Transverse Vibrations of Visco - Elastic Square Plate with Thickness and Temperature Variation

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Abstract

A mathematical model is developed with an aim that scientists and design engineers can make a use of it with a practical approach, for the welfare of the human beings. In which effect of thermal gradient is studied on vibration of homogeneous isotropic visco-elastic square plate having parabolic varying thickness. Using Rayleigh- Ritz procedure, frequency function is calculated for first mode of vibration for different values of thermal gradient and taper constants Parameter.

1. Introduction

In the course of time, the study of vibration of plates has acquired great importance in the field of research, engineering and space technology. The visco-elastic behaviors of some materials invigorated scientists for modern designs and analysis techniques and their application to many practical problems. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications.

Plates of various geometries are commonly used as structural elements in various fields of engineering such as civil, naval and mechanical. In particular, rectangular plates are widely used in ocean structures and aerospace industry. Plates with varying thickness possess a number of attractive features such as material saving, weight reduction, stiffness enhancing, high strength and also meet the desirability of economy. A thorough dynamic study of their behavior and characteristics is essential to assess and use the full potentials of plates. In the aeronautical field, analysis of plates with variable thickness has been of great interest due to their utility in aircraft wings.

The aim of present investigation is to study one dimensional parabolic thermal effect on the vibration of visco-elastic square plate with varying thickness parabolic in one direction. It is assumed that the plate is clamped on all the four edges and its temperature varies parabolic in one direction. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity. For various numerical values of thermal gradient and taper constants, frequencies for the first two modes of vibration are calculated with the help of latest software. All results are shown in Graphs.

2. Equation of Motion

The governing differential equation of transverse motion of a visco-elastic plate of variable thickness in Cartesian co-ordinates, as below:-

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \qquad (1)$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\tilde{D}D_1\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\tilde{D}D_1\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\tilde{D}D_1(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \qquad (2)$$

On substitution the values M_x, M_y and M_{yx} from equation (2) in (1) and taking w, as a product of two function, equal to $w(x,y,t)=W(x,y)T(t)$, equation (1) become:

$$[D_1(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}) + 2\frac{\partial D_1}{\partial x}(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}) + 2\frac{\partial D_1}{\partial y}(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y})$$
$$+ \frac{\partial^2 D_1}{\partial x^2}(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}) + \frac{\partial^2 D_1}{\partial y^2}(\frac{\partial^2 W}{\partial y^2} + \nu\frac{\partial^2 W}{\partial x^2}) + 2(1-\nu)\frac{\partial^2 D_1}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y}]/\rho h W = -\frac{\ddot{T}}{\tilde{D}T} \qquad (3)$$

Here dot denote differentiation with respect to t. taking both sides of equation (3) are equal to a constant p^2, we have

$$D_1\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right)$$
$$+ \frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\right) + \frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2} + \nu\frac{\partial^2 W}{\partial x^2}\right) + 2(1-\nu)\frac{\partial^2 D_1}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} - \rho h p^2 W = 0 \qquad (4)$$

which is a differential equation of transverse motion for visco-elastic square plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3 / 12(1-\nu^2) \qquad (5)$$

and corresponding two-term deflection function is taken as [5]

$$W = [(x/a)(y/a)(1-x/a)(1-y/a)]^2 \qquad (6)$$
$$[A_1 + A_2(x/a)(y/a)(1-x/a)(1-y/a)]$$

Assuming that the square plate of engineering material has a steady one dimensional parabolic temperature distribution i.e.

$$\tau = \tau_0(1 - x^2 / a^2) \quad (7)$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this

$$E = E_0(1 - \gamma\tau) \quad (8)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus variation (8) become

$$E = E_0[1 - \alpha(1 - x^2 / a^2)] \quad (9)$$

where, $\alpha = \gamma\tau_0 (0 \le \alpha < 1)$ thermal gradient. It is assumed that thickness also varies one dimensional parabolic as shown below:

$$h = h_0(1 + \beta_1 x^2 / a^2) \quad (10)$$

where, β_1 is taper parameter in x direction respectively and $h=h_0$ at x=y=0.

