Triple- diffusive convection in a micropolar ferrofluid in the presence of rotation

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Abstract

This paper deals with the theoretical investigation of the triple-diffusive convection in a micropolar ferrofluid layer heated and soluted below subjected to a transverse uniform magnetic field in thev presence of uniform vertical rotation. For a flat fluid layer contained between two free boundaries, an exact solution is obtained. A linear stability analysis theory and normal mode analysis method have been carried out to study the onset convection. The influence of various parameters like rotation, solute gradients, and micropolar parameters (i.e. coupling parameter, spin diffusion parameter and micropolar heat conduction parameter) has been analyzed on the onset of stationary convection. The critical magnetic thermal Rayleigh number for the onset of instability is also determined numerically for sufficiently large value of buoyancy magnetization parameter M_1(ratio of the magnetic to gravitational forces). The principle of exchange of stabilities is found to hold true for the micropolar fluid heated from below in the absence of micropolar viscous effect, microinertia, solute gradient and rotation. The oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia , solute gradient and rotation, which were non-existent in their absence. In this paper, an attempt is also made to obtain the sufficient conditions for the non-existence of overstability.

Keywords: Triple- diffusive convection; Micropolar ferrofluid; Thermal convection; Solute gradient; Vertical magnetic field; Rotation; Magnetization.

1. Introduction

Micropolar fluids are fluids with internal structures in which coupling between the spin of each particle and the microscopic velocity field is taken into account. They represent fluids consisiting of rigid, randomly oriented or spherical particles suspended in viscous medium, where the deformation of fluid particles is ignored (e.g. polymeric suspension, animal blood, liquid crystal). Micropolar fluids have been receiving a great deal of research focus and interest due to their application in a number of processes that occur in industry. Such applications include the extrusion of polymer fluids, solidification of liquid crystal, cooling of metallic plate in a bath, exotic lubricants and colloidal suspension solutions. Micropolar fluid theory was introduced by Eringen [1] in order to describe some physical systems, which do not satisfy the Navier- Stokes equation. The equations governing the micropolar fluid involve a spin vector and microinertia tensor in addition to the velocity vector. The theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood etc. The generalization of the theory including thermal effects has been developed by Kazakia and Ariman [2] and Eringen [3]. The theory of thermomicropolar convection began with Datta and Sastry[4] and interestingly continued by Ahmadi[5], Lebon and Perez-Garcia[6],Bhattacharya and Jena[7], Payne and Straughan [8], Sharma and Kumar [9,10] and Sharma and Gupta [11]. The above works give a good understanding of thermal convection in micropolar fluids.

In many situations involving suspensions, as in the magnetic fluid case, it might be pertinent to demand an Eringen micropolar description. This was suggested, in fact, by Rosenweig [12] in his monograph. An interesting possibilities in a planer micropolar ferrofluid flow with an AC magnetic field has been considered by Zahn and Greer [13].They examined a simpler case where the applied magnetic fields along and transverse to the duct axis are spatially uniform and varying sinusoidally with time. In a uniform magnetic field, the magnetization characteristic depends on particle spin but does not depend on fluid velocity. Micropolar ferrofluid stabilities have become an important field of research these days. A particular stability problem is Rayleigh-Bénard instability in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below has been given by Chadrasekhar [14]. For a ferrofluid, a thermo-mechanical interaction is predicted by Finlayson [15] in the presence of a uniform vertical magnetic field provided the magnetization is a function of temperature and magnetic field, and a temperature gradient is established across the fluid layer. The thermal convection in Newtonian ferro fluid has been studied by many authors [16-25].

Rayleigh-Bénard convection in a micropolar ferrofluid layer permeated by a uniform, vertical magnetic field with free-free, isothermal, spin-vanishing, magnetic boundaries has been considered by Abraham [26]. She observed that the micropolar ferro fluid layer heated from below is more stable as compared with the classical Newtonian ferrofluid.The effect of rotation on thermal convection in a micropolar fluids is important in certain chemical engineering and biochemical situations. Qin and Kaloni [27] have considered a thermal instability problem in a rotating micropolar fluid. They found that, depending upon the values of various micropolar parameters and the low values of the Taylor number, the rotation has a stabilizing effect. The effect of rotation on thermal convection in micropolar fluids has also been studied by Sharma and Kumar [28] , whereas the numerical solution of thermal instability of rotating micropolar fluid has been discussed by Sastry and Rao[29] without taking into account the rotation effect in angular momentum equation. But we also appreciate the work of Bhattacharyya and Abbas [30] and Qin and Kaloni ,they have considered the effect of rotation in angular momentum equation. More recently, Sunil et al., [31-33]] have studied the effect of rotation on the thermal convection problems in ferrofluid. .

In the standard Bénard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid, additionally has salt dissolved in it , then there are potentially two destabilizing sources for the density difference, the temperature field and salt field. The solution behavior in the double-diffusive convection problem is more interesting than that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Bénard problem. When temperature and two or more component agencies, or three different salts, are present then the physical and mathematical situation becomes increasingly richer. Very interesting results in triply diffusive convection have been obtained by Pearlstein et al., [34]. The results of Pearlstein et al., are remarkable. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker [35] derive the equations for non-Boussinesq convection in a multi- component fluid and investigate the situation analogous to that of Pearlstein et al., but allowing for a density non linear in the temperature field. Lopez et al., [36] derive the equivalent problem with fixed boundary conditions and show that the effect of the boundary conditions breaks the perfect symmetry. In reality the density of a fluid is never a linear function of temperature, and so the work of Straughan and Walker applies to the general situation where the equation of state is one of the density quadratic in temperature. This is important, since they find that departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.,. Suresh [37, 38] has studied the triple-diffusive convection in Walters'(Model B') fluid in the porous medium in hydromagnetics and effect of rotation on triple-diffusive convection in a magnetized ferrofluid with internal angular momentum saturating a porous medium.

In view of the recent increase in the number of non iso-thermal situations wherein magnetic fluid are put to use in place of classical fluids, we intend to extend our work to the problem of thermal convection in Eringen's micropolar fluid to the triple-diffusive convection in a mocropolar ferrofluid in the presence of rotation. In the present analysis, for mathematically simplicity, we have not considered the effect of rotation in angular momentum equation.

2. Mathematical formulation of the problem

Here we consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible thin micropolar ferromagnetic fluid heated and salted from below. The temperature T and solute concentrations C^1 and C^2 at the bottom and top surfaces $z = \mp \frac{1}{2}d$ are T_0 and T_1 ; C_0^1 and C_1^1; and C_0^2 and C_1^2 respectively, and a uniform temperature gradient $\beta (= |\frac{dT}{dZ}|)$ and uniform solute gradients $are\ \beta'$ $(= |\frac{dC^1}{dZ}|)$ and β'' $(= |\frac{dC^2}{dZ}|)$ are maintained. Both the boundaries are taken to be free and perfect conductors of heat. The fluid layer is assumed to occupy the layer $z \in (-d/2, d/2)$ with gravity acting in the negative z- direction and magnetic field , $\mathbf{H} = H_0^{ext}\hat{\boldsymbol{k}}$, where $\hat{\boldsymbol{k}} = (0,0,1)$, acts outside the layer. The whole system is assumed to rotate with angular velocity $\mathbf{\Omega} = (0,0,\Omega)$ along the vertical axis,which is taken as z-axis.

