Tuning of a PD-PI Controller used with First-order-Delayed Processes

Galal A. Hassaan
Emeritus Professor, Department of Mechanical Design & Production,
Faculty of Engineering, Cairo University,
Giza, EGYPT

***Abstract* - Time delayed processes require more attention in selecting reasonable controllers associated with them because of the poor performance of the control system associated with them . In this work, the PD-PI controller is examined to investigate its replacement to the classical PID controller. This research work has proven that the PD-PI results in a better performance for the closed-loop control system incorporating the PD-PI controller and a first-order delayed process.**

A first-order-delayed process of 50 s time constant and time delay between 2 and 16 seconds is controlled using simulation . The time delay effect is compensated using 4th order Pade approximation. The controller is tuned by minimizing the sum of square of error (ISE) of the control system using MATLAB. The MATLAB optimization toolbox is used assuming that the tuning problem is an unconstrained one. The result was reducing the maximum percentage overshoot, maximum percentage overshoot and settling time. The performance of the control system using an PD-PI controller using the present tuning technique is compared with that using a PID controller tuned by Ziegler-Nichols and Tavakoli tuning techniques.

Keywords: PD-PI controller , first-order-delayed process , controller tuning , control system performance.

I. INTRODUCTION

Time delayed processes are difficult to control than the un-delayed processes. This is simply because of the delayed characteristic of the system is compensated in the analysis by using polynomials in the Laplace operator s which means increasing the order of the closed-loop control system. This of course has its impact on the control system stability and performance. The PD-PI controller is one of the next generation of PID controllers where research and application is required to investigate its effectiveness compared with PID controllers when controlling such processes.

Zhang and Sun (1996) developed simple tuning rules for the Smith predictor used to control an integrator and a long time delay [1]. Wittenmark, Bastian and Nilsson (1998) described how to determine the time delays caused by synchronous and asynchronous control loops. They concluded that for large variation in time delay, it would be important to consider the delays in the controller design [2]. Lee, Lee and Park (2000) proposed a method for PID controller tuning based on process models for integrating and unstable processes with time delay giving better closed-loop performance [3]. Singh and Singhose (2002) presented techniques to shape the input to the system so as to minimize the residual vibration of the structure for precise position control [4]. Tan, Marquez and Chen (2003) proposed a modified IMC structure to control unstable processes with time delays. They tuned the modified IMC structure with an emphasis on the robustness of the structure [5]. Barraud, Creff and Petit (2004) presented a process model with delay variability exploring robustness properties of a wide panel of PI controllers [6]. Bozorg and Davison (2006) considered the stability of time delay processes having uncertain delays. They presented a numerical algorithm for the calculation of the time delay stability margins in the space of time delays [7]. Dostal, Gazdos and Bobal (2008) studied the design of controllers for integrating and unstable time delay systems using time delay approximation using the polynomial approach [8]. Liu, Zinoba and Shtessel (2009) considered a fully linearizable SISO system with output time delay using the Pade approximation and second-order sliding-mode control [9].

Ruscio (2010) presented analytical results concerning PI controller tuned to control integrator plus time delay plants. He proposed modified parameters for the Ziegler-Nichols tuning method [10]. Vrancic and Huba (2011) presented a tuning method based on characteristic areas and magnitude optimum criterion for some unstable processes using a 2DOF PI controller [11]. Dostal, Bobal and Babik (2012) proposed a method for the design of controllers for time delay systems having integrative or unstable properties. They used two methods for time delay approximations [12]. Acharya, Mitra and Halder (2013) approximated the delay term as a transfer function using Pade approximation and used the Bode integral to determine the PID controller parameters [13]. Sandaram and Padhy (2013) proposed a genetic algorithm based PI-PD controller for improving network utilization in TCP/IP networks. They used the ISTE criterion to tune the parameters of the PI-PD controller [14]. Shariati, Taghirad and Fatehi (2014) presented a neutral system approach for the design of a H∞ controller for input delay systems with uncertain time-invariant delay. Delay-dependent sufficient conditions for the existence of a H∞ PD and PI controllers in the presence of uncertain delay were derived in terms of matrix inequalities [15]. Brun et. al. (2014) studied the design of a control system for the fuel syst H∞em of a turbojet. They selected multi-loop strategy based on PID, finding which strategy was the best suitable [16].

