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Traveling Wave Solutions of the Extended Calogero-Bogoyavlenskii-Schiff Equation

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***Abstract*— Traveling wave solutions of the extended Calogero–Bogoyavlenskii–Schiff equation (ECBS) is investigated. The (G'/G)-expansion method is applied to find the explicit solutions. New traveling wave solutions of the ECBS equation are obtained. The solitary wave solutions show one-soliton, N-solitons, kink and periodic waves. Some of the resulting solutions are plotted.**

***Keywords*— Extended Calogero-Bogoyavlenskii-Schiff equation; (G'/G)-expansion method; Traveling wave solutions.**

I. INTRODUCTION

Nonlinear integrable equations are playing a substantial role in modeling most of the scientific and engineering applications [1-5], such as propagation of shallow water waves, optical fibers, condensed matter, electromagnetic, plasma and fluid mechanics. The study of higher dimensional nonlinear integrable equations is one of the most important recent studies [6-8]. Large assortments of mathematical methods of solutions are demoralized in studying these equations, some of these methods are the (G'/G)-expansion method [9-11], exponential function method [12, 13], Lax pairs [14, 15], Extended homoclinic test [16, 17], Hirota bilinear method [18, 19], Darboux transformation [20, 21], sine–cosine and tanh–coth methods [22].

Bogoyavlenskii and Schiff described the interaction of a Riemann waves along two spatial dimensions, by the nonlinear integrable equation Calogero–Bogoyavlenskii–Schiff (GBS) equation [23, 24]. Riemann waves' dynamics is one of the most important applications of physics and engineering; such as tsunami and tidal in rivers, magento-sound waves in plasmas, internal waves in oceans and optical tsunami in fibers.

Song et al. [25], introduced an integrable (2 + 1)-dimensional equation. This equation describes the interaction between Riemann wave promulgated along the y-axis and long wave promulgated along the x-axis as;

$$u_{tx} + u_x u_{xy} + \frac{1}{2} u_{xx} u_y + \frac{1}{4} u_{xxxy} + \frac{1}{4} \partial_x^{-1} u_{yyy} = 0 \qquad (1)$$

Where $u(x, y, t)$ is a function of space variables x, y and temporal variable t. Equation (1) is also obtained by unifying two directional generalization of the potential KdV equation and Calogero-Bogoyavlenskii-Shiff equation [26-28]. The explicit N-soliton solutions of equation (1) is obtained in [25]. Wazwaz [18], extended equation (1) by adding the planar flux term βu_{xy}, introducing the ECBS equation;

$$u_{tx} + u_x u_{xy} + \frac{1}{2} u_{xx} u_y + \frac{1}{4} u_{xxxy} + \frac{1}{4} \partial_x^{-1} u_{yyy} + \beta u_{xy} = 0 \quad (2)$$

He in [18], showed that the extension term does not disturb the integrability of the Song's equation (1), and found its N-soliton solutions using the simplified Hirota's method.

The purpose of this paper is to find the traveling wave solutions of the extended Calogero–Bogoyavlenskii–Schiff equation (2), using the (G'/G) expansion method. The paper is organized as follows: Section II, is devoted to summarize the (G'/G) expansion method. In section III, the (G'/G) expansion method is applied to the extended Calogero–Bogoyavlenskii–Schiff equation. Section IV, is devoted to discuss the results. The paper ends with conclusions in Section V.

II. MATHEMATICAL METHOD

(G'/G)-expansion method is excessively used in finding the traveling wave solutions of nonlinear evolution equations [29-32]. It can be summarized as; for any partial differential equation (PDE);

$$P(u, u_x, u_y, u_t, u_{xx}, u_{xy}, \ldots) = 0$$
(3)

Where p is a polynomial in u and its partial derivatives. Suppose the solution of the partial differential equation (2.1) is in the form;

$$u(x, y, t) = u(\eta), \qquad \eta = x + y \pm ct. \qquad (4)$$

The constant c is the velocity of the travelling wave. The PDE (3) is reduced to a nonlinear ODE; which can be integrated many times with setting the constants of integration equal to zero for simplicity. Consider the traveling wave solution of the final ODE in the form;

$$u(\eta) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i \qquad (5)$$

Where $G = G(\eta)$ satisfies the second order linear ODE;

$$G''(\eta) + \lambda\, G'(\eta) + \mu G(\eta) = 0 \qquad (6)$$

Where, $G' = \frac{dG}{d\eta}$, $G'' = \frac{d^2G}{d\eta^2}$, a_i, λ and μ are real constants to be determined.

The positive integer m is determined through balancing the highest order linear and nonlinear terms' derivatives appearing in the ODE. Substitute (5) and (6) into the final ODE, then collect all terms with the same order of (G'/G) and set each coefficient to zero yield a set of algebraic equations for a_i, c, μ and λ.

