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Transport of Aerosols in the Presence of Electric Field with Interphase Mass Transfer

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Abstract

We have developed a mathematical model for evaluating the unsteady convective dispersion of aerosols with inter phase mass transfer by poorly conducting couple stress fluid in a channel in the presence of a transverse electric field. The validity of time dependent dispersion coefficient is widened by using a generalized dispersion coefficient. The exact expression is obtained for the dispersion coefficient (K_2) which shows it decreases with increase in couple stress parameter a and reaction rate β. The anlaysis leads to the novel result for convection coefficient (K_1) and dispersion coefficient (K_2) (which is a measure of the longitudinal dispersion coefficient of the aerosol). It is found that the value of K_2 depends on the value of reaction rate (β) and electric field (w_e) whereas the values of K_1 is constant in both the cases. Finally, the effect of a,β and the electric field (w_e) on the axial distribution of the mean concentration θ_m is investigated in detail.

keywords: electrodes, generalized dispersion, aerosols, chemical reaction.

1. Introduction

The problem caused by the air pollution are complex and they affect natural processes, strongly influencing the ecological balance [2]. For this reason, it is important to study and understand the dispersion process of pollutants in the atmosphere in order to predict the possible impact of the pollution on the diverse ecosystems involved. Pollutants released from various sources into the atmosphere in the form of suspended ultrafine particles are aerosols [1] affect the environment directly or indirectly. The couple stress fluid theory developed by stokes [9] represents the simplest generalization of the classical viscous fluid theory that sustains couple stresses and the body couples.

The presence of earth's surface influences the concentration of ions, aerosols and radioactive particles, through its control over the wind, temperature and water vapour distributions [4], [8]. Atmospheric electricity abounds in the environment [7]. Recently it has become evident that lightening is a form of electricity associated with thunderstorms [10]. The resulting ground level concentration patterns have to be estimated for a wide variety of air quality analysis for social planning and industrial growth [3].

It is not always feasible to measure/monitor concentration of species at various vulnerable points of a city. However man insight in this regard could be achieved with the help of suitable mathematical models. In this paper, we developed a mathematical model to study the unsteady convective diffusion of atmospheric aerosols with interphase mass transfer in a couple stress fluid flow through a channel in the presence of electric field. The atmospheric flow and concentration change of aerosols are commonly described by a set of partial differential equations, which are mathematical formulation of one or more of the conservation law of physics. These include the equations of mass, momentum species along with Maxwell's equation which involve advection and diffusion terms as a main constituent. They have been solved using generalised dispersion model [5] with appropriate boundary conditions and the results have been depicted graphically.

2. Mathematical Formulation

The physical configuration shown in the figure 1 consists of an infinite horizontal couple stress fluid layer bounded on both sides by electro-conducting impermeable right plates embedded with electrodes located at y = ± h and electric potentials

$\phi = \frac{V}{h}x$ at y = -h and $\phi = \frac{V}{h}(x - x_0)$ at y = h

are maintained on these boundaries where V is potential.

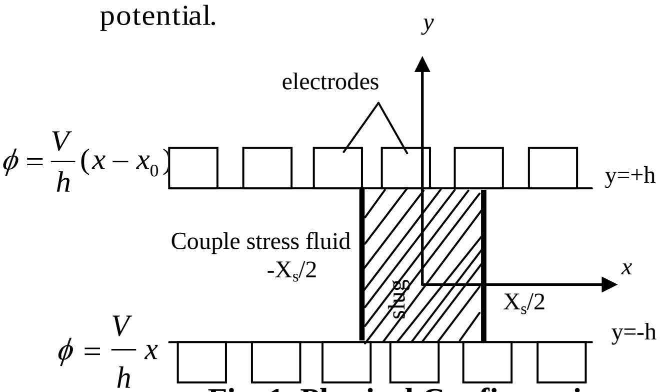

Fig. 1. Physical Configuration

In this paper, we make the assumption that the electrical conductivity (σ) is negligibly small and hence the magnetic field is negligible. This assumption makes the electric field $\vec{E}$, to be conservative.

i.e. $\vec{E} = -\nabla\phi$ (1)

The basic equations are

Conservation of mass for an incompressible flow

$\nabla.\vec{q} = 0$ (2)

Conservation of momentum

$$\rho\left(\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} + -\lambda\nabla^4\vec{q} + \rho_e\vec{E} \quad (3)$$

Where λ is a couple stress parameter.