The mathematical equations governing the motion of incompressible micropolar ferrofluids (utilizing Boussinesq approximation) the for the above model are as follows:

The continuity equation for an incompressible fluid is

$$\nabla . \boldsymbol{q} = 0 \tag{1}$$

The momentum and internal angular momentum equations are

$$\rho_0 \left[\frac{\partial}{\partial t} + (\mathbf{q}.\nabla)\right]\mathbf{q} = -\nabla p + \rho\mathbf{g} + \mu_0 (\mathbf{M}.\nabla)\boldsymbol{H} + (\zeta + \eta)\nabla^2 q + 2\zeta(\nabla \times \omega) + 2\rho_0 (\mathbf{q} \times \mathbf{\Omega}), \tag{2}$$

$$\rho_0 I \left[\frac{\partial}{\partial t} + (\mathbf{q}.\nabla)\right]\omega = 2\zeta(\nabla \times \boldsymbol{q} - 2\omega) + \mu_0 (\mathbf{M} \times \boldsymbol{H}) + (\lambda' + \eta')\nabla(\nabla . \omega) + \eta' \nabla^2 \omega \tag{3}$$

The temperature and solute concentration equations for an incompressible micropolar ferromagnetic fluid are

$$[\rho_0 C_{V,H} - \mu_0 \mathbf{H} \left(\frac{\partial \boldsymbol{M}}{\partial t}\right)_{V,H}] \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \boldsymbol{M}}{\partial t}\right)_{V,H} . \frac{DH}{Dt} = K_1 \nabla^2 T + \delta (\nabla \times \omega). \nabla T \tag{4}$$

$$[\rho_0 C_{V,H} - \mu_0 \mathbf{H} \left(\frac{\partial \boldsymbol{M}}{\partial C^1}\right)_{V,H}] \frac{DC^1}{Dt} + \mu_0 \left(C^1 \frac{\partial \boldsymbol{M}}{\partial C^1}\right)_{V,H} . \frac{DH}{Dt} = K'_1 \nabla^2 C^1 \tag{5}$$

$$[\rho_0 C_{V,H} - \mu_0 \mathbf{H} \left(\frac{\partial \boldsymbol{M}}{\partial C^2}\right)_{V,H}] \frac{DC^2}{Dt} + \mu_0 C^2 \left(\frac{\partial \boldsymbol{M}}{\partial C^2}\right)_{V,H} . \frac{DH}{Dt} = K''_1 \nabla^2 C^2 \tag{6}$$

In terms of temperature T and the concentrations C^1 and C^2, we suppose the density of the mixture is given by (known as density equation of state)

$$\rho = \rho_0 [1 - \alpha (T - T_a) + \alpha' (C^1 - C_a^1) + \alpha'' (C^2 - C_a^2)] \tag{7}$$

Where ρ, ρ_0, $\mathbf{q}, \omega$, t, p, η, ζ, λ', η', δ, I, μ_0, **B,** $C_{V,H}$, M, K_1, K'_1, K''_1, α, α', α'' are the fluid density ,reference density, velocity, microrotation, time, pressure, shear kinematic viscosity coefficient, coupling viscosity coefficient or vortex viscosity, bulk spin viscosity coefficient, shear spin viscosity coefficient, micropolar heat conduction coefficient , moment of inertia (microinertia constant),magnetic permeability, magnetic induction, heat capacity at constant volume and magnetic field, magnetization, thermal conductivity, solute conductivity, thermal expansion coefficient and concentration expansion coefficient analogous to the thermal expansion coefficient respectively. $T_a$is the average temperature given by
$T_a = (T_0+T_1)/2$ where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer and C_a^1 and $C_a^2$are the average concentrations given by $C_a^1 = (C_0^1+C_1^1)/2$ and $C_a^2 = (C_0^2+C_1^2)/2$, where C_0^1 , C_1^1 and C_0^2, $C_1^2$are the constant average concentrations of the lower and upper surfaces of the layer. The partial derivatives of **M** are the material properties that can be evaluated once the magnetic equation of state, such as (10) below is known. In writing equation (2), we also use the Boussinesq approximation by allowing the density to change only in the gravitational body force term.
Maxwell 's equation, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla . \mathbf{B} = 0, \tag{8a}$$

$$\nabla \times \mathbf{H} = 0, \tag{8b}$$

where the magnetic induction is given by
$B = \mu_0 (H + M)$.
We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\boldsymbol{H}}{H} M(H, T, C^1, C^2). \tag{9}$$

The magnetic equation of state is linearized about the magnetic field, H_0, an average temperature,
T_a, and average concentrations, $C_a^1$and C_a^2 to become

$$M = M_0 + \chi(H - H_0) - K_2 (T - T_a) + K_3(C^1 - C_a^1) + K_4(C^2 - C_a^2). \tag{10}$$

where magnetic susceptibility, pyromagnetic coefficient and salinity magnetic coefficients are defined by

$$\chi \equiv \left(\frac{\partial M}{\partial H}\right)_{H_0, T_a}; \quad K_2 \equiv -\left(\frac{\partial M}{\partial T}\right)_{H_0, T_a}; \quad K_3 \equiv \left(\frac{\partial M}{\partial C^1}\right)_{H_0, c_a^1} \text{ and } K_4 \equiv \left(\frac{\partial M}{\partial C^2}\right)_{H0, c_a^2} \text{ respectively.} \tag{11}$$

Here H_0 is the uniform magnetic field of the fluid layer when placed in an external magnetic field $H = H_0^{ext} \hat{k}$, where $\hat{k}$ is a unit vector in the z direction,
$H = |\mathbf{H}|$, $M = |\mathbf{M}|$and $M_0 = M(H_0, T_a, C_a^1, C_a^2)$.

The effect of rotation contributes two terms: (a) Centrifugal force $-(\rho_0/2)\,\mathrm{grad}|\boldsymbol{\Omega}\times \mathrm{r}|^2$ and (b) Coriolis force $2\rho_0\,(\mathbf{q}\times\boldsymbol{\Omega})$. In equation (2), $p = p_f - \frac{1}{2}\rho_0|\boldsymbol{\Omega}\times \mathrm{r}|^2$ is the reduced pressure, where p_f stands for fluid pressure.

The basic state is assumed to be quiescent state and is given by

$\mathbf{q} = \mathbf{q_b} = (0,0,0)$, $\boldsymbol{\omega} = \boldsymbol{\omega_b} = (0,0,0)$, $\rho = \rho_b(z)$, $p = p_b(z)$, $T = T_b(z) = -\beta z + T_a$, $C^1 = C^1{}_b(z) = -\beta' z + C^1{}_a$
$C^2 = C^2{}_b(z) = -\beta'' z + C^2{}_a$, $\beta = (T_0 - T_1)/d$, $\beta' = (C_1{}^1 - C_0{}^1)/d$, $\beta'' = (C_1{}^2 - C_0{}^2)/d$,

$$\mathbf{H_b} = \left[H_0 - \frac{K_2\beta z}{1+\chi} + \frac{K_3\beta' z}{1+\chi} + \frac{K_4\beta'' z}{1+\chi}\right]\hat{k},\quad \mathbf{M_b} = \left[M_0 + \frac{K_2\beta z}{1+\chi} - \frac{K_3\beta' z}{1+\chi} - \frac{K_4\beta'' z}{1+\chi}\right]\hat{k} \text{ and } H_0 + M_0 = H_0^{ext}, \quad (12)$$

where the subscript ' b ' denotes the basic state.