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III. TRAVELING WAVE SOLUTION OF THE ECBS EQUATION

This section presents, the application of (G'/G)-expansion method to find the explicit solutions of the extended Calogero–Bogoyavlenskii–Schiff equation (2). Differentiate equation (2) with respect to x, yields;

$$4u_{txx}+6u_{xx}u_{xy}+4u_{x}u_{xxy}+2u_{xxx}u_{y}+u_{xxxxy}+u_{yyy}+4\beta u_{xxy}=0 \qquad (7)$$

We now utilize the wave transformation equation (4) in reducing (7) to the nonlinear ODE;

$$(4\beta-4c+1)u'''+6(u'')^2+6\,u'\,u'''+u^{(5)}=0 \qquad (8)$$

Where dashes refer to the derivatives with respect to η. Let

$$(4\beta-4c+1)=\delta \qquad (9)$$

Then integrate (8) twice with respect to η yields;

$$u'''+3u'^2+\delta u'=0 \qquad (10)$$

Let $u'=v$, yields

$$v''+\delta v+3v^2=0 \qquad (11)$$

The balance between v'' and v^2, leads to $m = 2$, and the solution of equation (11) is written as;

$$v(\eta)=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2 \qquad (12)$$

Where $G=G(\eta)$ satisfies equation (6) and a_i, λ and μ, are real constants to be determined. Substitute from (12) using (6) into (11) yields;

$$(6a_2+3a_2^2)\left(\frac{G'}{G}\right)^4+(10a_2\lambda+6a_1a_2+2a_1)\left(\frac{G'}{G}\right)^3+(3a_1\lambda+4a_2\lambda^2+8a_2\mu+\delta a_2+3a_1^2+6a_0a_2)\left(\frac{G'}{G}\right)^2+(a_1\lambda^2+2a_1\mu+6a_2\lambda\mu+\delta a_1+6a_0a_1)\left(\frac{G'}{G}\right)+(a_1\lambda\mu+2a_2\mu^2+\delta a_0+3a_0^2)=0 \qquad (13)$$

Collecting all terms with the same order of (G'/G), and setting each coefficient to zero yielding; a set of algebraic equations for a_i, δ, μ and λ,

$$\begin{cases} 6a_2+3a_2^2=0 \\ 10a_2\lambda+6a_1a_2+2a_1=0 \\ 3a_1\lambda+4a_2\lambda^2+8a_2\mu+\delta a_2+3a_1^2+6a_0a_2=0 \\ a_1\lambda^2+2a_1\mu+6a_2\lambda\mu+\delta a_1+6a_0a_1=0 \\ a_1\lambda\mu+2a_2\mu^2+\delta a_0+3a_0^2=0 \end{cases} \qquad (14)$$

Solutions of this system of equations result in two cases;

A. Case 1; $a_0 = -2\mu$, $a_1 = -2\lambda$, $a_2 = -2$ and $\delta = 4\mu-\lambda^2$

The function $G(\eta)$ is found through the solution of equation (6) by setting, $\lambda^2-4\mu=\alpha$

1) The solution (12) For $\alpha > 0$ is;

$$v=-2\mu-2\lambda\left[\frac{-\lambda}{2}+\frac{\sqrt{\alpha}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}\right)\right]-2\left[\frac{-\lambda}{2}+\frac{\sqrt{\alpha}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}\right)\right]^2 \qquad (15)$$

Which can be simplified for $C_1 = 0$ and $C_2 = 1$, to;

$$v_1=-2\mu+\frac{\lambda^2}{2}-\frac{\alpha}{2}\coth^2\left(\frac{\sqrt{\alpha}}{2}\eta\right) \qquad (16)$$

And for $C_1 = 1$ and $C_2 = 0$;

$$v_2=-2\mu+\frac{\lambda^2}{2}-\frac{\alpha}{2}\tanh^2\left(\frac{\sqrt{\alpha}}{2}\eta\right) \qquad (17)$$

2) Equation (12) for $\alpha < 0$, is;

$$v=-2\mu-2\lambda\left[\frac{-\lambda}{2}+\frac{\sqrt{-\alpha}}{2}\left(\frac{C_1\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}\right)\right]-2\left[\frac{-\lambda}{2}+\frac{\sqrt{-\alpha}}{2}\left(\frac{C_1\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}\right)\right]^2 \qquad (18)$$

Which can be simplified for $C_1 = 0$ and $C_2 = 1$, to;

$$v_3=-2\mu+\frac{\lambda^2}{2}-\frac{\alpha}{2}\cot^2\left(\frac{\sqrt{-\alpha}}{2}\eta\right) \qquad (19)$$