Conservation of species

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = D\nabla^2 C \quad (4)$$

Conservation of charges

$$\frac{\partial\rho_e}{\partial t} + (\vec{q}.\nabla)\rho_e + \nabla.\vec{J} = 0 \quad (5)$$

Maxwell's equation

$$\nabla.\vec{E} = \frac{\rho_e}{\varepsilon_0}(\text{Gauss law}) \quad (6)$$

$$\nabla\times\vec{E} = 0\,(\text{Faraday 's law}) \quad (7)$$

The equation (7) is zero because in a poorly conducting fluid, induced magnetic field is negligible and there is no applied magnetic field.

$$\vec{J} = \sigma\,\vec{E}(\text{Ohm's law}) \quad (8)$$

The above equations are solved using the following boundary conditions on velocity and potential are,

No slip condition,

$$u = 0 \quad at \quad y = -h, h \quad (9)$$

The couple stress condition,

$$\frac{d^2u}{dy^2} = 0 \; at \; y = \pm h \quad (10)$$

$$\left.\begin{aligned} \phi &= \frac{V}{h}x \;\; at \;\; y = -h \\ \phi &= \frac{V}{h}(x - x_0) \;\; at \;\; y = h \end{aligned}\right\} \quad (11)$$

In Cartesian form, using the above approximation equation (3) becomes

$$0 = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \lambda\nabla^4 u + \rho_e E_x, \nabla^2 = \frac{\partial^2}{\partial y^2}$$

In a poorly conducting fluid, the electrical conductivity is assumed to vary linearly with temperature in the form

$$\sigma = \sigma_0[1 + \alpha_h(T_b - T_0)] \quad (12)$$

where α_h is the coefficient of volumetric expansion.

We assume the flow is fully developed and unidirectional in the x –direction. This means the velocity is independent of time and all physical quantities except pressure and concentration are independent of *x*, so that the velocity and temperature will be functions of y only. Using the following dimensionless quantities,

$$y^* = \frac{y}{h}; u^* = \frac{u}{\frac{\nu}{h}}; E_x^* = \frac{E_x}{\frac{V}{h}}; \rho_e^* = \frac{\rho_e}{\frac{\varepsilon_0 V}{h^2}};$$

$$P^* = \frac{P}{\rho\left(\frac{\nu}{h}\right)^2}; x^* = \frac{x}{h}$$

Where V is electric potential, we get electric potential through electrodes.

Equations (3) to (11) becomes,

$$\frac{d^4u}{dy^4}-\frac{h^2}{l^2}\frac{d^2u}{dy^2}-\frac{h^2}{l^2\mu}\rho_e E_x \in_0 V^2=\frac{-\rho v h^2}{l^2\mu}\frac{\partial p}{\partial x}$$

We assume that the fluid with pollutants is isotropic and homogenous so that molecular diffusivity D, viscosity μ are all constants [6].

$$\frac{\partial^4u}{\partial y^4}-a^2\frac{d^2u}{dy^2}-a^2weP_e\vec{E}_x=a^2P \qquad (13)$$

where

$$\text{we}=\frac{\in_0 V^2}{\mu};P=-\frac{\partial P}{\partial x},l=\sqrt{\frac{\lambda}{\mu}},a=\frac{h}{l}$$

is the couple stress parameter

Equation (5) becomes, $\nabla\cdot\vec{J}=0$ using equation (1) we get,

$$\sigma(\nabla^2\phi)+\nabla\phi.\nabla\sigma=0 \qquad (14)$$

The boundary conditions on velocity, couple stress and electric potentials after dimensionless are

$$u=0 \quad \text{at } y=\pm1 \qquad (15)$$

$$\frac{d^2u}{dy^2}=0 \qquad at \qquad y=\pm1 \qquad (16)$$

$$\left.\begin{aligned}\phi&=x \quad at\,y=-1\\ \phi&=x-x_0 \quad at\;y=1\end{aligned}\right\} \qquad (17)$$

The solution for ϕ, according to (14)depends on σ which in turn depends on the temperature T_b as in (12). In a poorly conducting fluid, σ<<1 and hence any perturbation on it is negligible and hence it depends on the conduction temperature T_b namely,

$$\frac{d^2T_b}{dy^2}=0 \qquad (18)$$

Hence the solution satisfying the boundary conditions

$$\left.\begin{aligned}T_b&=1\,at\,y=1\\ T_b&=-1\,at\,y=-1\end{aligned}\right\} \qquad (19)$$

is $T_b=y$ (20)

Therefore equation (12) becomes

$\sigma=\sigma_0[1+\alpha_h\Delta\, Ty]=\sigma_0(1+\alpha y)\approx\sigma_0 e^{\alpha y}$

$\sigma\approx e^{\alpha y}$ (Where $\alpha=\alpha_h\Delta T$) (21)

From equations (14) and (21) we get,

$$\frac{d^2\phi}{dy^2}+\alpha\frac{d\phi}{dy}=0 \qquad (22)$$

Its solution satisfying the boundary condition (17) is

$$\phi=x-\frac{x_0}{2\sin h\alpha}[e^{\alpha}-e^{-\alpha y}] \qquad (23)$$

using the dimensionless quantities and equation (23), equation (6), (7) and (8) reduce to

$$\rho_e=\nabla.\vec{\mathbf{E}}=-\nabla^2\phi=-\frac{x_0^2\alpha^2e^{-\alpha y}}{2\sinh\alpha};$$

$$\therefore\rho_eE_x=\frac{x_0^2\alpha^2e^{-\alpha y}}{2\sin h\alpha} \qquad (24)$$

