Two Dimensional Kamal Transform of Space of Distribution of Boehmians

*Arun J. Ghanwat
MES's, Shri Dnyaneshwar Mahavidyalaya Newasa,
Dist. Ahmednagar

#Dr. S. B. Gaikwad
P. G. Department of Mathematics, New Arts, Commerce and Science College,
Ahmednagar (Maharashtra).

Abstract:- **In this paper we have tried to define two dimensional Kamal transform namely the double Kamal transform of bounded support with the help of Zemanian technique. The integrable Boehmian of double Kamal transform is developed using Mikusinki's theory with some of the properties. The double Kamal transform is extended to generalized function, some properties n distribution sense are obtained and tried to extend this class to integrable Boehmians.**

Keywords: Convolution theorem, Double Kamal transform , Kamal Transform, Integrable Boehmian.

INTRODUCTION

In the field of engineering the ordinary and partial differential equations plays vital role. To solve the differential equation integral transforms are used suitably to convert into simpler form and obtain the solutions easily. The double Natural transform are studied in [2-4], [6] deals with applications of [2-4] where as [9] gives the applications of Kamal transform for single variable functions. The two dimensional Kamal transform of function of two variables can be used to solve partial differential equations.

Defintion 1: The Kamal transform for a single variable function of exponential order in the set

S = $\{f(t): \exists\, M, k_1, k_2 > 0, |f(t)| < Me^{|t|/\alpha_j}, t \in (-1)^j x[0,\infty)\}$; where the constant M is finite number, k_1, k_2 may be finite or infinite is defined by [1]

K [*f(t)*] = G(*v*) = $\int_0^\infty e^{\frac{-t}{v}} f(t)dt, t \geq 0, k_1 \leq v \leq k_2$.

The inverse Kamal transform is $f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\frac{t}{v}} G\left(\frac{1}{v}\right) dv, c \geq 0.$

Definition 2: The two dimensional Kamal transform namely the Double Kamal transform of a function $f(x,t), x, t \in \mathbb{R}_+$ of two variables is defined by [8] as K₂$\{f(x,t); (u,v)\}$= G(u,v)=$\int_0^\infty \int_0^\infty e^{-\left(\frac{x}{u}+\frac{t}{v}\right)} f(x,t)dxdt$, provided the integral exists and $f(x,t)$ can be expressed in the convergent series on a set A of functions continuous and of exponential order.

A = $\left\{f: |f(x,t)| < M\, e^{\left(|x|/\alpha_j + |t|/\beta_j\right)}, t \in (-1)^j \text{x}\,[0,\infty),\ j = 1,2; M, \alpha_j, \beta_j > 0\right\}$; where the constant M should be finite number, α_j, β_j may be finite or infinite and v is a variable of transform.

Let f, g be integrable functions of two variables then by using [7] the double convolution of f, g is given by $(f ** g)(x,t)$=$\int_0^\infty \int_0^\infty f(p,q)g(x-p, t-q)dpdq$.

DOUBLE KAMAL TRANSFORM OF BOUNDED SUPPORT

Let J be compact subset of I = $(0,\infty)$. C^∞ denote the test function space of all complex valued infinitely smooth function spaces of E (I x I) such that $\sup_{(x,t)\in JxJ}\left|\frac{\partial^{k+m}}{\partial x^k \partial t^m}\emptyset(x,t)\right| < \infty$, for k, m are non-zero positive integers. Then E (I x I) is a vector space under the usual operations. Also D (I x I)⊂ E (I x I). A sequence $\{\emptyset_v\}_{v=1}^\infty$ converges in E (I x I) and for each non-negative k, m, $(\frac{\partial^{k+m}}{\partial x^k \partial t^m}\emptyset_v)$ converges to $\left(\frac{\partial^{k+m}}{\partial x^k \partial t^m}\emptyset\right)$ uniformly on every compact subset I x I.

The space E (I x I) is complete and E'(I x I) dual of E(I x I) consists of distribution of compact support. By the definition of double Kamal transform the Kernel $e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}$ is smooth and satisfies $\sup_{(x,t)\in JxJ}\left|\frac{\partial^{k+m}}{\partial x^k \partial t^m}\left(e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\right)\right| < \infty$ (1) for all positive real numbers u, v.