And for $C_1 = 1$ and $C_2 = 0$, to;

$$v_4=-2\mu+\frac{\lambda^2}{2}+\frac{\alpha}{2}\tan^2\left(\frac{\sqrt{-\alpha}}{2}\xi\right) \qquad (20)$$

B. Case 2; $a_0=-\frac{\lambda^2}{3}-\frac{2}{3}\mu$, $a_1 = -2\lambda$, $a_2 = -2$ and $\delta=\lambda^2-4\mu$

1) For $\alpha > 0$, equation (12) becomes;

$$v=-\frac{\lambda^2}{3}-\frac{2}{3}\mu-2\lambda\left[\frac{-\lambda}{2}+\frac{\sqrt{\alpha}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}\right)\right]-2\left[\frac{-\lambda}{2}+\frac{\sqrt{\alpha}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\alpha}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\alpha}}{2}\eta\right)}\right)\right]^2 \qquad (21)$$

at $C_1 = 0$ and $C_2 = 1$, we have;

$$v_5=\frac{\lambda^2}{6}-\frac{2}{3}\mu-\frac{\alpha}{2}\coth^2\left(\frac{\sqrt{\alpha}}{2}\eta\right) \qquad (22)$$

And at $C_1 = 1$ and $C_2 = 0$, we have;

$$v_6=\frac{\lambda^2}{6}-\frac{2}{3}\mu-\frac{\alpha}{2}\tanh^2\left(\frac{\sqrt{\alpha}}{2}\eta\right) \qquad (23)$$

2) For $\alpha < 0$, equation (12) becomes;

$$v=-\frac{\lambda^2}{3}-\frac{2}{3}\mu-2\lambda\left[\frac{-\lambda}{2}+\frac{\sqrt{-\alpha}}{2}\left(\frac{C_1\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}\right)\right]-2\left[\frac{-\lambda}{2}+\frac{\sqrt{-\alpha}}{2}\left(\frac{C_1\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{-\alpha}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{-\alpha}}{2}\eta\right)}\right)\right]^2 \qquad (24)$$

$C_1 = 0$ and $C_2 = 1$, yields;

$$v_7=\frac{\lambda^2}{6}-\frac{2}{3}\mu+\frac{\alpha}{2}\cot^2\left(\frac{\sqrt{-\alpha}}{2}\eta\right) \qquad (25)$$

$C_1 = 1$ and $C_2 = 0$, yields;

$$v_8=\frac{\lambda^2}{6}-\frac{2}{3}\mu+\frac{\alpha}{2}\tan^2\left(\frac{\sqrt{-\alpha}}{2}\eta\right) \qquad (26)$$

IV. RESULTS AND DISCUSSION

This section is motivated to identify and plot the traveling wave solutions of ECBS equation (2), for the two cases;

A. *Traveling wave solutions for case 1*

Integrating equation (16) with respect to η yields, a one-soliton solution u_1 of the ECBS equation;

$$u_1=\sqrt{\alpha}\coth\left(\frac{\sqrt{\alpha}}{2}\eta\right)=\sqrt{\alpha}\coth\left(\frac{\sqrt{\alpha}}{2}(x+y+\delta t)\right) \qquad (27)$$

This solution is plotted in Fig.1 for $\alpha = 2$, $t = 2$ and $\delta=-1$.

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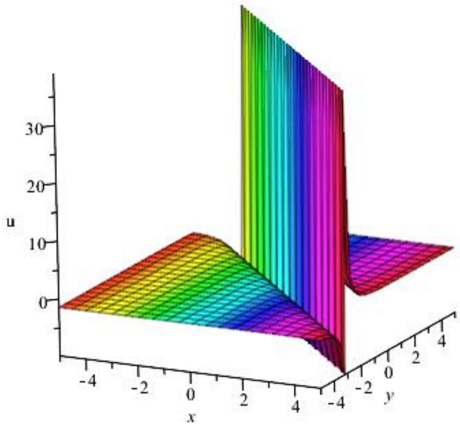

Fig.1. One soliton solution of EBCS equation for α =2, t =2 and δ= -1

Integrating (17) with respect to η yields, a kink-soliton solution u_2 of the ECBS equation, as shown in Fig.2, for $\alpha = 2$, $t = 0.5$ and $\delta = -2$.

$$u_2 = \sqrt{\alpha}\tanh\left(\frac{\sqrt{\alpha}}{2}\eta\right) = \sqrt{\alpha}\tanh\left(\frac{\sqrt{\alpha}}{2}(x + y + \delta t)\right)$$
(28)