II. ANALYSIS PROCESS:

Large number of processes having non-oscillating step time response can be approximated by a first-order plus delay time model. Such a process has the transfer function:

$$G_p(s) = K\, e^{-Tds} / (1 + Ts) \qquad (1)$$

Where:

K = process gain
T_d = process delay time
T = process time constant

Controller:

The controller used in this study is a proportional+derivative (PD) - proportional + integral (PI) controller. In this controller, The PD and PI parts of the controller are connected in series. The input to the PD part is the system error, while the input of the PI part is the output of the PD part [17]. The block diagram of the closed-loop control system incorporating the PD-PI controller is shown in Fig.1 [17].

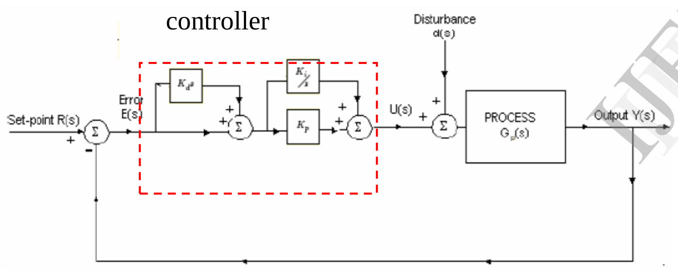

Fig.1 PD-PI controlled process.

The controller transfer function is, $G_c(s)$ is:

$$G_c(s) = (1/s)\, [K_{pc}K_d s^2 + (K_{pc} + K_iK_d)s + K_i] \qquad (2)$$

Where:

K_{pc} = Proportional gain
K_i = Integral gain
K_d = Derivative gain

i.e. the controller has 3 parameters to be identified to control the process and produce a satisfactory performance.

Control System Transfer Function:

The controller and process are cascaded in the forward path of the unity feedback control system. To be able to analyze the closed-loop control system using linear control theory, the exponential term in the process transfer function of Eq.1 has to be dealt with. Padi approximation gives the solution [5,8,9,10,13,18].

Vajta (2000) presented 5 Padi approximations for the delayed term $e^{-Td}s$ from 1st-oder to 5th-order. From the work of Vajta one can conclude that for better representation of the delay term, the order has to be ≥ 4. A forth order Pade approximation is [18]:

$$e^{-Tds} = (-T_d^3 s^3 + 60T_d^2 s^2 - 360T_d s + 840) / (T_d^4 s^4 + 16T_d^3 s^3 + 120T_d^2 s^2 + 480T_d s + 840) \qquad (3)$$

Combining Eqs.1 and 2, the process transfer function becomes:

$$G_p(s) = (b_0s^3 + b_1s^2 + b_2s + b_4) / \{a_0s^5 + a_1s^4 + a_2s^3 + a_3s^2 + a_4s + a_5\} \qquad (4)$$

Where:

$b_0 = -T_d^3$
$b_1 = 60T_d^2$
$b_2 = -360T_d$
$b_3 = 840$
$a_0 = TT_d^4$
$a_1 = 16TT_d^3 + T_d^4$
$a_2 = 120TT_d^2 + 16T_d^3$
$a_3 = 480TT_d + 120T_d^2$
$a_4 = 840T + 480T_d$
$a_5 = 840$

The open-loop transfer function of the control system for the unity feedback control system of Fig.1, G(s) is given by:

$$G(s) = (c_0s^5 + c_1s^4 + c_2s^3 + c_3s^2 + c_4s + c_5) / \{a_0s^6 + a_1s^5 + a_2s^4 + a_3s^3 + a_4s^2 + a_5s\} \qquad (4)$$

where:

$c_0 = b_0K_{pc}K_d$
$c_1 = b_0(K_{pc} + K_iK_d) + b_1K_{pc}K_d$
$c_2 = b_0K_i + b_1(K_{pc} + K_iK_d) + b_2K_{pc}K_d$
$c_3 = b_1K_i + b_2(K_{pc} + K_iK_d) + b_3K_{pc}K_d$
$c_4 = b_2K_i + b_3(K_{pc} + K_iK_d)$
$c_5 = b_3 K_i$

The closed-loop transfer function of the control system, M(s) is;

$$M(s) = (c_0s^5 + c_1s^4 + c_2s^3 + c_3s^2 + c_4s + c_5) / \{a_0s^6 + (a_1+c_0)s^5 + (a_2+c_1)s^4 + (a_3+c_2)s^3 + (a_4+c_3)s^2 + (a_5+c_4)s + c_5\} \qquad (5)$$

System Step Response:

A unit step response is generated by MATLAB using the numerator and deniminator of Eq. 5 providing the system response c(t) as function of time [19].