The solution of equation (13) satisfying the condition (15), (16) is

$$u=b_0y+b_1-\frac{P}{2}y^2+b_2\sinh ay+$$
$$b_3\cosh ay+b4e-^{\alpha y} \qquad (25)$$

Where b_0

$$=\frac{a_0\sinh(\alpha)}{\alpha^2-a^2}\left(\frac{1}{\alpha^2}-\frac{1}{a^2}\right);\ b_1=P\left(\frac{1}{2}-\frac{1}{a^2}\right)+\left(\frac{a_0\cosh(\alpha)}{\alpha^2-a^2}\right)\left(\frac{1}{a^2}-\frac{1}{\alpha^2}\right);$$

$$b_2=\frac{\sin h\alpha}{\sinh a}\frac{a_0}{a^2(\alpha^2-a^2)};$$

$$b_3=\frac{1}{a_2\cosh a}\left[P-\frac{a_0\cosh\alpha}{\alpha^2-a^2}\right];$$

$$b_4=\frac{a_0}{\alpha^2(\alpha^2-a^2)}$$

$$and\, a_0 = \frac{W_e\alpha^2 x_0}{2\sin h(\alpha)}$$

The average velocity is given by,

$$\bar{u} = \frac{1}{2}\int_0^1 u\,dy = b_1 - \frac{P}{6} - \frac{b_3\sinh a}{a} + \frac{b_4\sinh\alpha}{\alpha} \qquad (26)$$

Into this flow, introduced a slug input of concentration C(*0,x,y*) and the local concentration C(*t,x,y*) of the solute which satisfies the convective diffusion equation

$$\frac{\partial C}{\partial t} + u(y,t)\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \qquad (27)$$

along with the initial condition

$$C(0,x,y) = C_0\psi_1(x)Y_1(y) \qquad (28a)$$

Where C_0 is a reference concentration. The corresponding boundary condition at the channel walls are

$$\left.\begin{aligned} -D\frac{\partial C}{\partial y} &= K_s C \; at \quad y = h \quad and \\ D\frac{\partial C}{\partial y} &= K_s C \quad at \quad y = -h \end{aligned}\right\} (28b)$$

Where K_s is the reaction rate constant catalysed by the walls.

$$C(t,\infty,y) = \frac{\partial C}{\partial x}(t,\infty,y) = 0 \qquad (28c)$$

On introducing the following non-dimensional quantities,

$$\tau = \frac{Dt}{h^2};\theta = \frac{C}{C_0}; X = \frac{Dx}{h^2\bar{u}};\xi = \frac{y}{h};$$

$$U(\tau,\xi) = \frac{u}{\bar{u}}; P_e = \frac{h\bar{u}}{D};\beta \frac{K_s H}{D}$$

Equation (27) and (28) become

$$\frac{\partial\theta}{\partial\tau} + U\frac{\partial\theta}{\partial X} = \frac{1}{P_e^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial\xi^2} \qquad (29)$$

Where U=$\frac{u}{\bar{u}}$,$\bar{u}$ is the average velocity of the flow, $P_e$is the Peclet number and β is the dimensionless reaction rate constant, and the initial conditions reduce to

$$\theta(0,X,\xi) = \psi(X)Y(\xi), \qquad (30a)$$

$$\left.\begin{aligned} \frac{\partial\theta}{\partial\xi} &= -\beta\theta \qquad at\,\xi = 1, \\ \frac{\partial\theta}{\partial\xi} &= \beta\theta \qquad at\,\xi = -1, \end{aligned}\right\} \qquad (30b)$$

$$\theta(\tau,\infty,\xi) = \frac{\partial\theta}{\partial X}(\tau,\infty,\xi) = 0 \qquad (30c)$$

In the generalized dispersion model [5], we let

$$\theta(\tau,X,\xi) = \sum_k f_k(\tau,\xi)\frac{\partial^k\theta_m}{\partial X^{k'}} \qquad (31)$$

Where

$\theta_m = \frac{1}{2}\int_{-1}^{1}\theta(\tau,X,\xi)\,d\xi\, is$ the dimensionless average concentration. (31a)

Equation (29) is multiplied throughout by $\frac{1}{2}$ and integrated with respect to y from -1 to 1. Introducing the definition of θ_m, we get

$$\frac{\partial\theta m}{\partial\tau} = \frac{1}{P_e^2} - \frac{\partial^2\theta_m}{\partial X^2} + \frac{1}{2}\left[\frac{\partial\theta}{\partial\xi}\right]_{-1}^{1} - \frac{1}{2}\frac{\partial}{\partial x}\int_{-1}^{1} U(\tau,\xi)\theta\, d\xi \qquad (32)$$

Using equation (31) in (32), the dispersion model for θ_m is obtained as

$$\frac{\partial\theta m}{\partial\tau} = \sum_{i=0}^{\infty} K_i(\tau)\frac{\partial^i\theta_m}{\partial X^i} \qquad (33)$$

Where K_i's are given by

$$Ki(\tau) = \frac{\delta_{i2}}{P_e^2} + \frac{1}{2}\frac{\partial f_i}{\partial\xi}(\tau,1) - \frac{1}{2}\int_{-1}^{1} f_{i-1}(\tau,\xi)U(\tau,\xi)\,d\xi \qquad (34)$$