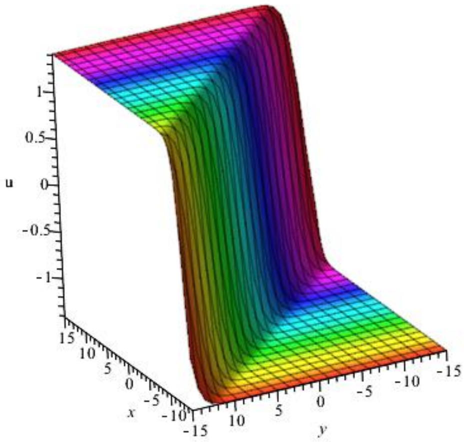

Fig.2. Kink solution of ECBS equation for δ =2, t = 0.5 and δ=-2.

Integrating (19) with respect to η yields, a periodic solution u_3 of the ECBS equation, as presented in Fig.3, for $\alpha = 2$, $t = 1$ and $\delta = 1$.

$$u_3 = \sqrt{-\alpha}\cot\left(\frac{\sqrt{-\alpha}}{2}\eta\right) = \sqrt{-\alpha}\cot\left(\frac{\sqrt{-\alpha}}{2}(x + y + \delta t)\right)$$
(29)

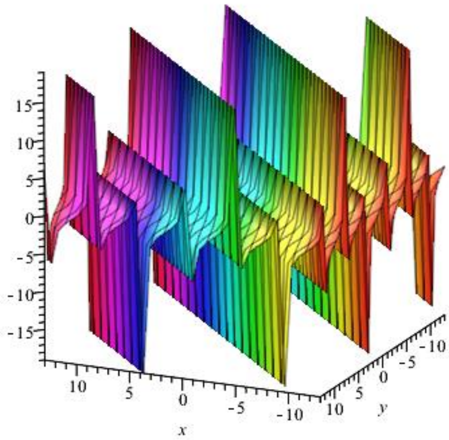

Fig.3. Periodic solution of ECBS equation for α = 2, t =1 and δ= 1.

Integrating (20) with respect to η yields, an N-soliton solution u_4 of the ECBS equation, as presented in Fig.4, for $\alpha = 2$, $t = 0.2$ and $\delta = 1$.

$$u_4 = -\sqrt{-\alpha}\tan\left(\frac{\sqrt{-\alpha}}{2}\eta\right) = -\sqrt{-\alpha}\tan\left(\frac{\sqrt{-\alpha}}{2}(x + y + \delta t)\right) \quad (30)$$

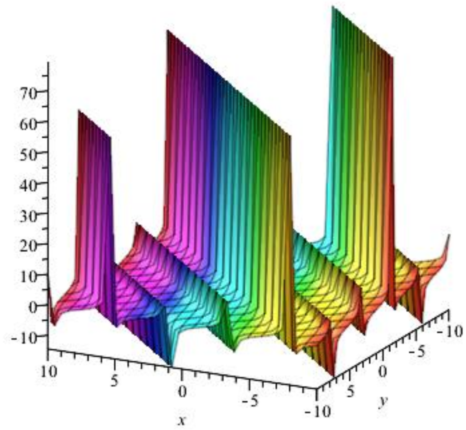

Fig.4. N-soliton solution of ECBS equation for α = 2, t = 0.2 and δ = 1.

B. *Traveling wave solutions for case 2*

Integrating (22) with respect to η yields, a one-soliton solution u_5 of the ECBS equation;

$$u_5 = -\frac{\alpha}{3}\eta + \sqrt{\alpha}\coth\left(\frac{\sqrt{\alpha}}{2}\eta\right) \quad (31)$$

Integrating (23) with respect to η yields;

$$u_6 = -\frac{\alpha}{3}\eta + \sqrt{\alpha}\tanh\left(\frac{\sqrt{\alpha}}{2}\eta\right) \quad (32)$$

Integrating (25) with respect to η yields,

$$u_7 = -\frac{\alpha}{3}\eta + \sqrt{-\alpha}\cot\left(\frac{\sqrt{-\alpha}}{2}\eta\right) \quad (33)$$

Integrating (26) with respect to η yields,

$$u_8 = -\frac{\alpha}{3}\xi - \sqrt{-\alpha}\tan\left(\frac{\sqrt{-\alpha}}{2}\eta\right) \quad (34)$$

V. CONCLUSIONS

In this paper, the (G'/G)-expansion method is effectively employed to reveal many explicit solutions for the extended Calogero-Bogoyavlenskii-Schiff equation. The presented solutions include a variety of one-soliton, kink, periodic and N-soliton solutions. These aggregations of traveling wave solutions are applicable for different conditions. The results help in studying the wave behaviors in several applications as, deep oceans and nonlinear optics.

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