III. CONTROLLER TUNING

The sum of square of error (ISE) is used an objective function, F of the optimization process. Thus:

$$F = \int [c(t) - c_{ss}]^2 \, dt \qquad (6)$$

where c_{ss} = steady state response of the system = 1 for a unit step input.

The performance of the control system is judged using three time-based specifications:

(a) Maximum percentage overshoot, OS_{max}
(b) Maximum percentage undershoot, US_{max}
(c) Settling time, T_s

Tuning Results:

The MATLAB command "*fminunc*" is used to minimize the optimization objective function given by Eq.6 without any parameters ot functional constraints [20]. The results are as follows:

Controller parameters:

The optimally tuned controller parameters depend on the process delay time, T_d. For a process time constant of 50 seconds, and a delay time from 4 to 16 seconds, the PD-PI controller parameters are given in Table 1.

Table 1: PD-PI tuned controller parameters

T_d (s)	K_{pc}	K_i	K_d
4	5.59992	0.23976	2.04608
8	3.94550	0.166098	2.39727
12	3.23192	0.09426	1.66965

The time response of the closed-loop control system to a unit step input is shown in Fig.2, for 4, 8 and 12 s delay time.

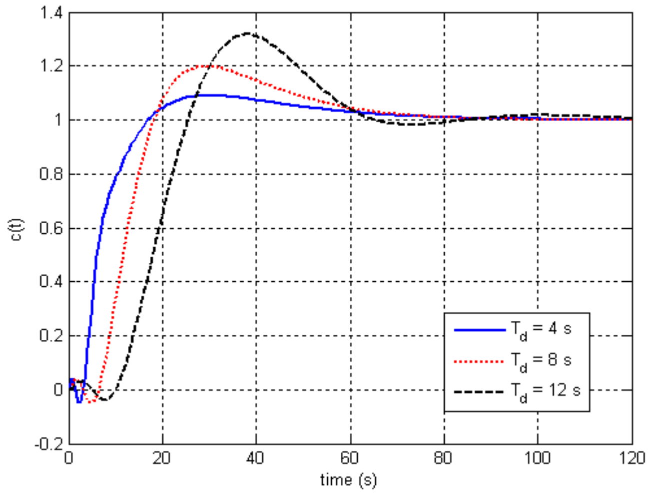

Fig.2 Step response of the PD-PI controlled first-order- delayed process.

The characteristics of the control system using the tuned PD-PI controller will be compared later with PID controller tuned using different techniques.

IV. COMPARISON WITH PID CONTROLLER

Using a conventional PID controller instead of the proposed PD-PI controller to control the first-order-delayed process is investigated for sake of comparison and assessment. There are different techniques that may be used in tuning PID controllers used with a first-order-delayed process. One of those techniques are Ziegler-Nichols [21] and Tavakoli [22].

The PID controller parameters tuned using Ziegler-Nichols and Tavakoli techniques are given in Table 2 for T = 50 and T_d = 4, 8 and 12 seconds.

Table 2: PID controller parameters.

	Ziegler- Nichols tuning			Tavakoli tuning		
T_d (s)	K_{pc}	K_i	K_d	K_{pc}	K_i	K_d
4	15.0612	1.8825	30.1205	5.9538	0.7530	8.3668
8	7.5301	0.4706	30.1205	3.8000	0.2196	4.1834
12	5.0201	0.2092	30.1205	2.8345	0.1116	2.7889

The time response of the control system using the proposed PD-PI controller is compared with that using PID controller tuned by Ziegler-Nichols and Tavakoli techniques.

ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

It is shown in Figs.3, 4 and 5 for T_d = 4, 8 and 12 seconds respectively.