3. The perturbation equations and normal mode analysis method

We now examine the stability of the basic state, and assume that the perturbation quantities are small. We write

$$\mathbf{q} = \mathbf{q_b} + \mathbf{q}',\ \boldsymbol{\omega} = \boldsymbol{\omega_b} + \omega',\ \rho = \rho_b + \rho',\ p = p_b(z) + p',\ T = T_b(z) + \theta,\ C^1 = C^1{}_b(z) + \gamma,\ C^2 = C^2{}_b(z) + \gamma',$$
$$H = H_b(z) + H' \text{ and } M = M_b(z) + M' \quad (13)$$

where $\mathbf{q}' = (u, v, w)$, $\boldsymbol{\omega}' = (\omega_1, \omega_2, \omega_3)$, ρ', θ, γ, γ', $\mathbf{H}'$, $\mathbf{M}'$ are perturbation in velocity q, spin ω, pressure p, temperature T, concentrations C^1 and C^2, magnetic field intensity **H**, and magnetization **M,** respectively. The change in density ρ', caused mainly by the perturbations θ, γ, and γ' in temperature and concentrations, respectively, is given by

$$\rho' = -\rho_0(\alpha\,\theta - \alpha'\gamma - \alpha''\gamma'). \quad (14)$$

Then, the linearized perturbation equations (by neglecting second-order small quantities) of the micropolar ferromagnetic fluid become

$$\rho_0\frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} + (1+\chi)\nabla^2 u + 2\zeta\Omega_1' + 2\rho_0\Omega_V, \quad (15)$$

$$\rho_0\frac{\partial v}{\partial t} = -\frac{\partial p'}{\partial y} + \mu_0(M_0 + H_0)\frac{\partial H_2'}{\partial z} + (1+\chi)\nabla^2 v + 2\zeta\Omega_2' - 2\rho_0\Omega_u, \quad (16)$$

$$\begin{aligned}\rho_0\frac{\partial w}{\partial t} = &-\frac{\partial p'}{\partial z} + \mu_0(M_0 + H_0)\frac{\partial H_3'}{\partial z} + (1+\chi)\nabla^2 w + 2\zeta\Omega_3 - \mu_0\frac{K_2\beta}{1+\chi}\{H_3'(1+\chi) - K_2\theta\},\\ &+\mu_0\frac{K_3\beta'}{1+\chi}\{H_3'(1+\chi) + K_3\gamma\} + \mu_0\frac{K_4\beta''}{1+\chi}\{H_3'(1+\chi) + K_4\gamma'\} - \mu_0\frac{K_2K_3}{1+\chi}(\beta'\theta + \beta\gamma)\\ &+\mu_0\frac{K_3K_4}{1+\chi}(\beta''\gamma + \beta\gamma') - \mu_0\frac{K_2K_4}{1+\chi}(\beta''\theta + \beta\gamma) + \rho_0 g(\alpha\,\theta - \alpha'\gamma - \alpha''\gamma'),\end{aligned} \quad (17)$$

$$\rho_0 I\frac{\partial w'}{\partial t} = 2\zeta(\nabla\times\boldsymbol{q}' - 2\omega') + \mu_0(\mathbf{M_b}\times\boldsymbol{H}' + \boldsymbol{M}'\times\boldsymbol{H_b}) + (\lambda' + \eta')\nabla(\nabla.\omega') + \eta'\nabla^2\omega' \quad (18)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (19)$$

$$\rho C_1\frac{\partial\theta}{\partial t} - \mu_0 T_0K_2\frac{\partial}{\partial t}\left(\frac{\partial\phi 1'}{\partial z}\right) = K_1\nabla^2\theta + [\rho C_1\beta - \mu_0 T_0K_2{}^2\beta/(1+\chi)]\omega - \delta\beta\,\Omega_3', \quad (20)$$

$$\rho C_1'\frac{\partial\gamma}{\partial t} - \mu_0 C_0{}^1K_3\frac{\partial}{\partial t}\left(\frac{\partial\phi 2'}{\partial z}\right) = K_1'\nabla^2\gamma + [\rho C_1'\beta' - \mu_0 C_0{}^1K_3{}^2\beta'/(1+\chi)]\omega, \quad (21)$$

$$\rho C_1''\frac{\partial\gamma'}{\partial t} - \mu_0 C_0{}^2K_2\frac{\partial}{\partial t}\left(\frac{\partial\phi 3'}{\partial z}\right) = K_1''\nabla^2\gamma' + [\rho C_1''\beta'' - \mu_0 C_0{}^2K_2{}^2\beta''/(1+\chi)]\omega, \quad (22)$$

where $\rho C_1 = \rho_0 C_{V,H} + \mu_0 K_2\mathbf{H_0}$; $\rho C_1' = \rho_0 C_{V,H} - \mu_0 K_3\mathbf{H_0}$; $\rho C_1'' = \rho_0 C_{V,H} - \mu_0 K_4\mathbf{H_0}$,

Equation (9) and Equation (10) yield

$$H_3' + M_3' = (1+\chi)H_3' - K_2\theta, \quad (23)$$
$$H_3' + M_3' = (1+\chi)H_3' - K_3\gamma, \quad (24)$$
$$H_3' + M_3' = (1+\chi)H_3' - K_4\gamma' \quad (25)$$
$$H_i' + M_i' = (1 + M_0/H_0)H_i' \quad i=1,2,3. \quad (26)$$

where, we have assumed $K_2(T_b - T_a) \ll (1+\zeta)H_0$; $K_3\beta' d \ll (1+\zeta)H_0$; $K_4\beta'' d \ll (1+\zeta)H_0$;

$\Omega' = (\Omega_1', \Omega_2', \Omega_3') = (\nabla \times \omega')$

Thus the analysis is restricted to physical situation in which the magnetization induced by temperature and concentration variations is small compared to that induced by the external magnetic field. Eq. (7b) means that we can write $H' = \nabla (\phi_1' - \phi_2' - \phi_3')$, where ϕ_1' is the perturbed magnetic potential and ϕ_2', ϕ_3' are the perturbed magnetic potentials analogous to solute.

Eliminating u, v, p′ between Eq. (15)- (17), using Eq. (18), and taking curl once on Eq. (3) and considering only kth component, we obtain

$$\begin{aligned}(\rho_0 \frac{\partial}{\partial t} - (\zeta + \eta)\nabla^2) = & -\mu_0 \frac{K_2\beta}{1+\chi}\nabla_1^2\{(1+\zeta)\frac{\partial}{\partial z}(\phi_1' - \phi_2' - \phi_3') - K_2\theta\} \\ & + \mu_0\frac{K_3\beta'}{1+\chi}\nabla_1^2\{(1+\zeta)\frac{\partial}{\partial z}(\phi_1' - \phi_2' - \phi_3') - K_3\gamma\} \\ & + \mu_0\frac{K_4\beta''}{1+\chi}\nabla_1^2\{(1+\zeta)\frac{\partial}{\partial z}(\phi_1' - \phi_2' - \phi_3') - K_4\gamma'\} \\ & - \mu_0\frac{K_2K_3}{1+\chi}\nabla_1^2(\beta'\theta + \beta\gamma) + \mu_0\frac{K_4K_3}{1+\chi}\nabla_1^2(\beta''\gamma + \beta'\gamma') \\ & - \mu_0\frac{K_2K_4}{1+\chi}\nabla_1^2(\beta''\theta + \beta'\gamma) + \rho_0 g\nabla_1^2(\alpha\theta - \alpha'\gamma - \alpha''\gamma') \\ & + 2\zeta\nabla^2\Omega_3' - 2\rho_0\Omega\frac{\partial\zeta_1}{\partial Z}\end{aligned} \tag{27}$$

$$\rho_0 I\frac{\partial\Omega 3'}{\partial t} = -2\zeta(\nabla^2\omega + 2\Omega_3') + \eta'\nabla^2\Omega_3'. \tag{28}$$

The vertical component of the vorticity equation is

$$\rho_0\frac{\partial\zeta_1}{\partial t} = 2\rho_0\Omega\frac{\partial w}{\partial z} + (\zeta + \eta)\nabla^2\zeta_1 \tag{29}$$

Where $\zeta_1 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ stands for the z-component of the vorticity

From Eq. (20), we have

$$(1+\chi)\frac{\partial^2\phi 1'}{\partial z^2} + (1+\frac{M_0}{H_0})\nabla^2\phi_1' - K_2\frac{\partial\theta}{\partial Z} = 0, \tag{30}$$

$$(1+\chi)\frac{\partial^2\phi 2'}{\partial z^2} + (1+\frac{M_0}{H_0})\nabla^2\phi_2' - K_3\frac{\partial\gamma}{\partial Z} = 0, \tag{31}$$

$$(1+\chi)\frac{\partial^2\phi 3'}{\partial z^2} + (1+\frac{M_0}{H_0})\nabla^2\phi_3' - K_4\frac{\partial\gamma'}{\partial Z} = 0, \tag{32}$$

We analyze the normal mode technique. This can be written

$$f(x, y, z, t) = f(z, t)\exp i(k_x x + k_y y), \tag{33}$$

where $f(z, t)$ represent $W(z,t)$, $\Theta(z,t)$, $Z(z,t)$, $\Gamma(z,t)$, $\Psi(z,t)$, $\phi_1(z,t)$, $\phi_2(z,t)$, $\phi_3(z,t)$, $\Omega_3(z,t)$; k_x, k_y are the wave numbers along the x- and y-directions, respectively and $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number.