Where $f_{-1}=0$ and δ_{i2} is the Kroneckar delta defined by

$$\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$$

The exchange coefficient $K_0(\tau)$ accounts for the nonzero solute flux at the channel wall and negative sign indicates the depletion of solute in the system with time caused by the irreversible reaction, which occurs at the channel wall. The presence of non-zero solute flux at the walls of the channel also affects the higher order K_i due to the explicit appearance of $\partial f_i/\partial\xi\ (\tau,1)$ in equation (34). Equation (33) can be truncated after the term involving K_2 without causing serious error because K_3, K_4, etc. become negligibly small compared to K_2.The resulting model for the mean concentration is

$$\frac{\partial\theta_m}{\partial\tau}=K_0(\tau)\theta_m+K_1(\tau)\frac{\partial\theta_m}{\partial X}+K_2(\tau)\frac{\partial^2\theta_m}{\partial X^2} \qquad (35)$$

To solve this equation, we need the coefficients $K_i(\tau)$ in addition to the appropriate initial and boundary conditions. For this, the corresponding function f_k must be determined. So, substituting (31) into (29) and using equation (32), the following set of differential equations for f_k are generated.

$$\frac{\partial f_k}{\partial\tau}=\frac{\partial_2 f_k}{\partial\xi^2}-Uf_{k-1}+\frac{1}{P_e^2}f_{k-2}$$
$$-\sum_{i=0}^{k}K_i f_{k-1}\ (k=0,1,2,\ldots) \qquad (36)$$

where $f_{-1}=f_{-2}=0$.

We note that to evaluate K_i's we need to know the f_k's which are obtained by solving

(36) for f_k's subject to the boundary conditions.

$$f_k(\tau,0)=\text{finite} \qquad (37a)$$

$$\frac{\partial f_k}{\partial\xi}(\tau,1)=-\beta f_k(\tau,1) \qquad (37b)$$

$$\frac{\partial f_k}{\partial\xi}(\tau,0)=0 \qquad (37c)$$

$$\frac{1}{2}\int_{-1}^{1}f_k(\tau,\xi)d\xi=\delta k_0,\ (k=0,1,2) \qquad (37d)$$

The function f_0 and the exchange coefficient K_0 are independent of the velocity field and can be solved easily. It should be pointed out here that a simulatenous solution has to be obtained from these two quantities since K_0, which can be obtained from (34) as

$$K_0(\tau)=\frac{1}{2}\left[\frac{\partial f_0}{\partial y}\right]_{-1}^{1} \qquad (38)$$

Substituting k =0 in (36) we get the differential equation or f_0 as

$$\frac{\partial f_0}{\partial\tau}=\frac{\partial^2 f_0}{\partial\xi^2}-f_0K_0 \qquad (39)$$

We now derive an initial condition for f_0 using (31a) by taking $\tau=0$ in that equation to get

$$\theta_m(0,X)=\frac{1}{2}\int_{-1}^{1}\theta(0,X,\xi)d\xi \qquad (40)$$

Substituting $\tau=0$ in (31) and setting $f_k(\xi)=0$ (k=1,2,3) gives us the initial condition for f_0 as

$$f_0(0,\xi)=\frac{\theta(0,X,\xi)}{\theta_m(0,X)} \qquad (41)$$

We note that the left hand side of (41) is function of ξ only and the right hand side is a function of both X and ξ. Thus clearly the initial concentration distribution must be a separable function of X and ξ. This is the justification for the chosen form of θ $(0,X,\xi)$ in (41). Substituting (30) into (41), we get

$$f_0(0,\xi)=\frac{\psi(\xi)}{\frac{1}{2}\int_{-1}^{1}\psi(\xi)d\xi} \qquad (42)$$

The solution of the reaction diffusion equation (39) with these conditions may be formulated as

$$f_0(\tau,\xi) = g_0(\tau,\xi)\exp\left[-\int_0^t K_0(\eta)d\eta\right] \qquad (43)$$

From which it follows that $g_0(\tau,\xi)$ has to satisfy

$$\frac{\partial g_0}{\partial \tau} = \frac{\partial^2 g_0}{\partial \xi^2} \qquad (44)$$

along with the conditions

$$f_0(0,\xi) = g_0(0,\xi) = \frac{\psi(\xi)}{\frac{1}{2}\int_{-1}^{1}\psi(\xi)d\xi} \qquad (45a)$$

$g_0(\tau,0)$= finite (45b)

$$\frac{\partial g_0}{\partial \xi}(\tau,1) = -\beta g_0(\tau,1) \qquad (45c)$$

The solution of (44) subject to conditions (45) is

$$g_0(\tau-\xi) = \sum_{n=0}^{\infty} A_n \cos(\mu_n\xi)\exp(-\mu_n^2\tau) \qquad (46)$$