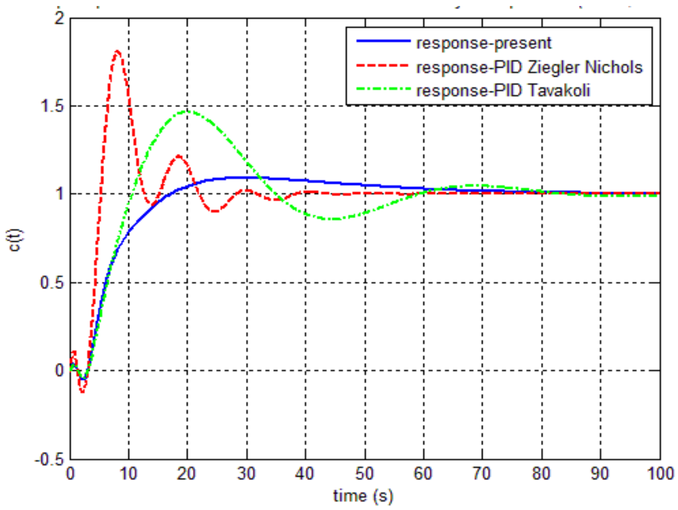

Fig.3 Step response of the PD-PI and PID controlled first-order-delayed process (T_d = 4 s)

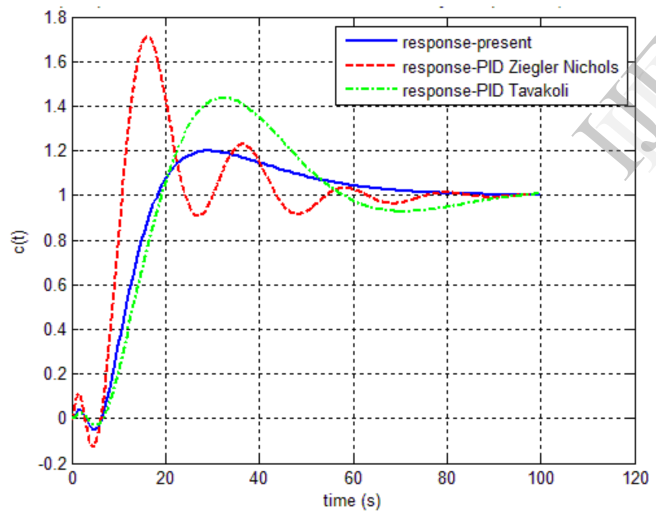

Fig.4 Step response of the PD-PI and PID controlled first- order-delayed process (T_d = 8 s)

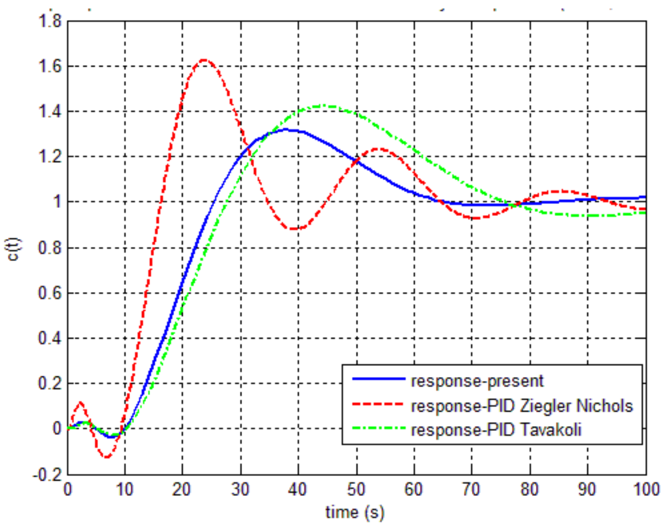

Fig.5 Step response of the PD-PI and PID controlled first-order-delayed process (T_d = 12 s)

The performance parameters of the control system using the PD-PI controller and PID controller are compared in Table 3.

Table 3: Performance comparison

T_d (s)	Method	OS (%)	US (%)	T_s (s)
4	Present	9	0	50
4	Z-N	80	10	26
4	Tavikoli	47	14	54
8	Present	20	0	58
8	Z-N	72	8	50
8	Tavikoli	44	7	80
12	Present	31.5	1.5	62
12	Z-N	62.5	13	74
12	Tavikoli	42	6	100

Z-N: Ziegler-Nichols

ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

V. DISCUSSIONS

- Fourth-order Pade approximation is used to deal with the process delay.
- It was possible to produce a step time response without any overshoot for delay time ≤ 8 seconds.
- Maximum percentage undershoot was ≤ 1.5 % fit T_d ≤ 12 seconds .
- The settling time using the PD-PI controller was less than that associated with the PID controller tuned by Ziegler-Nichols for $T_d \geq 8$ seconds.
- The settling time using the PD-PI controller was less than that associated with the PID controller tuned by Tavakoli for $4 \leq T_d \leq 12$ seconds.