Put the value of E & h from equation (9) & (10) in the equation (5), one obtain

$$D_1 = [E_0[1 - \alpha(1 - x^2 / a^2)]h_0^3(1 + \beta_1 x^2 / a^2)^3] / 12(1 - v^2) \quad (11)$$

3. Solution of Equation of Motion

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^* - T^*) = 0 \quad (12)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions. Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{array}{ll} W = W,_x = 0, & x = 0, a \\ W = W,_y = 0, & y = 0, a \end{array}\right\} \quad (13)$$

Now assuming the non-dimensional variables as $X = x/a, Y = y/a, \overline{W} = W/a, \overline{h} = h/a$ (2.14)

The kinetic energy T* and strain energy V* are [2]

$$T^* = (1/2)\rho p^2 \overline{h}_0 a^5 \int_0^1\int_0^1 [(1 + \beta_1 X^2)\overline{W}^2] dY dX \quad (15)$$

and

$$V^* = Q\int_0^1\int_0^1 [1 - \alpha(1 - X^2)](1 + \beta_1 X^2)^3 \{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1 - v)(\overline{W},_{XY})^2\} dY dX \quad (16)$$

where, $Q = E_0 h_0^3 a^3 / 24(1 - v^2)$

Using equations (15) & (16) in equation (12), one get

$$(V^{**} - \lambda^2 T^{**}) = 0 \quad (17)$$

where,

$$V^{**} = \int_0^1\int_0^1 [1 - \alpha(1 - X^2)](1 + \beta_1 X^2)^3 \{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1 - v)(\overline{W},_{XY})^2\} dY dX \quad (18)$$

and

$$T^{**} = \int_0^1\int_0^1 [(1 + \beta_1 X^2)\overline{W}^2] dY dX \quad (19)$$

Here, $\lambda^2 = 12\rho(1 - v^2)a^2 / E_0 h_0^2$ is a frequency parameter.

Equation (19) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \quad , n = 1, 2 \quad (20)$$

On simplifying (2.17), one gets

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , n = 1, 2 \quad (21)$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \quad (22)$$

With the help of equation (22), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) & λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

4. Result and Discussion

The frequency Equation (22), one can find a quadratic equation in λ^2 from which two roots can be determined. The frequency parameter λ corresponding to the first two modes of vibration of clamped square plate have been computed for various values of temperature gradient (α) and taper constant β_1. All calculations are carried out with the help of latest Matrix Laboratory computer software

In Fig. 1: - It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for β_1=0.0, β_1=0.4 and β_1=0.8 for both modes of vibrations.

In Fig. 2 :- Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_1 from 0.0 to 1.0 for α=0.2 , α=0.4 and α=0.8 respectively. Note that in this case the value of frequency increased.

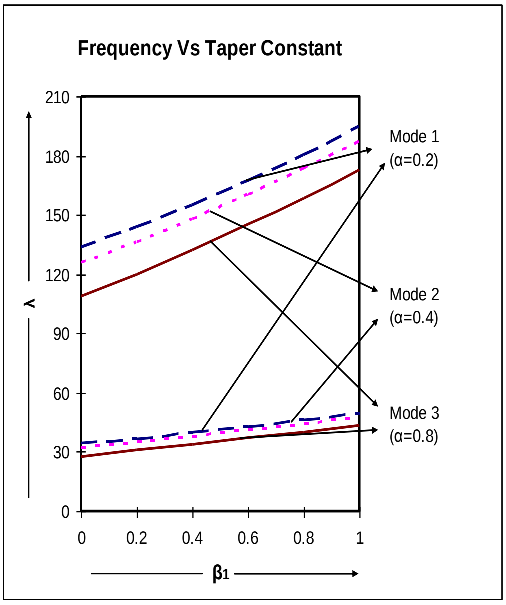

Fig. 1:- Frequency vs. Thermal Gradient

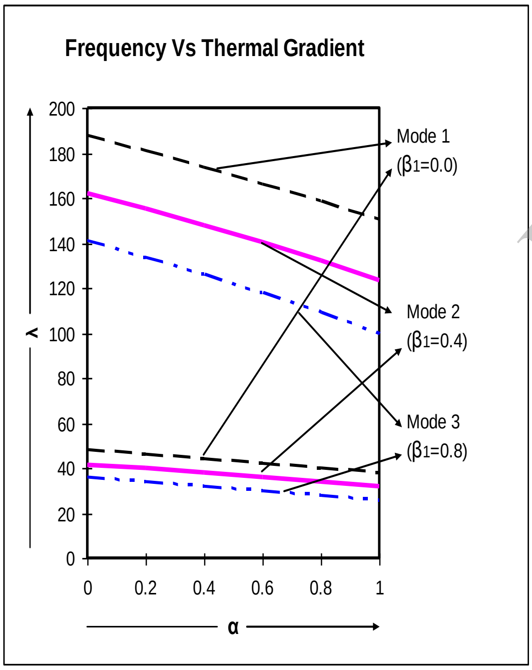

Fig. 2:- Frequency vs. Taper Constant

5. Conclusion

Motive is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field & increase strength, durability and efficiency of mechanical design and structuring with a practical approach .Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers/researchers/practitioners.

6. References

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