Where μ_n's are the roots of

$\mu_n\tan\mu_n=\beta$, n=0,1,2,3…….. (47)

and A_n's are given by

$$A_n = \frac{2\int_{-1}^{1}\psi(\xi)\cos\mu_n\xi d\xi}{\left(1+\frac{\sin 2\mu_n}{2\mu_n}\right)\int_{-1}^{1}\psi(\xi)d\xi} \qquad (48)$$

Now from (43) is follows that

$$f_0(\tau,\xi) = \frac{2g_0(\tau,\xi)}{\int_{-1}^{1} g_0(\tau,\xi)d\xi} = \frac{\sum_{n=0}^{9} A_n\exp(-\mu_n^2\tau)\cos\mu_n\xi}{\sum_{n=0}^{9}\frac{A_n}{\mu_n}\exp(-\mu_n^2\tau)\sin\mu_n} \qquad (49)$$

The first ten roots of the transcendental equation (47) are obtained using mathematica and are given in Table 1. We find that these ten roots ensured convergence of the series seen in the expansions for f_0 and K_0. Having obtained f_0, we get K_0 from (38) in the form

$$K_0(\tau) = -\frac{\sum_{n=0}^{9} A_n\mu_n\exp(-\mu_n^2\tau)\sin\mu_n}{\sum_{n=0}^{9}\frac{A_n}{\mu_n}\mu_n\exp(-\mu_n^2\tau)\sin\mu_n} \qquad (50)$$

$K_0(\tau)$ is independent of velocity distribution.

As τ→∞, we get the asympotic solution for K_0 from (50) as

$K_0(\infty) = -\mu_0^2$ (51)

Where μ_0 is the first root of the equation (47). Physically this represents first order chemical reaction coefficient having obtained $K_0(\infty)$, we can now get $K_1(\infty)$, from (34) with (i=1) knowing $f_0(\infty,\xi)$ and $f_1(\infty,\xi)$. Likewise $K_2(\infty)$, $K_3(\infty)$,… require the knowledge of K_0, K_1, f_0, f_1 *and* f_2. Equation (49) in limit τ→∞ reduces to

$$f_0(\infty,\xi) = \frac{\mu_0}{\sin\mu_0}\cos(\mu_0\xi) \qquad (52)$$

We then find f_1, K_1, f_2, and K_2. For asymptotically long times, i.e., τ→ ∞, (34) and (36) give us K_i's and f_k's as

$$K_i(\infty) = \frac{\delta_{i2}}{P_e^2} - \beta f_i(\infty,1) - \int_{-1}^{1} U f_{i-1}(\infty,\xi)d\xi,$$

$$(i=1,2,3) \qquad (53)$$

$$\frac{d^2 fk}{d\xi^2} + \mu_0^2 fk = (U+K_1)f_{k-1} - \left(\frac{1}{P_e^2} - K_2\right) f_{k-2}, \qquad (K=1,2) \qquad (54)$$

The f_k's must satisfy the conditions (31a) and this permits the eigen function expansion in the form

$$f_k(\infty,\xi) = \sum_{j=0}^{9} B_{j,k}\cos(\mu_j\xi), \quad k=1,2,3,.... \qquad (55)$$

Substituting (55) in equation (54) and multiplying the resulting equation by Cos $(\mu_j\xi)$ and integrating with respect to ξ from -1 to 1, we get after simplification

$$B_{j,k} = \frac{1}{(\mu_j^2 - \mu_0^2)}\left[\frac{1}{P_e^2}B_{j,k-2} - \sum_{i=1}^{K} k_i B_{j,k-i} - \left(1+\frac{\sin 2\mu_j}{2\mu_j}\right)^{-1}\sum_{j=0}^{9} B_{j,k-1} I(j,1)\right] \quad (56)$$

$$k = (2)$$

Where

$$I(j,l) = \int_{-1}^{1} U\cos\mu_j\xi.\cos\mu_l\xi d\xi = I(l,j) \quad (57)$$

$$B_{j,-1} = 0, B_{j,0} = 0 \quad for \quad j = 1\ to \quad 9$$

The first expansion coefficient $B_{0,k}$ in (55) can be expressed in terms of $B_{j,k}$ (j=1 to 9) by using the conditions (37) as

$$B_{0,k} = -\left(\frac{\mu_0}{\sin\mu_0}\right)\sum_{j=1}^{9} B_{j,k}\frac{\sin\mu_j}{\mu_j}, (k = 1,2,3,...) \quad (59)$$

Further, from (55) and (52) we find that

$$B_{0,0} = \frac{\mu_0}{\sin\mu_0} \quad (60)$$

Substituting i=1 in (53) and using (57), (58) and (60) in the resulting equation, we get

$$K_1(\infty) = -\frac{I(0,0)}{\left[1+\frac{\sin 2\mu_0}{2\mu_0}\right]} \quad (61)$$

Substituting i=2 in (53) and using (56), (57) and (60) in the resulting equation, we get

$$K_2 = \frac{1}{P_e^2} - \frac{Sin\mu_0}{\mu_0\left(1+\frac{\sin 2\mu_0}{2\mu_0}\right)}\sum_{j=0}^{9} B_{j,1}.I_{j,0} \quad (62)$$

$$\text{Where}Bj-1 = -(\mu_j^2 - \mu_0^2)^{-1}\left(1+\frac{\sin 2\mu_0}{2\mu_0}\right)^{-1}\frac{\mu_0}{\sin\mu_0}.I(j,0)$$