Following the normal mode analysis, the linearized perturbation dimensionless equations are

$$\begin{aligned}\{\frac{\partial}{\partial t^*} - (1+N_1)(D^2 - a^2)\}(D^2 - a^2)W = & a\sqrt{R}[(M_1 - M_4)D\phi_1^* - (1+M_1 - M_4)T^*] \\ & + a\sqrt{S_1}[(M_1' - M_4')D\phi_2^* + (1 - M_1' + M_4')C^{1*}] + a\sqrt{S_2}[(M_1'' - M_4'')D\phi_3^* \\ & + (1 - M_1'' + M_4'')C^{2*}] + 2N_1(D^2 - a^2)\Omega_3^* - \sqrt{T_A}DZ^*\end{aligned} \tag{34}$$

$$\{\frac{\partial}{\partial t^*} - (1+N_1)(D^2 - a^2)\}Z^* = \sqrt{T_A}DW^* \tag{35}$$

$$I'\frac{\partial\Omega_3^*}{\partial t^*} = -2N_1\{(D^2 - a^2)W^* + 2\Omega_3^*\} + N_3(D^2 - a^2)\Omega_3^*, \tag{36}$$

$P_r\frac{\partial T^*}{\partial t^*}$ - $P_r M_2 \frac{\partial}{\partial t^*}(D\phi_1^*) = (D^2 - a^2)T^*$ +a $\sqrt{R}$ (1- M_2) W^* - a $\sqrt{R}$ $N_5 \Omega_3^*$, (37)

$Ps_1 \frac{\partial C^{1^*}}{\partial t^*}$ - Ps_1 $M_2' \frac{\partial}{\partial t^*}(D\phi_2^*) = (D^2 - a^2) C^{1^*}$ +a $\sqrt{S_1}$ (1- M'_2) W^* , (38)

$Ps_2 \frac{\partial C^{2^*}}{\partial t^*}$ - $Ps_2 M_2'' \frac{\partial}{\partial t^*}(D\phi_3^*) = (D^2 - a^2) C^{2^*}$ +a $\sqrt{S_2}$ (1- M_2'') W^* , (39)

$D^2 \phi_1^*$ - $a^2 M_3 \phi_1^*$ - D T^* = 0, (40)

$D^2 \phi_2^*$ - $a^2 M_3 \phi_2^*$ - D C^{1^*} = 0, (41)

$D^2 \phi_3^*$ - $a^2 M_3 \phi_3^*$ - D C^{2^*} = 0, (42)

Where the following non dimension quantities and non dimensionless parameters are introduced:

$t' = \frac{vt}{d^2}$, $W^* = \frac{Wd}{v}$, $\phi_1^* = \frac{(1+\chi)K_1 a\sqrt{R}}{K_2\,\rho C_1 \beta v d^2}\phi_1$, $\phi_2^* = \frac{(1+\chi)K_1' a\sqrt{S_1}}{K_3\,\rho C_1'\beta' v d^2}\phi_2$, $\phi_3^* = \frac{(1+\chi)K_1'' a\sqrt{S_2}}{K_4\,\rho C_1''\beta'' v d^2}\phi_3$,
$R_1 = \frac{g\alpha\,\beta\, d^4\,\rho C_1}{v K_1}$, $S_1 = = \frac{g\alpha'\,\beta'\,d^4\,\rho C_1'}{v K_1'}$, $S_2 = \frac{g\alpha''\,\beta''\,d^4\,\rho C_1''}{v K_1''}$, $T^* = \frac{K_1 a\sqrt{R}}{\rho C_1 \beta v d}\Theta$, $C^{1^*} = \frac{K_1' a\sqrt{S_1}}{\rho C_1'\beta' v d}\Gamma$,
$C^{2^*} = \frac{K_1'' a\sqrt{S_2}}{\rho C_1''\beta'' v d}\Psi$, a = k d, $z' = \frac{z}{d}$, $D = \frac{\partial}{\partial Z^*}$, $P_r = \frac{v}{K_1}\rho C_1$, $P_{s_1} = \frac{v}{K_1'}\rho C_1'$,
$P_{s_2} = \frac{v}{K_1''}\rho C_1''$, $T_A = (\frac{2\Omega d^2}{v})^2$, $M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha\rho_0 g}$, $M_1' = \frac{\mu_0 K_3^2 \beta'}{(1+\chi)\alpha'\rho_0 g}$, $M_1'' = \frac{\mu_0 K_4^2 \beta''}{(1+\chi)\alpha''\rho_0 g}$,
$M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_1}$ $M_2' = \frac{\mu_0 C_0^1 K_3^2}{(1+\chi)\rho C_1'}$, $M_2'' = \frac{\mu_0 C_0^2 K_4^2}{(1+\chi)\rho C_1''}$, $M_3 = \frac{(1+\frac{M_0}{H_0})}{(1+\chi)}$, $M_4 = \frac{\mu_0 K_2 K_3 \beta'}{(1+\chi)\alpha\rho_0 g}$,
$M_4' = \frac{\mu_0 K_3 K_4 \beta''}{(1+\chi)\alpha'\rho_0 g}$, $M_4'' = \frac{\mu_0 K_2 K_4 \beta}{(1+\chi)\alpha''\rho_0 g}$, $M_5 = \frac{M_4}{M_1} = \frac{M_1'}{M_4'} = \frac{K_3\beta'}{K_2\beta} = \frac{M_1''}{M_4''} = \frac{K_4\beta''}{K_2\beta}$, $N_1 = \frac{\zeta}{\eta}$,
$N_3 = \frac{\eta'}{\eta d^2}$, $N_5 = \frac{\delta}{\rho C_1 d^2}$, $I' = \frac{I}{d^2}$, and $\Omega_3^* = \frac{\Omega_3 d^3}{v}$.

4. Exact solution for free boundaries

Here the simplest boundary conditions chosen, namely free-free, no- spin, isothermal with infinite magnetic susceptibility χ in the perturbed field keep the problem analytically tractable and serve the purpose of providing a qualitative insight in to the problem. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus the exact solution of Eqs. (34)- (42) subject to the boundary conditions are

$W^* = D2W = T^* = C^{1^*} = C^{2^*} = \Omega 3^* = D\phi 1^* = D\phi 2^* = D\phi 3^* = 0$ at $z = \pm\frac{1}{2}$, (43)

is written in the form

$W^* = A_1 e^{\sigma t^*}\cos\pi z^*$, $T^* = B_1 e^{\sigma t^*}\cos\pi z^*$, $D\,\phi_1^* = C_1 e^{\sigma t^*}\cos\pi z^*$, $\Omega_3^* = D_1 e^{\sigma t^*}\cos\pi z^*$,
$D\,\phi_2^* = E_1 e^{\sigma t^*}\cos\pi z^*$, $\phi_1^* = (\frac{C_1}{\pi})\, e^{\sigma t^*}\sin\pi z^*$, $\phi_2^* = (\frac{E_1}{\pi})\, e^{\sigma t^*}\sin\pi z^*$, $C^{1^*} = = F_1 e^{\sigma t^*}\cos\pi z^*$
$C^{2^*} = = G_1 e^{\sigma t^*}\cos\pi z^*$, $D\phi_3^* = H_1 e^{\sigma t^*}\cos\pi z^*$, $\phi_3^* = (\frac{H_1}{\pi})\, e^{\sigma t^*}\sin\pi z^*$ (44)