Hence for every $f \in$ E'(I x I) we define the distributional double Kamal transform of compact support with the help of Zemanian [10] as 𝕂 $(f(x,t),(u,v))$= $\langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$ for $(u,v) \in$ IxI (2)

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Theorem 1: The distributional double Kamal transform $\mathbb{K}$ is linear.

Proof: For any constants *c*, *d* we have

$$\mathbb{K}\,(cf(x,y)+dg(x,t)) = \langle cf(x,t)+g(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = \langle cf(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle + \langle dg(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$$
$$= c\,\langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle + d\,\langle g(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = c\,\mathbb{K}(f(x,t)) + d\,\mathbb{K}(g(x,t)).$$

Theorem 2: Let h be distribution in E'(I x I) and let g be defined by

$$g(x,t) = \begin{cases} h(x-\lambda_1, t-\lambda_2), & x \geq \lambda_1,\ t \geq \lambda_2 \\ 0, & x < \lambda_1,\ t < \lambda_2 \end{cases}$$

Then $\mathbb{K}g(x,t) = e^{-\left(\frac{\lambda_1}{u}+\frac{\lambda_2}{v}\right)}\mathbb{K}\,[h(x,t)]$

Proof: Here $g(x,t)$ is a member of E'(I x I), by translation property

$$\mathbb{K}g(x,t) = \langle h(x-\lambda_1, t-\lambda_2),\ e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = \langle h(x,t), e^{-\left(\frac{x-\lambda_1}{u}+\frac{t-\lambda_2}{v}\right)}\rangle = \langle h(x,t), e^{\frac{-x}{u}}.e^{\frac{\lambda_1}{u}}.e^{\frac{-t}{v}}.e^{\frac{\lambda_2}{v}}\rangle$$
$$= e^{-\left(\frac{\lambda_1}{u}+\frac{\lambda_2}{v}\right)}\langle h(x,t),\ e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = e^{-\left(\frac{\lambda_1}{u}+\frac{\lambda_2}{v}\right)}\mathbb{K}\,[h(x,t)].$$

Theorem 3: Let $f(x,t) \in$ E'(I x I) and $\mathbb{K}$ [h(x,t)] be the distributional double Kamal transform then $\frac{\partial^k}{\partial x^k}\mathbb{K}\,[f(x,t)] = \langle f(x,t), \frac{\partial^k}{\partial x^k}e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$ and $\frac{\partial^n}{\partial t^n}\mathbb{K}\,[f(x,t)] = \langle f(x,t), \frac{\partial^n}{\partial t^n}e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$; where $\frac{\partial^k}{\partial x^k}$ and $\frac{\partial^n}{\partial t^n}$ are k^{th} and n^{th} derivatives w. r. t. *x* & *t.*

Theorem 4: Let $f(x,t) \in$ E'(I x I) and $\mathbb{K}$ [f(x,t)] be the distributional double Kamal transform then

$$\mathbb{K}\left(e^{-\alpha x-\beta t}f(x,t)\right)[(u,v)] = \mathbb{K}\left[f(x,t), \left(\frac{u}{1+\alpha u}, \frac{v}{1+\beta v}\right)\right]$$

Proof:

$$\mathbb{K}[f(x,t)](u,v) = \langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle \therefore \mathbb{K}\left(e^{-\alpha x-\beta t}f(x,t)\right)[(u,v)] = \langle e^{-\alpha x-\beta t}f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$$
$$= \langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}e^{-\alpha x-\beta t}\rangle = \langle f(x,t),\ e^{\frac{-x}{u}-\alpha x-\frac{t}{v}-\beta t}\rangle = \langle f(x,t),\ e^{-\frac{(1+\alpha u)x}{u}}.e^{-\frac{(1+\beta v)t}{v}}\rangle$$
$$= \mathbb{K}\left[f(x,t), \left(\frac{u}{1+\alpha u}, \frac{v}{1+\beta v}\right)\right].$$

Theorem 5: Let $f(x,t) \in$ E'(I x I) and $\mathbb{K}$ [f(x,t)] be the distributional double Kamal transform of $f(x,t)$ then $\mathbb{K}[f(\alpha x, \beta t)] = \frac{1}{\alpha\beta}\mathbb{K}\left[f(x,t)\left(\frac{1}{\alpha}.\frac{1}{\beta}\right)(u,v)\right].$