Using the asymptotic coefficients $K_0(\infty)$,$K_1(\infty)$ and $K_2(\infty)$, in (33). One can determine the mean concentration distribution as a function of X,τ and the parameters of the problem a and *β*. This distribution is valid only for long time and is a gross approximation at short and moderate times. The initial conditions for solving (33) can be obtained from (30a) by taking the cross-sectional average. Since we are making long time evaluations of the coefficients, we note that the side effect is independent of θ_m on the initial concentration distribution. In view of this, the solution of (33) with asymptotic coefficients can be written as

$$\theta_m(\tau, X) = \frac{1}{2P_e\sqrt{\pi K_2(\infty)\tau}} \exp\left[K_0(\infty)\tau - \frac{(X + K_1(\infty)\tau)^2}{4K_2(\infty)\tau}\right] \quad (63)$$

Where

$$\theta_m(\tau,\infty) = 0, \frac{\partial\theta_m}{\partial X}(\tau,\infty) = 0 \quad (64)$$

3. Results and Discussions

We have modeled the solvent as a couple-stress fluid and studied dispersion of solute in a couple-stress fluid bounded by electrodes considering heterogenous chemical reaction, on the interphase. The walls of the channel act as catalysts to the first order reaction and the effect of wall reaction on dispersion is discussed. The problem brings into focus three important dispersion coefficients namely the exchange coefficient ($-K_0$), the convective coefficient ($-K_1$) and diffusive coefficient (K_2). The exchange coefficient arises only due to the interphase mass transfer and hence it is unaffected by the flow and thus is independent of the couple-stress parameter a. The asymptotic values of these three coefficients are plotted in figure 3 to 6 for various value of a and reaction rate parameter *β*. From these figures we predict the following.

Figure 2 is a plot of the velocity against the non-dimensional transverse coordinate η for different values of electric number W_e. It is seen that the velocity profile is a parabolic curve. We note that the velocity increases with the increase in electric number. From figure 3, it is evident that $– K_0(\infty)$ increases with increase in the value of the wall reaction parameter *β* and is unaffected by the couple-stress parameter *a*.

The convective coefficient -K_1 is plotted in figure 4 versus wall reaction parameter β for different couple stress parameter a with a fixed value of slip parameter α = 0.1. From these figures we conclude that increase in β as well as decrease in a is to increase $-K_1$. This is advantageous in maintaining the laminar of flow. Figure 5 is a plot of dispersion coefficient K_2 against electric number W_e for different values of β. From this we conclude that increase in β and W_e is to decrease the effective dispersion coefficient K_2. The scaled dispersion coefficient K_2-P_e^{-2} is plotted versus β in figure 6 for different values of a. From this figure, it is clear that the increase in a and β, the effective dispersion coefficient decreases. These are useful in the control of dispersion of a solute.

The cross sectional average concentration θ_m is plotted versus x in figures 7 and 8 respectively for different values of a,β and for fixed values of the other parameters given in these figures. It is clear that the increase in β and a increases θ_m as expected on the physical grounds. θ_m is also plotted in figures 9 and 10 against the dimensionless time τ for different values of β and a for fixed values of the other parameters given in these figures. We note that the peak of θ_m decreases with an increase in β occuring at the lower interval of time τ. We also note that the peak increases with an increase in a but occurs at almost at the same interval of time τ. These informations are useful to understand the transport of solute at different times.

The proposed model and analysis presented here also suggests that to remove the pollutants from the atmosphere, external species may be introduced in the atmosphere which can interact with the pollutant and remove it by some suitable removal processes based upon the physical and chemical properties of the pollutants as well as that of externally introduced species.

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Table 1: Roots of the equation $\mu_n \tan \mu_n = \beta$

	μ_0	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9
10^{-2}	0.099834	3.14477	6.28478	9.42584	12.5672	15.7086	18.8501	21.9916	25.1331	28.2747
0.05	0.22176	3.15743	3.15743	6.29113	9.43008	12.5703	15.7111	21.9934	25.1347	28.2761
10^{-1}	0.311053	3.1731	6.29906	9.43538	12.5743	15.7143	18.8549	21.9957	25.1367	28.2779
0.5	0.653271	3.29231	6.36162	9.47749	12.606	15.7397	18.876	22.0139	25.1526	28.292
1.0	0.860334	3.42562	6.4373	9.52933	12.6453	15.7713	18.9024	22.2126	25.1724	28.3096
5.0	1.31384	4.03357	6.9096	9.89275	12.9352	16.0107	19.1055	22.2126	25.3276	28.4483
10.0	1.42887	4.3058	7.22811	10.2003	13.2142	16.2594	19.327	22.4108	25.5064	28.6106
100.0	1.55525	4.66577	7.77637	10.8871	13.9981	17.1093	20.2208	23.3327	26.445	29.5577