Where A_1, B_1, C_1, D_1, E_1, F_1, G_1, and H_1 are constants and σ is the growth rate, in, general, a complex constant. Substituting eq. (49) in equations. (40)- (47) and dropping asterisks for convenience, we get following equations:

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$$\{\sigma + (1+N_1)(\pi^2+a^2)\}(\pi^2+a^2)A_1 + a\sqrt{R}\,[(M_1 - M_4)C_1 - (1+M_1-M_4)B_1] + a\sqrt{S_1}\,[(M_1' - M_4')E_1 + (1-M_1'+M_4')F_1] + a\sqrt{S_2}\,[(M_1''-M_4'')H_1 + (1-M_1''+M_4'')G_1] - 2N_1(\pi^2+a^2)D_1 + \frac{T_A}{\{\sigma + (1+N_1)(\pi^2+a^2)\}}\pi^2 A_1 = 0, \tag{45}$$

$$-2N_1(\pi^2+a^2)A_1 + I'\sigma + 4N_1 + N_3(\pi^2+a^2)\}D_1 = 0, \tag{46}$$

$$(1-M_2)\,a\sqrt{R}\,A_1 - (\pi^2+a^2+P_r\sigma)B_1 + (P_r M_2\sigma)C_1 - a\sqrt{R}\,N_5 D_1 = 0, \tag{47}$$

$$(1-M_2')\,a\sqrt{S_1}\,A_1 - (\pi^2+a^2+PS_1\sigma)F_1 + (PS_1 M_2'\sigma)E_1 = 0, \tag{48}$$

$$(1-M_2'')\,a\sqrt{S_2}\,A_1 - (\pi^2+a^2+PS_2\sigma)G_1 + (PS_2 M_2''\sigma)H_1 = 0, \tag{49}$$

$$-\pi\,B_1 + (\pi^2+a^2M_3)C_1 = 0, \tag{50}$$

$$-\pi^2 F_1 + (\pi^2+a^2M_3)E_1 = 0, \tag{51}$$

$$-\pi^2 G_1 + (\pi^2+a^2M_3)H_1 = 0. \tag{52}$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1, D_1, E_1, F_1, G_1, and H_1 in equations (45) – (52) must vanish. This determinant on simplification yields

$$iT_5\,\sigma_i^5 + T_4\,\sigma_i^4 - iT_3\,\sigma_i^3 - T_2\,\sigma_i^2 + iT_1\,\sigma_i + T_0 = 0. \tag{53}$$

Here

$$T_5 = b\,L_4'\,L_3'\,L_2'\,I_1,$$

$$T_4 = b\,[\,bL_4'\,L_1(L_3' + L_2')\,I_1 + \{(1+N_1)b\,I_1 + 4N_1 + N_3'b\}\,L_4'\,L_3'\,L_2' + L_3'\,L_2' I_1\,],$$

$$\begin{aligned}T_3 = {} & b^3 L_1\,[\,L_4'\,(L_3' + L_2')(1+N_1) + (L_4' L_1 + L_2')\,]\,I_1 + b^2\,[\,L_4'\,L_1(4N_1 + N_3'b)(L_3' + L_2') \\ & + L_3'\,\{(1+N_1)\,L_2' + L_1\}\,I_1] + b\,[L_3'\,L_2'\big(4N_1 + N_3'\,b\big)\,] + L_4'\,L_3'\,L_2' \\ & \{(1+N_1)(4N_1 + N_3'b)\big(b^3 + \tfrac{T_{A_1}}{(1+N_1)^2}\big) - 4N_1^2 b^3\} + [-L_4'\,L_3'\,L_2\,(1-M_2)(xR_1) \\ & + L_2'\,(1-M_2')(x\,S_1) + \{L_3' + L_1'(L_1 - 1\}\,L_4' L_5\,(x\,S_1)(x\,S_2)]]I_1,\end{aligned}$$

$$\begin{aligned}T_2 = {} & b^4\,[(L_4'\,L_1^2 + L_2'\,L_1)(1+N_1)\,]I_1 + b^3\,[\,(L_4'\,L_1 + L_2')L_1(4N_1 + N_3'b) + L_3'\,L_1\,(1+N_1)\,I_1] \\ & + b^2\,[\,L_3'\,L_1\,(4N_1 + N_3'\,b)\,] + b[\{\,L_2'\,L_3(1-M_2')(x\,S_1) - (1-M_2)(L_4'\,L_1 + L_3')L_2(xR_1)\}\,I_1 \\ & + \{L_5\,L_4\,L_1^2 - L_4 L_1'\,(1-M_2'')(L_1 - 1)\}\,(xS_1)(x\,S_2)\,I_1 + L_4'\,L_3'\,L_2\,(2N_1N_5')\,(xR_1)\,] \\ & + [\,L_4'L_1(L_2' + L_3') + L_3'\,L_2']\{(1+N_1)\,(4N_1 + N_3'\,b)\big(b^3 + \tfrac{T_{A_1}}{(1+N_1)^2}\big) - 4N_1^2 b^3\} + [\{L_1\,L_4\,L_3' \\ & + (L_5\,L_4\,L_1'(L_1 - 1)\}(4N_1 + N_3'\,b)(xS_1)(x\,S_2) - L_4'\,L_3'\,L_2\,(1 - M_2)\big(4N_1 + N_3'\,b\big)(xR_1) \\ & + (1-M_2')\,L_4'\,L_3\{L_2'(4N_1 + N_3'\,b) + I_1\}\,(xS_1)\,.\end{aligned}$$

$$\begin{aligned}T_1 = {} & b^4\,[L_1^2\,(4N_1 + N_3'\,b)] \\ & + b^2[\{(1-M_2')I_1\,L_1L_3 - (1-M_2'')I_1\,L_1L_4(L_1 - 1)\}\,(xS_1) + \{2N_1N_5'\,(L_4'\,L_1 + L_1'\,L_3) - L_1L_2(1-M_2)I_1\,\}(xR_1)] \\ & + b\,[\,(\,L_4'L_1^2 + L_2'\,L_1 + L_3'\,L_1\,)\,\{\,\{(1+N_1)\,(4N_1 + N_3'\,b)\big(b^3 + \tfrac{T_{A_1}}{(1+N_1)^2}\big) - 4N_1^2 b^3\,\} \\ & + \{L_5L_4L_1^2 - (1-M_2'')\,L_4L_1'(L_1 - 1)\,\}(4N_1 + N_3'\,b)(xS_1)(x\,S_2) + (\,L_2'\,L_3 + L_4'\,L_1L_3) \\ & \{\,(1-M_2')\,(4N_1 + N_3'\,b)(xS_1)\} - (1-M_2)\,L_2\,L_1\,(L_4' + L_3')\,\big(4N_1 + N_3'\,b\big)(xR_1)].\end{aligned}$$

$$\begin{aligned}T_0 = {} & b^3[L_1(2N_1N_5')(xR_1) + b^2\{\,[(1+N_1)\,(4N_1 + N_3'\,b)\big(b^3 + \tfrac{T_{A_1}}{(1+N_1)^2}\big) - 4N_1^2 b^3] + [(1-M_2')\,L_1L_3\,(xS_1) \\ & - (1-M_2'')\,L_1L_4(L_1 - 1)\,)(xS_1)(x\,S_2)\}\big(4N_1 + N_3'\,b\big) - (1-M_2)\,L_1L_2\,\big(4N_1 + N_3'\,b\big)\,(xR_1)]\}\,.\end{aligned} \tag{54}$$

where

$R_1 = \frac{R}{\pi^4}$, $T_{A_1} = \frac{T_A}{\pi^4}$ $S_1 = \frac{S_1}{\pi^4}$, $S_2 = \frac{S_2}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $I_1 = \pi^2 I'$ $i\sigma_i = \frac{\sigma}{\pi^2}$, $N_3' = \pi^2 N_3$, $N_5' = \pi^2 N_5$, $b = 1+x$,
$L_1 = (1+xM_3)$, $L_1' = Ps_1\,(1+x\,M_3)$, $L_2 = [1+xM_3 + xM_3\,M_1\,(1-M_5)]$, $L_2' = P_r\,[(1-M_2) + xM_3\,]$,
$L_3 = [1+x\,M_3' + x\,M_3\,M_1'(\frac{1}{M_5'} - 1)]$, $L_3' = Ps_1(\,1 - M_2' + x\,M_3\,)$, $L_4 = [1+x\,M_3 + x\,M_3\,M_1''(\frac{1}{M_5'} - 1)]$,
$L_4' = Ps_2(1 - M_2'' + x\,M_3)$, $L_5 = P_r\,(1-M_2'')$.

