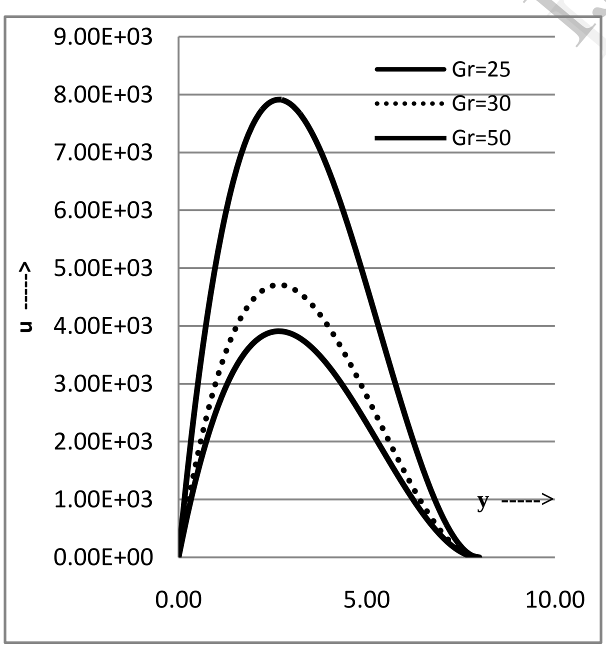

Figure 2: Variation of u with y for different Gr

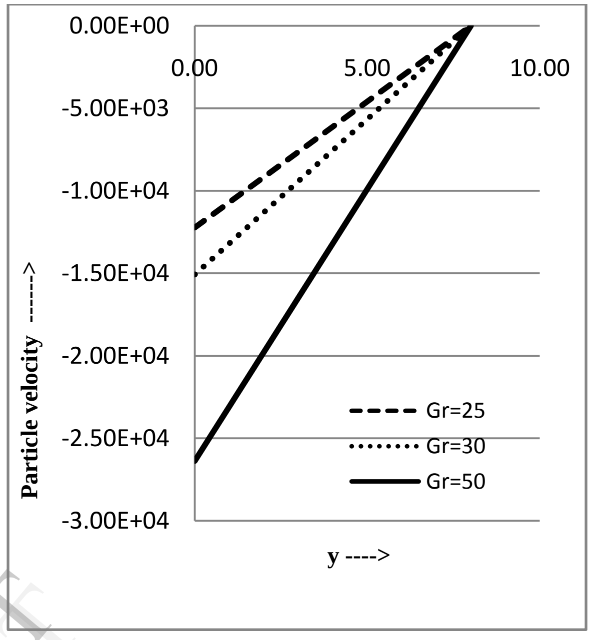

Figure 3: Variation of particle velocity with y for different Gr

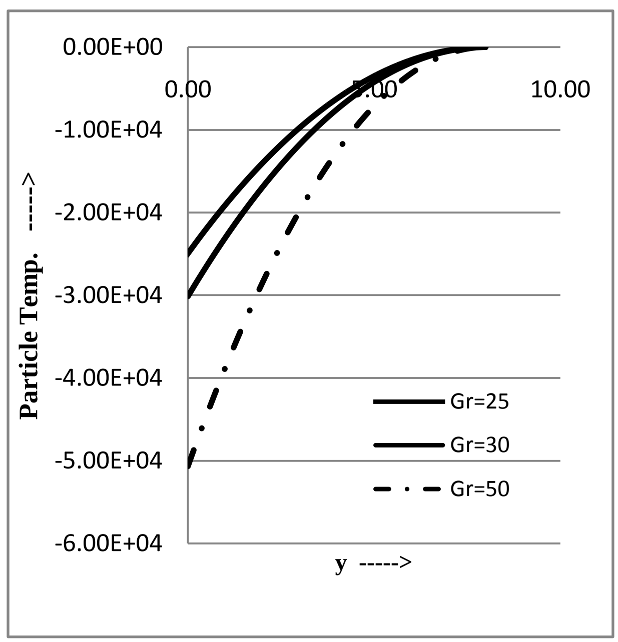

Figure 4: Variation of particle temperature with y for different Gr.

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Table 1: Magnitude of carrier fluid velocity with and without SPM

y	With out SPM	With SPM
0.00E+00	0.00E+00	0.00E+00
2.00E-01	2.22E+01	6.27E+02
6.00E-01	6.00E+01	1.69E+03
1.00E+00	8.95E+01	2.53E+03
1.40E+00	1.11E+02	3.14E+03
1.80E+00	1.26E+02	3.57E+03
2.20E+00	1.35E+02	3.81E+03
2.60E+00	1.38E+02	3.91E+03
3.00E+00	1.37E+02	3.86E+03
3.40E+00	1.31E+02	3.71E+03
3.80E+00	1.22E+02	3.45E+03
4.20E+00	1.11E+02	3.13E+03
4.60E+00	9.71E+01	2.74E+03
5.00E+00	8.22E+01	2.32E+03
5.40E+00	6.67E+01	1.88E+03
5.80E+00	5.13E+01	1.45E+03
6.20E+00	3.67E+01	1.04E+03
6.60E+00	2.36E+01	6.67E+02
7.00E+00	1.28E+01	3.61E+02
7.40E+00	4.87E+00	1.37E+02
7.80E+00	5.71E-01	1.61E+01
8.00E+00	1.63E-10	4.59E-09