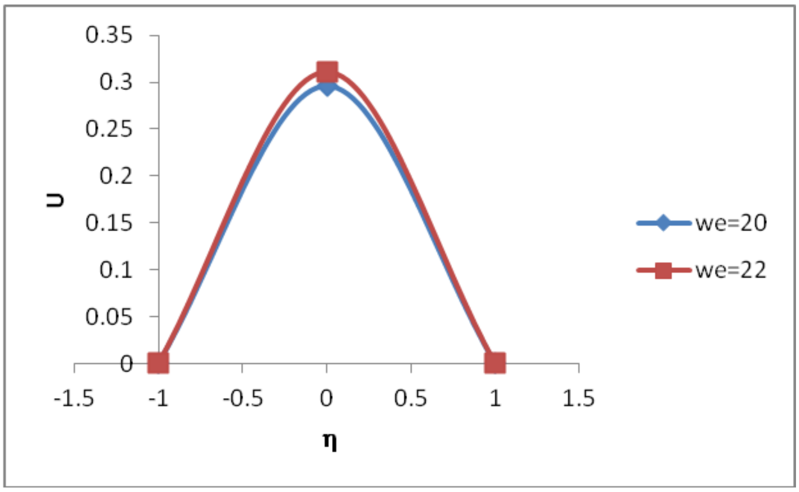


Figure 2. Velocity profiles of aerosols or different W_e

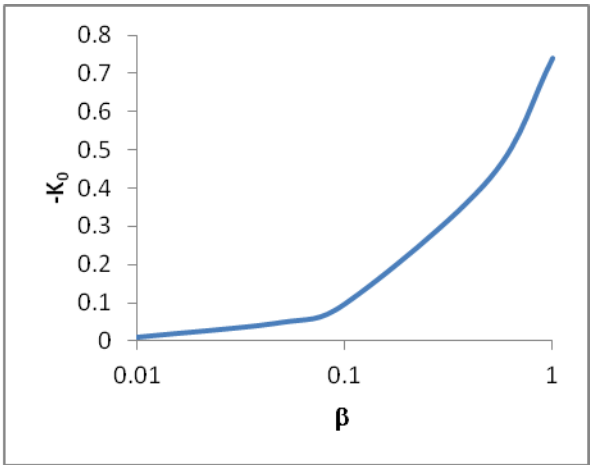


Figure 3. Plots of exchange coefficient versus reaction rate parameter β

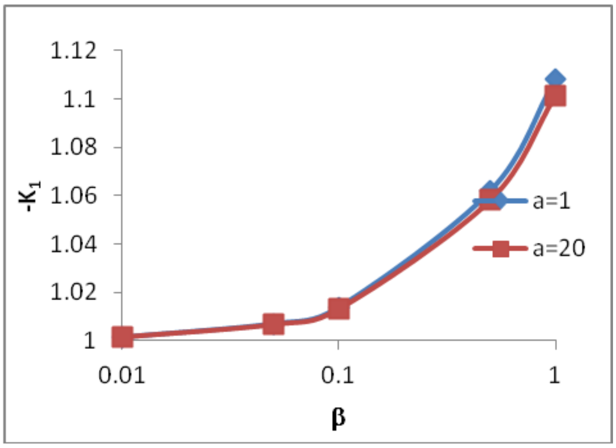


Figure 4. Plot of convective coefficient – K, with wall reaction parameter β for different values of a.

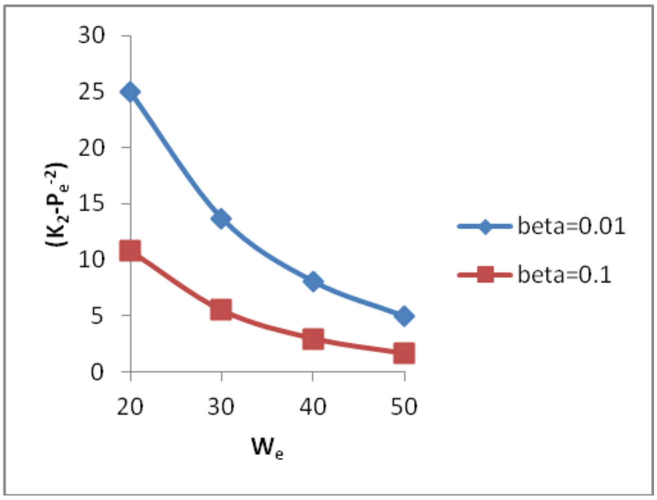
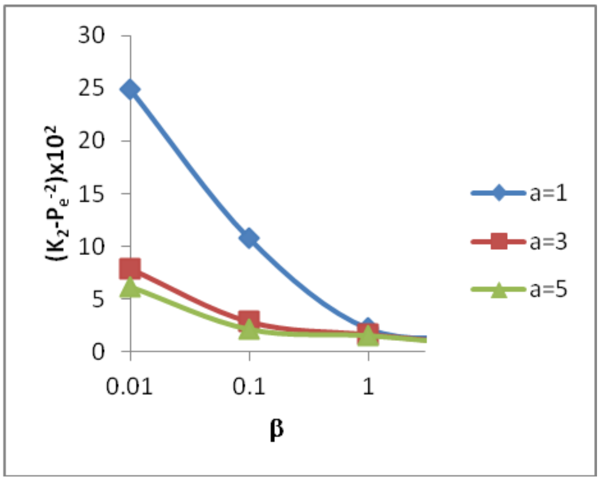
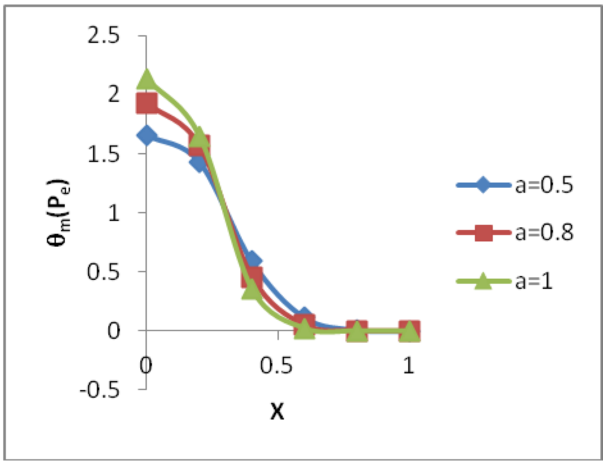