5. Results and discussion

5.1 The case of stationary convection

When the instability sets in as stationary convection in the case $M_2 \cong 0$, $M_2' \cong 0$, the marginal state will be characterized by $\sigma_i = 0$ [14], then the Rayleigh number R_1 is given by

$$R_1 = \frac{(1+xM_3)\left\{(1+N_1)(4N_1+N_3' b)\left[b^3+\frac{T_{A_1}}{(1+N_1)^2}\right]-4N_1^2 b^3\right\}+\left[\left\{1+xM_3+x M_1' M_3\left(\frac{1}{M_5}-1\right)\right\}(xS_1)+\left\{1+xM_3+x M_1'' M_3\left(\frac{1}{M_5}-1\right)\right\}(xS_2)\right](4N_1+N_3' b)}{X\{1+xM_3+x M_1 M_3(1-M_5)\}\{4N_1+b(N_3'-2N_1N_5')\}} \quad (55)$$

which expresses the modified Rayleigh number R_1 as a function of dimension less wave number x, buoyancy magnetization parameter M_1, the non-buoyancy magnetization parameter M_3 , solute gradient parameters S_1 and S_2, ratio of the salinity effect on magnetic field to pyromagnetic coefficient M_5, coupling parameter N_1(coupling between vorticity and spin effects), spin diffusion parameter $N_3'$and micropolar heat conduction parameter N_5'(coupling between spin and heat fluxes). The parameters N_1 and N_3' measure the micropolar viscous effect and micropolar diffusion effect, respectively.

The classical results in respect of Newtonian fluids can be obtained as the limiting case of present study. Setting $N_1 = 0$ and $S_1 = 0$, and keeping N_3' arbitrary in equation (55), we get

$$R_1 = \frac{(1+xM_3)\{(1+x)^3+T_{A_1}\}}{x\{1+xM_3(1+M_1)\}} \quad (56)$$

which is the expression for the Rayleigh number of ferromagnetic fluids (Finlayson [15]).
Setting $M_3 = 0$ in equation (56), we get

$$R_1 = \frac{\{(1+x)^3+T_{A_1}\}}{x}, \quad (57)$$

the classical Rayleigh Bénard result [14], for the Newtonian fluid case.
Before we investigate the effects of various parameters, we first make some comments on the parameters N_1, N_3' and N_5' arising due to suspended particles. Assuming the Clausius- Duhem inequality, Eringen [31] presented certain thermodynamic restrictions which lead to non-negativeness of N_1, N_3' and N_5'. It is obvious that couple stress comes into play at small values of N_3'. This supports the condition that $0 \leq N_1 \leq 1$ and that $N_3'$is small positive real number. The parameter N_5' has to finite because the increasing of concentration has to be practically stop somewhere and hence it has to positive, finite real number. The range of the values for the other parameters is as in classical ferroconvection problem involving Newtonian ferromagnetic fluid [28-30]. $M_1'$and M_1'' is the effect of magnetization due to salinity. This is allowed to vary 0.1 to 0.5 taking values less than the magnetization parameter M_3. M_5 represents the ratio of the salinity effects on the magnetic field to pyromagnetic coefficient. This is varied between 0.1 to 0.5. The salinity Rayleigh numbers S_1 and S_2 varied from 0 to 500.

To investigate the effect of solute gradients, non-buoyancy magnetization coefficient, coupling parameter, spin parameter, and micropolar heat conduction parameter, we examine the behavior of $\frac{dR_1}{dT_A}, \frac{dR_1}{dS_1}, \frac{dR_1}{dS_2}, \frac{dR_1}{dM_3}, \frac{dR_1}{dN_1}, \frac{dR_1}{dN_3'}$ *and* $\frac{dR_1}{dN_5'}$ analytically. Equation (55) gives

$$\frac{dR_1}{dT_A} = \frac{L_1(4N_1+N_3' b)}{x(1+N_1)L_2\{4N_1+b(N_3'-2N_1N_5')\}}$$

which is positive if

$$N_3' > 2N_1N_5', \tag{58}$$

which shows that the rotation has a stabilizing effect when condition (58) holds. In the absence of micropolar viscous effect ($N_1 = 0$), the rotation always has a stabilizing effect on the system.

$$\frac{dR_1}{dS_1} = \frac{\left[1+\mathrm{x}M_3+\mathrm{x}\,M_1'M_3\left(\frac{1}{M_5}-1\right)\right](4N_1+N_3'\mathrm{b})}{L_2\{4N_1+\mathrm{b}(N_3'-2N_1N_5')\}}, \tag{59}$$

$$\frac{dR_1}{dS_2} = \frac{\left[1+\mathrm{x}M_3+\mathrm{x}\,M_1''M_3\left(\frac{1}{M_5}-1\right)\right](4N_1+N_3'\mathrm{b})}{L_2\{4N_1+\mathrm{b}(N_3'-2N_1N_5')\}}, \tag{60}$$

This shows that,for a stationary convection,the stable solute gradients have stabilizing effect, if

$$N_3' > 2N_1N_5'. \tag{61}$$

In the absence of micropolar viscous effect (coupling parameter N_1), stable solute gradients always have stabilizing effect, on the system. Equation (55) also yields

$$\frac{dR_1}{dM_3} = -\frac{(1-M_5)b\left[b^2M_1\{(1+N_1)N_3'\mathrm{b}+4N_1\}+\frac{(N_3'\mathrm{b}+4N_1)T_{A_1}}{b(1+N_1)}\right]+\left[(\mathrm{x}S_1)\left(M_1-\frac{M_1'(1+\mathrm{x}M_3)}{M_5}\right)+(\mathrm{x}S_2)\left(M_1-\frac{M_1''(1+\mathrm{x}M_3)}{M_5}\right)\right](4N_1+N_3'\mathrm{b})}{\{1+\mathrm{x}M_3+\mathrm{x}M_1M_3(1-M_5)\}^2\{4N_1+\mathrm{b}(N_3'-2N_1N_5')\}}, \tag{62}$$

Which is negative, if

$$N_3' > 2N_1N_5',\ M_4 > M_1'(1+\mathrm{x}M_3) \text{ and } M_4 > M_1''(1+\mathrm{x}M_3). \tag{63}$$

This shows that the non-buoyancy magnetization has a destabilizing effect when conditions (63) hold. In the absence of micropolar viscous effect ($N_1 = 0$) and the effect on magnetization due to salinity ($M_1' = 0$ and $M_1'' = 0$), the non-buoyancy magnetization always has a destabilizing effect on the system.

It follows from equation (55) that

$$\frac{dR_1}{dN_1} = \frac{\begin{gathered} bL_1\left[b^4N_3'\left\{2N_5'+\frac{N_3'}{(1+N_1)^2}\left[\mathrm{b}(1+N_1)^2-\frac{T_{A_1}}{\mathrm{b}^2}\right]\right\}+\frac{T_{A_1}}{b(1+N_1)2}\left\{4\mathrm{b}N_1N_3'+2(\mathrm{b}N_5'-2)\{4N_1^2+\mathrm{b}N_3'+(2N_1+1)\}\right\}\right] \\ +(2b^2N_3'N_5')\left[(\mathrm{x}S_1)\left\{1+\mathrm{x}M_3+\mathrm{x}\,M_1'M_3\left(\frac{1}{M_5}-1\right)\right\}+(\mathrm{x}S_2)\left\{1+\mathrm{x}M_3+\mathrm{x}\,M_1''M_3\left(\frac{1}{M_5}-1\right)\right\}\right]\end{gathered}}{\mathrm{x}L_2\{4N_1+\mathrm{b}(N_3'-2N_1N_5')\}^2}, \tag{64}$$

which is positive if

$$T_{A_1} < 1 \text{ and } N_5' > 2. \tag{65}$$

This shows that coupling parameter always has a stabilizing effect when condition (65) hold. In the absence of rotation ($T_{A_1} = 0$), (65) yield that $\frac{dR_1}{dN_1}$ is always positive, implying thereby the stabilizing effect of coupling parameter. Thus , the stabilizing behavior of coupling parameter is virtually unaffected by magnetization parameters but it is significantly affected by micropolar heat conduction N_5' and by Taylor T_{A_1}.