VI. REFERENCES

[1] W. Zhang and Y. Sun, "Modified Smith predictor for controlling integral time delay processes", Industrial Engineering Chemestry Research, Vol.35, 1996, pp.2769-2772.
[2] B. Wittenmark, B. Bastian and I. Nilsson, "Analysis of time delays in synchronous and asynchronous control loops", 37th CDC, Tampa, December 1998.
[3] Y. Lee, J. Lee and S. Park, "PID controller tuning for integrating and unstable processes with time delay", Chemical Engineering Science, Vol.55, 2000, pp.3481-3493.
[4] T. Smith and W. Singhose, "Tutorial on input shaping / time delay control of maneuvering flexible structures", Proceedings of the American Control Conference, Anchorage, AK, 2002, pp.1707-1713.
[5] W. Tan, H. Marquez and T. Chen, "IMC design for unstable processes with time delays", J. of Process Control, Vol.13, 2003, pp.203-213.
[6] J. Barroud, Y. Creff and N. Petit, "PI controllers performances for a process model with varying delay", Control Conference, University of Path, UK, 6-9 September 2004.
[7] M. Bozorg and E. Davison, "Control of time delay processes with uncertain delays: time delay stability margins", J. of Process Control, Vol.16, 2006, pp.403-408.
[8] P. Dostal, F. Gazdos and V. Bohal, "Design of controllers for unstable time delay systems: part II: Integrating and unstable systems", J. of Electrical Engineering, Vol.59, No.1, 2008.
[9] G. Liu, A. Zinober and Y. Shtessel, "Second-order SM approach to SISO time-delay system output tracking", IEEE Transactions on Industrial Electronics, Vol.56, No.9, 2009, pp.3638-3645.
[10] D. Ruscio, "On tuning PI controllers for integrating plus time delay systems", Modeling, Identification & Control, Vol.31, No.4, 2010, pp.145-164.
[11] D. Vrancic and M. Huba, "Design of feedback control for unstable processes with time delay", Proceddings of the 18th International Conference on Process Control, Slovakia, June 14-17, 2011, pp.100-105.
[12] P. Dostal, V. Bobal and Z. Babik, "Control of unstable and integrating time delay systems using time delay approximations", WSEAS Transactions on Systems, Vol.11, No.10, 2012, pp.586-595.
[13] A. Achorya, D. Mitra and K. Halder, "Stability analysis of delayed system using Bode integral", Internation Conference on Computer Communication & Informatics, India, January 2013.
[14] R, Sundaram and P. Padhy, "GA-based PI-PD controller for TCP routers", International Journal for Machine Learning and Computing, Vol.3, No.4, August 2013, pp.361-364.
[15] A. Shariati, H. Taghirad and A. Fatehi, "A neutral system approach to H∞ PD/PI design of processes with uncertain input delay", Journal of Process Control, Vol.24, No.3, March 2014, pp.144-157.
[16] L. Brun, E. Godoy, D. Beauvois, G. Pache and R. Neguera, "Modeling and analysis of a turbojet fuel system", I nternational Journal of Computer Theory and Engineering, Vol.6. No.3, June 2014, pp.260-266.
[17] T. Jain and M. Nigram, "Optimization of PD-PI controller using swarm intelligence", J. of Theoretical and Applied Information Technology, Vol4, No.11, 2008, pp.1013-1018.
[18] M. Vajta, "Some remarks on Pade approximations", 3rd TEMPUS INTCOM Symposium, September 9-14, Hungary, 2000.
[19] ------, "Control system toolbox", MathWorks, September 2012.
[20] ------, "The MOSEK optimization toolbox for MATLAB", MOSEK, Denmark, 1998-2012.
[21] J. Ziegler and N. Nichols, "Optimum settings for automatic controllers", Transactions ASME, Vol.64, 1942, pp.759-768.
[22] S. Tavakoli and M. Tavakoli, "Optimum tuning of PID controllers for first-order plus time delay models using dimensional analysis", The 4th International Conference on Control and Automation, 10-12 June,2003, Montreal, Canada, pp.942-946.