Proof: $\mathbb{K}[f(\alpha x,\beta t)] = \langle f(\alpha x,\beta t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = \langle f(x,t), \frac{1}{\alpha}.\frac{1}{\beta}\ e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = \frac{1}{\alpha\beta}\langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$

$$= \frac{1}{\alpha\beta}\mathbb{K}\left[f(x,t)\left(\frac{1}{\alpha}.\frac{1}{\beta}\right)(u,v)\right].$$

Theorem 6: Let $f(x,t) \in$ E'(I x I) and $\mathbb{K}$ [f(x,t)] be the distributional double Kamal transform of $f(x,t)$ then

(1) $\mathbb{K}\,[xf(x,t)](u,v) = u^2\frac{\partial}{\partial x}\mathbb{K}\,[f(x,t)] + u\mathbb{K}\,[f(x,t)](u,v)$

(2) $\mathbb{K}\,[tf(x,t)](u,v) = v^2\frac{\partial}{\partial x}\mathbb{K}\,[f(x,t)] + v\mathbb{K}\,[f(x,t)](u,v)$

Proof: Let $f(x,t) \in$ E'(I x I). Then by theorem 3 and equation (2) we have $\frac{\partial}{\partial x}\mathbb{K}\,[f(x,t)] = \frac{\partial}{\partial x}\left(\langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle\right) =$ $\langle f(x,t), \frac{\partial}{\partial x}(e^{-\left(\frac{x}{u}+\frac{t}{v}\right)})\rangle = \langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\left(\frac{x}{u^2}\right) - e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\left(\frac{1}{u}\right)\rangle = \frac{1}{u^2}\langle xf(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle - \frac{1}{u}\langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$

$$= \frac{1}{u^2}\ \mathbb{K}\,[xf(x,t)](u,v) - \frac{1}{u}\mathbb{K}\,[f(x,t)](u,v)$$

$$\therefore u^2\frac{\partial}{\partial x}\mathbb{K}\,[f(x,t)] = \mathbb{K}\,[xf(x,t)](u,v) - u\mathbb{K}\,[f(x,t)](u,v)$$

$$\therefore\ \mathbb{K}\,[xf(x,t)](u,v) = u^2\frac{\partial}{\partial x}\mathbb{K}\,[f(x,t)] + u\mathbb{K}\,[f(x,t)](u,v)$$

Similarly we can prove the part (2).

Theorem 7: Let $f(x,t) \in$ E'(I x I) and $\mathbb{K}$ [f(x,t)] be the distributional double Kamal transform of $f(x,t)$ then $\mathbb{K}\{f(x-\mu,t-\rho).H(x-\mu,t-\rho)\} = e^{-\left(\frac{\mu}{u}+\frac{\rho}{v}\right)}\mathbb{K}\{f(x-\mu,t-\rho)\}(u,v)$; where *H*(*x*, *t*) is Heaviside unit step function defined by

$$H(x-\mu,t-\rho) = \begin{cases} 1, & x > \mu, t > \rho \\ 0, & x < \mu, t < \rho \end{cases}$$

Proof: By using the definition of distributional double Kamal transform

$$\mathbb{K}\{f(x-\mu,t-\rho).H(x-\mu,t-\rho)\} = \langle f(x-\mu,t-\rho).H(x-\mu,t-\rho, e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$$

For $x > \mu, t > \rho$

$$\mathbb{K}\{f(x-\mu,t-\rho).H(x-\mu,t-\rho)\} = \langle f(x-\mu,t-\rho), e^{-\left(\frac{x+\mu}{u}+\frac{t+\rho}{v}\right)}\rangle$$
$$= e^{-\left(\frac{\mu}{u}+\frac{\rho}{v}\right)}\langle f(x-\mu,t-\rho), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle$$
$$= e^{-\left(\frac{\mu}{u}+\frac{\rho}{v}\right)}\mathbb{K}\{f(x-\mu,t-\rho)\}(u,v).$$

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CONVOLUTION OF DOUBLE KAMAL TRANSFORM

Let $f(x,t), g(x,t)$ be distributions of compact support in E'(I x I). We define the convolution of $f(x,t), g(x,t)$ as $\langle (f ** g)(x,y), \emptyset(x,y)\rangle = \langle f(x,t), \langle g(J_1,J_2), \emptyset(x+J_1, y+J_2)\rangle\rangle$; (3) where $\emptyset \in$ E(I x I) if $\langle g(J_1,J_2), \emptyset(x+J_1, y+J_2)\rangle \in$ E (I x I).