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Table 2: Variation of Skin friction & Nusselt number

x	Variation of Skin friction (c_f) with SPM			Variation Nusselt number (Nu) with SPM		
	Pr=0.71	Pr=1.0	Pr=7.0	Pr=0.71	Pr=1.0	Pr=7.0
1.01E-01	5.17E-06	-8.40E-06	-2.07E-06	5.80E-04	-9.42E-04	-2.32E-04
5.05E+00	1.24E-05	-9.18E-05	-7.36E-08	6.94E-02	-5.15E-01	-4.13E-04
1.00E+01	1.32E-05	-1.71E-04	-4.56E-08	1.47E-01	-1.90E+00	-5.06E-04
1.49E+01	1.33E-05	-2.24E-04	-4.24E-08	2.20E-01	-3.72E+00	-7.05E-04
1.99E+01	1.33E-05	-2.64E-04	-4.12E-08	2.93E-01	-5.83E+00	-9.11E-04
2.48E+01	1.33E-05	-2.96E-04	-4.05E-08	3.66E-01	-8.16E+00	-1.12E-03
2.98E+01	1.33E-05	-3.23E-04	-4.01E-08	4.39E-01	-1.07E+01	-1.33E-03
3.47E+01	1.33E-05	-3.47E-04	-3.97E-08	5.12E-01	-1.34E+01	-1.53E-03
3.97E+01	1.33E-05	-3.68E-04	-3.95E-08	5.85E-01	-1.62E+01	-1.74E-03
4.46E+01	1.33E-05	-3.87E-04	-3.93E-08	6.58E-01	-1.92E+01	-1.95E-03
4.96E+01	1.33E-05	-4.04E-04	-3.91E-08	7.31E-01	-2.23E+01	-2.16E-03
5.45E+01	1.33E-05	-4.18E-04	-3.90E-08	8.04E-01	-2.54E+01	-2.36E-03
5.95E+01	1.33E-05	-4.31E-04	-3.89E-08	8.77E-01	-2.85E+01	-2.57E-03
6.44E+01	1.33E-05	-4.41E-04	-3.88E-08	9.50E-01	-3.16E+01	-2.78E-03
6.94E+01	1.33E-05	-4.49E-04	-3.87E-08	1.02E+00	-3.46E+01	-2.98E-03
7.43E+01	1.33E-05	-4.56E-04	-3.86E-08	1.10E+00	-3.77E+01	-3.19E-03
7.93E+01	1.33E-05	-4.61E-04	-3.85E-08	1.17E+00	-4.06E+01	-3.39E-03
8.42E+01	1.33E-05	-4.65E-04	-3.85E-08	1.24E+00	-4.35E+01	-3.60E-03
8.92E+01	1.33E-05	-4.68E-04	-3.84E-08	1.31E+00	-4.64E+01	-3.81E-03
9.41E+01	1.33E-05	-4.70E-04	-3.83E-08	1.39E+00	-4.92E+01	-4.01E-03
9.91E+01	1.33E-05	-4.72E-04	-3.83E-08	1.46E+00	-5.20E+01	-4.22E-03

Table 3: Variation of Skin friction & Nusselt number with and without SPM

x	Skin Friction (c_f)		Nusselt number (Nu)	
	Without SPM	With SPM	Without SPM	With SPM
1.01E-01	0.00E+00	5.17E-06	0.00E+00	5.80E-04
5.05E+00	1.18E-05	1.24E-05	2.77E-01	6.94E-02
1.00E+01	1.01E-05	1.32E-05	4.74E-01	1.47E-01
1.49E+01	9.13E-06	1.33E-05	6.43E-01	2.20E-01
1.99E+01	8.50E-06	1.33E-05	7.99E-01	2.93E-01
2.48E+01	8.04E-06	1.33E-05	9.44E-01	3.66E-01
2.98E+01	7.68E-06	1.33E-05	1.08E+00	4.39E-01
3.47E+01	7.38E-06	1.33E-05	1.21E+00	5.12E-01
3.97E+01	7.14E-06	1.33E-05	1.34E+00	5.85E-01
4.46E+01	6.93E-06	1.33E-05	1.46E+00	6.58E-01
4.96E+01	6.75E-06	1.33E-05	1.58E+00	7.31E-01
5.45E+01	5.66E-06	1.33E-05	2.66E+00	8.04E-01
5.95E+01	5.11E-06	1.33E-05	3.60E+00	8.77E-01
6.44E+01	4.72E-06	1.33E-05	4.55E+00	9.50E-01
6.94E+01	4.47E-06	1.33E-05	5.35E+00	1.02E+00
7.43E+01	4.27E-06	1.33E-05	6.12E+00	1.10E+00
7.93E+01	4.11E-06	1.33E-05	6.85E+00	1.17E+00
8.42E+01	3.98E-06	1.33E-05	7.57E+00	1.24E+00
8.92E+01	3.86E-06	1.33E-05	8.25E+00	1.31E+00
9.41E+01	3.85E-06	1.33E-05	8.32E+00	1.39E+00

9.91E+01	3.84E-06	1.33E-05	8.39E+00	1.46E+00

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