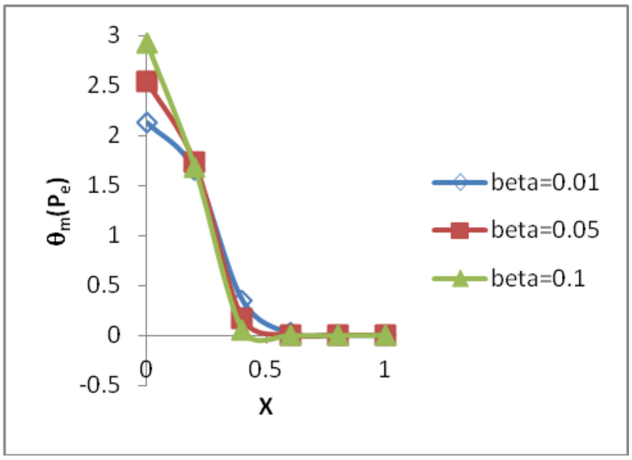
Figure 5. Effect of electric number W_e on depersion coefficient K_2 for different values of β



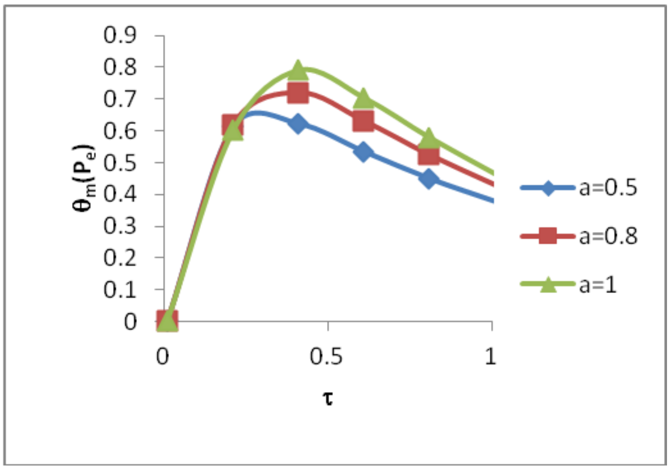
Figures 6. Plots of scaled dispersion coefficient $K_2(\tau)$- P_e^{-2} against β for different values of a.



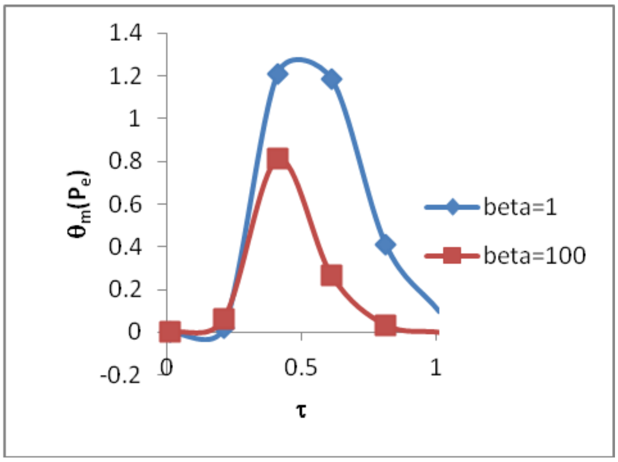
Figures 7. Mean concentration (θ_m) varying along axial distance x for different values of a and for fixed $\tau = 0.06$, $\beta=0.01$



Figures 8. Mean concentration (θ_m) varying along axial distance x for different values of β and for fixed $\tau = 0.06$, a=1



Figures 9. Mean concentration (θ_m) varying along dimensionless time τ for different values of a at x=0.6, β=0.01



Figures 10. Mean concentration (θ_m) varying along dimensionless time τ for different values of β at x=0.6, a=1