Equation (55) gives number

$$\frac{dR_1}{dN_3'} = -\frac{2b^4L_1N_1\left[N_5'\mathrm{b}+N_1(\mathrm{b}N_5'-2)+\frac{T_{A_1}N_5'}{\mathrm{b}^2(1+N_1)}\right]}{\mathrm{x}L_2\{4N_1+\mathrm{b}(N_3'-2N_1N_5')\}^2}, \tag{66}$$

which is negative if $N_5' > 2$.

This shows that the spin diffusion has a stabilizing effect when condition (66) holds.

Equation (55) also gives

$$\frac{dR_1}{dN_5'} = \frac{2b^2L_1N_1\left\{b^2[(1+N_1)N_3'\mathrm{b}+4N_1]+\frac{(N_3'\mathrm{b}+4N_1)T_{A_1}}{b(1+N_1)}\right\}+2b(4N_1+bN_3')\left[(\mathrm{x}S_1)\left\{1+\mathrm{x}M_3+\mathrm{x}\,M_1'M_3\left(\frac{1}{M_5}-1\right)\right\}+(\mathrm{x}S_2)\left\{1+\mathrm{x}M_3+\mathrm{x}\,M_1''M_3\left(\frac{1}{M_5}-1\right)\right\}\right]}{\mathrm{x}L_2\{4N_1+\mathrm{b}(N_3'-2N_1N_5'\}^2}, \tag{67}$$

which is always positive.

This shows that the micropoalr heat conduction always has a stabilizing effect.
For sufficiently large values of M_1 [15], we obtain the results for the magnetic mechanism

$R_m = R_1 M_1$

$$= \frac{b^2\,(1+xM_3)\left\{(1+N_1)(4N_1+N_3'\,b)\{b+\frac{T_{A_1}}{b^2(1+N_1)^2}\}-4N_1^2\right\}+(4N_1+bN_3')\left[(xS_1)\left\{1+xM_3+x\,M_1'M_3\left(\frac{1}{M_5}-1\right)\right\}+(xS_2)\left\{1+xM_3+x\,M_1''M_3\left(\frac{1}{M_5}-1\right)\right\}\right]}{x^2\,M_3\,(1-M_5)\{4N_1+b(N_3'-2N_1N_5')\}}, \quad (68)$$

where R_m is the magnetic thermal Rayleigh number.
As a function of x, R_m given by equation (70) attains its maximum when

$$P_6x^6+P_5x^5+P_4x^4+P_3x^3+P_2x^2+P_1x+P_0=0. \quad (69)$$

The coefficients P_0, P_1, P_2, P_3, P_4, P_5, P_6 being quite lengthy , have not been written here and are evaluated numerical calculation.
The values of critical wave number for the onset of instability are determined numerically using Newtonian Raphson method by the condition $\frac{dR_m}{dx} = 0$. With x_1 determined as a solution of equation (69), equation (68) will give the required critical magnetic thermal Rayleigh number N_c which depend upon M_3, S_1, S_2 and micropolar parameters N_1, N'_3 and N_5' .

5.2 Principle of exchange of stabilities

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of micropolar parameters and solute gradients. Equating the imaginary parts of equation (53), we obtain

$$\begin{aligned}
&\sigma_i\,[\{b\,L_4'\,L_3'\,L_2'\,I_1\}\,\sigma_i^4-\{b^4\,[(L_4'\,L_1^2+L_2'\,L_1)(1+N_1)I_1+b^3\,[\,(L_4'\,L_1+L_2')L_1(4N_1+N_3'b)+L_3'\,L_1\,(1+N_1)\,I_1]\\
&\quad+b^2\,[\,L_3'\,L_1\,(4N_1+N_3'\,b)\,]+b[\{\,L_2'\,L_3(1-M_2')(x\,S_1)-(1-M_2)(L_4'\,L_1+L_3')L_2(xR_1)\}\,I_1\\
&\quad+\{L_5\,L_4\,L_1^2-L_4L_1'\,(1-M_2'')(L_1-1)\}\,(xS_1)(x\,S_2)\,I_1+L_4'\,L_3'\,L_2\,(2N_1N_5')\,(xR_1)\,]\\
&\quad+[\,L_4'L_1(L_2'+L_3')+L_3'\,L_2']\{(1+N_1)\,(4N_1+N_3'\,b)(\,b^3+\tfrac{T_A}{(1+N)^2})-4{N_1}^2b^3\,\}+[\{L_1\,L_4\,L_3'\\
&\quad+(L_5\,L_4\,L_1'(L_1-1)\}(4N_1+N_3'\,b)(xS_1)(\,x\,S_2)-L_4'\,L_3'\,L_2\,(1-M_2)\big(4N_1+N_3'\,b\big)(xR_1)\\
&\quad+(1-M_2')\,L_4'\,L_3\{L_2'(4N_1+N_3'\,b)+I_1\}\,(xS_1)\}\,\sigma_i^2+\{\,b^4\,[L_1^2\,(4N_1+N_3'\,b)]\\
&\quad+b^2[\{(1-M_2')I_1\,L_1L_3-(1-M_2'')I_1\,L_1L_4(L_1-1)\}\,(xS_1)+\{2N_1N_5'\,(L_4'\,L_1+L_1'\,L_3)-L_1L_2(1-M_2)I_1\,\}(xR_1)]\\
&\quad+b\,[\,(\,L_4'L_1^2+L_2'\,L_1+L_3'\,L_1\,)\,\{\,\{(1+N_1)\,(4N_1+N_3'\,b)(\,b^3+\tfrac{T_A}{(1+N)^2})-4{N_1}^2b^3\,\}\\
&\quad+\{L_5L_4L_1^2--(1-M_2'')\,L_4L_1'(L_1-1)\,\}(4N_1+N_3'\,b)(xS_1)(\,x\,S_2)+(\,L_2'\,L_3+L_4'\,L_1L_3)\\
&\quad\{\,(1-M_2')\,(4N_1+N_3'\,b)(xS_1)\}-(1-M_2)\,L_2\,L_1\,(L_4'+L_3')\,\big(4N_1+N_3'\,b\big)(xR_1)\}=0,
\end{aligned} \quad (70)$$

It is evident from equation (70) that σ_i may be either zero or non-zero, meaning that the modes may be either oscillatory or oscillatory. In the absence of micropolar viscous effect (N_1=0), microinertia (I_1= 0) and solute gradients (S_1=0, S_2 =0 , $L_3' = 0$, and $L_4' = 0$), we obtain the result as

$$\sigma_i[L_1+L_2') = 0. \quad (71)$$

Here the quantity inside the bracket is positive definite because the typical values of M_2 are $+10^{-6}$ [15].Hence

$$\sigma_i = 0, \quad (72)$$

which implies that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for micropolar ferromagnetic fluid heated from below, in the absence of micropolar viscous effect, microinertia and solute gradients. Thus from equation (71), we conclude that the oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia and solute gradient, which are non-existent in their absence. Thus, it is important to note that the Taylor number T_{A_1} , gives significant contribution in developing the oscillatory modes in the stability analysis.