Theorem 8: Let $f(x,t) \in$ E'(I x I) and $\Psi \in$ E(I x I). If $\emptyset(x,y) = \langle g(J_1,J_2), \Psi(x+J_1, y+J_2)\rangle$; where $k, m = 1,2,3,\ldots$. Then Ψ is infinitely differentiable and $\frac{\partial^{k+m}}{\partial x^k \partial y^m}\big(\Psi(t)\big) = \langle g(J_1,J_2), \frac{\partial^{k+m}}{\partial x^k \partial y^m}(\Psi(x+J_1, y+J_2)\rangle$

This result is proved in [11], Pp-130.

Theorem 9: (The Convolution theorem):- If $Ꝁ[f(x,t)]$, $Ꝁ[g(x,t)]$ are distributional transforms of $f(x,t)$, $g(x,t)$ respectively then $Ꝁ[(f ** g)(x,t)] = Ꝁ[f(x,t)].Ꝁ[g(x,t)]$

Proof: $Ꝁ[(f ** g)(x,t)] = \langle (f ** g)(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle = \langle f(x,t), \langle g(J_1,J_2), e^{-\left(\frac{x+J_1}{u}+\frac{t+J_2}{v}\right)}\rangle\rangle$

$= \langle f(x,t), \langle g(J_1,J_2), e^{-\frac{x}{u}-\frac{J_1}{u}-\frac{t}{v}-\frac{J_2}{v}}\rangle\rangle = \langle f(x,t), \langle g(J_1,J_2), e^{-\frac{x}{u}-\frac{t}{v}}.e^{\frac{-J_1}{u}-\frac{J_2}{v}}\rangle\rangle$

$= \langle f(x,t), e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}\rangle . \langle g(J_1,J_2), e^{-\left(\frac{J_1}{u}+\frac{J_2}{v}\right)}\rangle = Ꝁ[f(x,t)].Ꝁ[g(x,t)]$.

INTEGRABLE BOEHMIAN FOR DOUBLE KAMAL TRANSFORM

Let L^1be the space of Lebesgue integrable functions on $\mathbb{R}_+$; $\mathbb{R}_+$ is the set of positive real numbers then by [5], a sequence $\{\delta_n\}_{n=1}^{\infty}$ of continuous real valued functions over $\mathbb{R}_+$x $\mathbb{R}_+$ is called a delta sequence if and only if

(1) $\int_0^\infty \int_0^\infty \delta_n(x,y) = 1$

(2) $\int_0^\infty \int_0^\infty |\delta_n| <$ K, for some positive K $\in \mathbb{R}$ and all $n \in \mathbb{N}$.

(3) $\int_{|(x,y)|<\rho} |\delta_n(x,y)|\, dxdy \to 0$ as $n \to \infty$ for every $\rho > 0$.

The space of all integrable Boehmians is denoted by $B_{L^1 x L^1}$, which is a convolution algebra with

(1) $\Lambda\left[\frac{f_n}{\delta_n}\right] = \left[\frac{\Lambda f_n}{\delta_n}\right]$

(2) $\left[\frac{f_n}{\delta_n}\right] + \left[\frac{g_n}{\Psi_n}\right] = \frac{(f_n ** \Psi_n)+(g_n ** \delta_n)}{(\delta_n ** \Psi_n)}$

(3) $\left[\frac{f_n}{\delta_n}\right] ** \left[\frac{g_n}{\Psi_n}\right] = \left[\frac{f_n ** g_n}{\delta_n ** \Psi_n}\right]$

Lemma: If $\left[\frac{f_n}{\delta_n}\right] \in B_{L^1 x L^1}$ then the sequence K$[f_n(x,t)] = \int_0^\infty \int_0^\infty e^{-\left(\frac{x}{u}+\frac{t}{v}\right)} f_n(x,t)dxdt$ converges to each compact set J x J in $\mathbb{R}_+$x $\mathbb{R}_+$.

Proof: We have $\{\delta_n\}_{n=1}^{\infty}$ then its double Kamal transform $Ꝁ(\delta_n) = \widetilde{\delta_n}$ converges uniformly to the function of (u, v). Hence for each J x J, $\widetilde{\delta_n}$ is positive on J x J and

K$[f_n(x,t)]$ = K$\left[f_n(x,t)\frac{\widetilde{\delta_k}}{\widetilde{\delta_k}}\right]$ (4)

$= K\frac{f_k}{\widetilde{\delta_k}}\widetilde{\delta_k}$ on J x J i.e. K$f_n \to$ K$\frac{f_k}{\widetilde{\delta_k}}$ as $n \to \infty$ on J x J. ($\widetilde{\delta} \to 1$ as $n \to \infty$).

Now, we will define the double Kamal transform of an integrable Boehmians as

$Ᵹ\left[\frac{f_n}{\delta_n}\right] = \lim_{x\to\infty} Ꝁ f_n$ (5); where the limit is over compact subsets of $\mathbb{R}_+$x $\mathbb{R}_+$.

Thus, we get a continuous function of double Kamal transform of an integrable Boehmians.

Theorem 10: The double Kamal transform of an integrable Boehmians is well defined.

Proof: Let $\alpha_1 = \left[\frac{f_n}{\emptyset_n}\right]$, $\alpha_2 = \left[\frac{g_n}{\Psi_n}\right] \in B_{L^1 x L^1}$ such that $\alpha_1 = \alpha_2$. Then

$\left[\frac{f_n}{\delta_n}\right] = \left[\frac{g_n}{\Psi_n}\right] \Rightarrow f_n ** \Psi_m = g_m ** \emptyset_n, m, n \in$ $\mathbb{N}$. Appling double Kamal transform

$Ꝁ(f_n ** \Psi_m) = Ꝁ(g_m ** \emptyset_n) = Ꝁ(g_n ** \emptyset_m)$

From equation (5) and theorem (9)

$\lim_{x\to\infty} Ꝁ f_n = \lim_{x\to\infty} Ꝁ g_n \Rightarrow Ᵹ\left[\frac{f_n}{\emptyset_n}\right] = Ᵹ\left[\frac{g_n}{\Psi_n}\right]$.

Theorem 11: The double Kamal transform of an integrable Boehmian Ᵹ is linear on compact sets.

Proof: Let $\alpha, \beta \in B_{L^1 x L^1}$ be such that

$\alpha = \left[\frac{f_n}{\emptyset_n}\right]$, $\beta = \left[\frac{g_n}{\Psi_n}\right]$ then $\alpha + \beta = \frac{(f_n ** \Psi_n)+(g_n ** \emptyset_n)}{(\emptyset_n ** \Psi_n)}$

$\therefore Ᵹ(\alpha+\beta) = \lim_{x\to\infty} Ꝁ(f_n ** \Psi_n) + \lim_{x\to\infty} Ꝁ(g_n ** \emptyset_n)$

$= \lim_{n\to\infty} Ꝁ(f_n) + \lim_{n\to\infty} Ꝁ(g_n)$

$= Ᵹ(\alpha) + Ᵹ(\beta)$.

Also, $Ᵹ(c\alpha) = Ᵹ\left[\frac{cf_n}{\delta_n}\right] = c \lim_{x\to\infty} f_n$

= c Ᵹ (α); where c$\in \mathbb{C}$ and $\alpha = \left[\frac{f_n}{\emptyset_n}\right]$.