5.2 The case of overstability

The present section is devoted to find the possibility that the observed instability may really be overstability. Since we wish to determine the Rayleigh number for the onset of instability through state of pure oscillations, it is suffices to find conditions for which (53) will admit of solutions with σ_i real.

Equating real and imaginary parts of (53) and eliminating R_1 between them, we obtain

$$A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{73}$$

Where, $c_1 = \sigma_i^2$, Since σ_i is real for overstability, the three values of $c_1 (= \sigma_i^2)$ are positive. The product of roots of equation (73) is $-\frac{A_0}{A_3}$, where

$$A_3 = -\,b\, L_4'^2 L_3'^2 L_2' I_1 \{ [L_1 + (1+N_1) L_2'] (1-M_2) + (2N_1 N_3') L_2' \}, \tag{74}$$

$$\begin{aligned} A_0 = {} & \{ b^3 L_1 (2N_1 N_5') - b^2 (1-M_2) L_1 L_2 \left(4N_1 + N_3' b\right) \}\{ b^4 L_1^2 \left(4N_1 + N_3' b\right) + b^2 [\{(1-M_2') I_1 L_1 L_3 - (1-M_2'') I_1 \\ & L_1 L_4 (L_1 - 1)\} (xS_1)] + b\Big\{ [(L_4' L_1^2 + L_2' L_1 + L_3') \{(1+N_1)(4N_1 + N_3' b)(b^3 + \tfrac{T_A}{(1+N)^2}) - 4N_1^2 b^3 \}] \\ & \{L_5 L_4 L_1^2 - -(1-M_2'') L_4 L_1' (L_1 - 1) \}(4N_1 + N_3' b)(xS_1)(xS_2) + (L_2' L_3 + L_4' L_1 L_3) \\ & \{(1-M_2')(4N_1 + N_3' b)(xS_1)\}]\Big\} - b^2 \Big\{ [L_1^2 \{(1+N_1)(4N_1 + N_3' b)(b^3 + \tfrac{T_A}{(1+N)^2}) - 4N_1^2 b^3 \}] \\ & + [\{(1-M_2') L_1 L_3 (xS_1) - (1-M_2'') L_1 L_4 (L_1 - 1))(xS_1)(xS_2)\}\left(4N_1 + N_3' b\right)]\Big\} \\ & \Big\{ b^2 [2N_1 N_5' (L_4' L_1 + L_1' L_3) - L_1 L_2 (1-M_2) I_1](xR_1) - b(1-M_2) L_2 L_1 (L_4' + L_3') \left(4N_1 + N_3' b\right)(xR_1)]\Big\}. \end{aligned} \tag{75}$$

The coefficients A_2 and A_1 being quite lengthy and not needed in the discussion of overstability , has not been written here.

Since σ_i is real for overstability, the three values of $c_1 (= \sigma_i^2)$ are positive. The product of roots of equation (73) is $-\frac{A_0}{A_3}$, and if this is to be negative, then A_3 and A_0 are of the same sign. Now, the product is negative if

$N_3' (1-M_2) > 4N_1 N_5'$, $N_1 (1-M_2) > T_{A_1} I_1 N_5'$, $N_3' L_2' > I_1 L_1$, $L_1 > N_1 L_2'$, $L_2' > L_1$ and $L_2' > L_3' \left(1 + \frac{N_3' L_2'}{I_1 L_1}\right)$, $L_2' > L_4' \left(1 + \frac{N_3' L_2'}{I_1 L_1}\right)$,

i.e. if $N_3' > \frac{4N_1 N_5'}{(1-M_2)}$, $T_{A_1} < \frac{N_1 (1-M_2)}{I_1 N_5'}$, $N_3' > \frac{I_1 (1+xM_3)}{P_r (1-M_2)}$, $\frac{1}{P_r} > N_1$, $P_r > \frac{(1+xM_3)}{(1-M_2)}$, $P_r > P_{S_1}\left(1 + \frac{\eta' P_r}{\eta I}\right)$ and $P_r > PS_1 \left(1 + \frac{\eta' P_r}{\eta I}\right) + P_r M_2$, $P_r > P_{S_2}\left(1 + \frac{\eta' P_r}{\eta I}\right)$ and $P_r > P_{S_2}\left(1 + \frac{\eta' P_r}{\eta I}\right) + P_r M_2$,

which implies that

$$N_3' > \max \left\{ \frac{4N_1 N_5'}{(1-M_2)}, \frac{I_1 (1+xM_3)}{P_r (1-M_2)} \right\}, \frac{1}{N_1} > P_r > \frac{(1+xM_3)}{(1-M_2)}, \quad K_1 < K_1' \left[\frac{\rho C_1 - \frac{\mu_0 T_0 K_2^2}{(1+\chi)}}{\rho C_1' \left(1 + \frac{\eta' P_r}{\eta I}\right)} \right] \text{ and } K_1 < K_1'' \left[\frac{\rho C_1 - \frac{\mu_0 T_0 K_2^2}{(1+\chi)}}{\rho C_1'' \left(1 + \frac{\eta' P_r}{\eta I}\right)} \right], \; T_{A_1} < \frac{N_1 (1-M_2)}{I_1 N_5'},$$

However $P_r > P_{S_1} \left(1 + \frac{\eta' P_r}{\eta I}\right)$ and $P_r > P_{S_2} \left(1 + \frac{\eta' P_r}{\eta I}\right)$ are already satisfied in above condition because the typical values of M_2 are $+\,10^{-6}$ [15].

Thus, for $N_3' > \max\{\frac{4N_1N_5'}{(1-M_2)}, \frac{I_1(1+xM_3)}{P_r(1-M_2)}\}$, $\frac{1}{N_1} > P_r > \frac{(1+xM_3)}{(1-M_2)}$, $T_{A_1} < \frac{N_1(1-M_2)}{I_1N_5'}$,

$K_1 < K_1' [\frac{\rho C_1 - \frac{\mu_0 T_0 K_2^2}{(1+\chi)}}{\rho C_1'(1+\frac{\eta' P_r}{\eta I})}]$ and $K_1 < K_1'' [\frac{\rho C_1 - \frac{\mu_0 T_0 K_2^2}{(1+\chi)}}{\rho C_1''(1+\frac{\eta' P_r}{\eta I})}]$,

overstability cannot occur and the principle of the exchange of stabilities is valid. Hence the above conditions are the sufficient conditions for the non existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. Rotation contributes two more conditions i.e.

$P_r > \frac{(1+xM_3)}{(1-M_2)}$, $T_{A_1} < \frac{N_1(1-M_2)}{I_1N_5'}$,

for the non-existence of overstability. In rotating non-magnetic fluid and in the absence of microrotation, above condition reduces to $P_r > 1$, which is in good agreement with the result obtained earlier [14].

5. Conclusions

In this paper, the effect of rotation on triple –diffusive convection in a micropolar ferrofluid layer heated and soluted from below subjected to a transverse uniform magnetic field has been investigated. The behavior of various parameters like rotation parameter,solute gradients, non-buoyancy magnetization, coupling parameter, spin diffusive parameter and micropolar heat conduction on the onset of convection has been analyzed analytically and numerically. The results show that for the state of stationary convection, the non-buoyancy magnetization, spin diffusive parameter have destabilizing effect under certain condition(s), whereas the rotation, coupling parameter and solute gradients have a stabilizing effect under certain condition(s). However, the micropolar heat conduction always has a stabilizing effect. The principle of exchange of stabilities is found to hold true for the micropolar ferrofluid heated from below in the absence of micropolar viscous effect, microinertia, rotation and solute gradient. Thus oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia, rotation and solute gradients, which were non-existent in their absence. In addition the presence of rotation, solute gradients, coupling between vorticity and spin effect (micropolar viscous effect) and microinertia may bring overstability in the system. Finally, we conclude that the rotation and micropolar parameters have a profound influence on triple-diffusive convection in a micropolar ferrofluid layer heated and soluted from below.The micropolar rotating ferrofluid stabilities do deserve a fresh look as related microgravity environment applications.

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