Theorem 12: (Convolution theorem for integrable Boehmians)

Let $\left[\frac{f_n}{\emptyset_n}\right], \left[\frac{g_n}{\Psi_n}\right] \in B_{L^1xL^1}$ then $\mathfrak{K}\left(\left[\frac{f_n}{\emptyset_n}\right] ** \left[\frac{g_n}{\Psi_n}\right]\right) = \mathfrak{K}\left(\left[\frac{f_n}{\emptyset_n}\right]\right).\mathfrak{K}\left(\left[\frac{g_n}{\Psi_n}\right]\right)$

Proof:

$$\mathfrak{K}\left(\left[\frac{f_n}{\emptyset_n}\right] ** \left[\frac{g_n}{\Psi_n}\right]\right) = \mathfrak{K}\left[\frac{(f_n ** g_n)}{(\emptyset_n ** \Psi_n)}\right] = \lim_{x\to\infty} K(f_n ** \Psi_n) = \lim_{x\to\infty}(K(f_n).K(g_n))$$

$$= \lim_{x\to\infty} K(f_n).\lim_{x\to\infty} K(g_n) = \mathfrak{K}\left(\left[\frac{f_n}{\emptyset_n}\right]\right).\mathfrak{K}\left(\left[\frac{g_n}{\Psi_n}\right]\right).$$

Theorem 13: If $\gamma = \left[\frac{h_n}{\delta_n}\right] \in B_{L^1xL^1}$ and $\mathfrak{K}(\gamma) = 0$ in $\mathbb{C}$ then $\gamma = 0$ in $B_{L^1xL^1}$.

Proof: Let $\gamma = \left[\frac{h_n}{\delta_n}\right]$ and $\mathfrak{K}(\gamma) = 0$ then by the definition of $\mathfrak{K}$, we have

$\lim_{x\to\infty} Kh_n = 0$ on compact sets, again by definition of Kamal transform $h_n \to 0$ a.e. in $B_{L^1xL^1}$ and $\frac{h_n}{\delta_n}$ is zero quotient of function.

Equivalently $\gamma = \left[\frac{h_n}{\delta_n}\right]$ is zero equivalence class in $B_{L^1xL^1}$.

Theorem 14: The generalized double Kamal transform $\mathfrak{K}$ is one-one mapping from $B_{L^1xL^1}$ into space $\mathbb{C}$ of continuous functions.

Proof: Let $\mathfrak{K}(\gamma_1) = \mathfrak{K}(\gamma_2)$, $\gamma_1, \gamma_2 \in B_{L^1xL^1}$. $\Rightarrow \mathfrak{K}(\gamma_1 - \gamma_2) = 0$ in $\mathbb{C}$, theorem (13)

$\Rightarrow \gamma_1 - \gamma_2 = 0 \Rightarrow \gamma_1 = \gamma_2$.

Theorem 15: The generalized double Kamal transform $\mathfrak{K}$: $B_{L^1xL^1} \to \mathbb{C}$ is continuous with respect to Δ- converence.

Proof: Let $\gamma_n \xrightarrow{\Delta} \gamma$ as $n \to \infty$ in $B_{L^1xL^1}$

Then the Δ- convergence $\Rightarrow ((\gamma_n - \gamma) ** \delta_n) = \left[\frac{f_n ** \delta_n}{\delta_n}\right] \to 0$, for some $f_n \in B_{L^1xL^1}$, $\delta_n \in \Delta$ and $f_n \to 0$ as $n \to \infty$.

$\mathfrak{K}\{(\gamma_n - \gamma) ** \delta_n\} = \mathfrak{K}\left[\frac{f_n ** \delta_n}{\delta_n}\right] = \lim_{n\to\infty} K(f_n ** \delta_n) = \lim_{n\to\infty} Kf_n \to 0$ as $n \to \infty$ in $\mathbb{C}$.

As $f_n \to 0$, $\mathfrak{K}(\gamma_n)$ - $\mathfrak{K}(\gamma) \to 0$ as $n \to \infty \Rightarrow \mathfrak{K}(\gamma_n) \to \mathfrak{K}(\gamma)$ as $n \to \infty$.

RESULTS

We have tried to obtain double Kamal transform of bounded support. The linearity property, Convolution is studied in distributional view and also Boehmian spaces are studied. The convergence δ and Δ are defined in double Kamal transformation.

CONCLUSIONS

In this paper, the two dimensional Kamal transform namely double Kamal transform of bounded support is obtained. Also the extended double Kamal transform of distributional space along with some of its properties is studied. The convolution of double Kamal transform of Boehmians and extension of it to integrable Boehmians is derived.

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