# Transmission Line Fault Analysis using Bus Impedance Matrix Method

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Abstract -- The fault analysis is done for the three phase symmetrical fault and the unsymmetrical faults. The unsymmetrical faults include single line to ground, line to line and double line to ground fault. The method employed is bus impedance matrix which has certain advantages over thevenin's equivalent method. The advantage of this approach over conventional method is to make the analysis of three typical un-symmetrical faults, namely single-line-to-ground fault, line-to-line fault and double-line-to-ground fault more unified. So it is unnecessary to cumbersomely connect three sequence networks when calculating the fault voltages at each bus and fault currents flowing from one bus to its neighboring bus.

Keywords- Bus impedance matrix; fault analysis; fault impedance; thevenin's equivalent.

## I. INTRODUCTION

The steady state operating mode of a power system is balanced 3-phase ac. However due to sudden external or internal changes in the system, this condition is disrupted. When the insulation of the system fails at one or more points or a conducting object comes in contact with a live point, a short circuit or fault occurs. A fault involving all the three phases is known as symmetrical (balanced) fault while one involving only one or two phases is known as unsymmetrical fault. Majority of the faults are unsymmetrical. Fault calculations involve finding the voltage and current distribution throughout the system during the fault. It is important to determine the values of system voltages and currents during fault conditions so that the protective devices may be

set to detect the fault and isolate the faulty portion of the system.

## II. FAULTS IN A THREE PHASE SYSTEM

- 1. Symmetrical three-phase fault
- 2. Single line-to-ground fault (SLG)

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3. Line-to-line fault (LL)

4. Double line-to-ground fault (DLG)

The most common type of faults by far is the SLG fault, followed in frequency of occurrence by the LL fault, DLG fault, and three-phase fault.

Out of the above four faults, two are of the line-toground faults. Most of these occur as a result of insulator flashover during electrical storms. The balanced threephase fault is the rarest in occurrence and the least complex in so far as the fault current calculations are concerned. The other three unsymmetrical faults will require the knowledge and use of symmetrical components. Unsymmetrical faults cause unbalanced currents to flow in the system. The method of symmetrical components is a very powerful tool which makes the calculations of unsymmetrical faults almost as easy as the calculations of a three-phase fault.

To analyze un-symmetrical faults, one needs to develop positive-, negative-, and zero-sequence networks of the power system under study, based on which one further need to work out the impedance of three thevenin equivalent circuits as viewed from faulty point. Then the positive-, negative- and zero-sequence components of phase-a faulty-point-to-ground current can be calculated. To calculate three-phase currents flowing from one bus to its neighboring bus and three-phase voltages at each bus, one needs to connect three sequence networks uniquely for each type of fault. This may make circuit drawing very cumbersome. Furthermore by using the network with three sequence networks connected, it is impossible to appreciate the impedance matrix approach to calculate the sequence voltage at each bus when fault occurs.

To overcome these two drawbacks, paper [1] introduces a new approach to unify the analysis of three typical unsymmetrical faults, namely single-line-to-ground fault, line-to-line fault and double-line-to-ground fault. This new method allows the analysis of three typical un-symmetrical faults to share all steps except one. The only different step is how to calculate the positive-, negative-, and zerosequence components of phase-a-to-ground fault current at faulty point. It also makes impedance matrix approach more understandable when used to calculate the sequence voltages at each bus. All the above four faults (1, 2, 3, 4) are being solved using the bus impedance matrix.

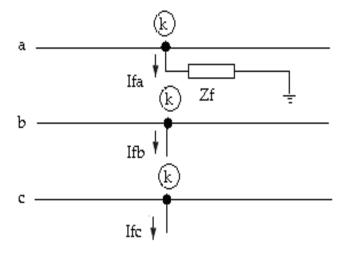


Fig. 1. Single line to ground fault

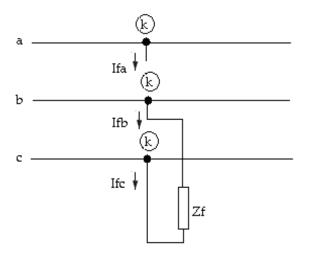


Fig. 2. Line to line fault

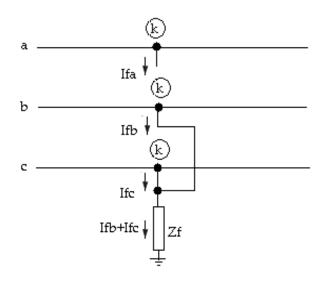


Fig. 3. Double line to ground fault

#### III. BUS IMPEDANCE MATRIX METHOD

We can work out a universal representation of all three typical un-symmetrical faults. This representation is valid with the imposition of different fault conditions for each typical un-symmetrical fault, such as for the single-line-to-ground fault, such as for the single-line-to-ground fault, the fault conditions being  $V_{ka}=Z_f I_{fa}$ ,  $I_{fb}=I_{fc}=0$ .

In the following formulation, per-unit system is adopted. Zero-sequence voltage at each bus contributed by equivalent current source is determined by

$$\begin{bmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & ... & Y_{1k}^{(0)} & ... & Y_{1n}^{(0)} \\ Y_{21}^{(0)} & Y_{22}^{(0)} & ... & Y_{2k}^{(0)} & ... & Y_{2n}^{(0)} \\ \vdots & \vdots & ... & \vdots & ... & \vdots \\ Y_{k1}^{(0)} & Y_{k2}^{(0)} & ... & Y_{kk}^{(0)} & ... & Y_{kn}^{(0)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{n1}^{(0)} & Y_{n2}^{(0)} & ... & Y_{nk}^{(0)} & ... & Y_{nn}^{(0)} \end{bmatrix} \begin{bmatrix} V_{1f}^{(0)} \\ V_{2f}^{(0)} \\ \vdots \\ V_{kf}^{(0)} \\ \vdots \\ V_{kf}^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I \\ I \\ I \\ 0 \end{bmatrix}$$

where

$$\mathbf{Y}^{(0)} = \begin{bmatrix} \mathbf{Y}_{11}^{(0)} & \mathbf{Y}_{12}^{(0)} & \dots & \mathbf{Y}_{1k}^{(0)} & \dots & \mathbf{Y}_{1n}^{(0)} \\ \mathbf{Y}_{21}^{(0)} & \mathbf{Y}_{22}^{(0)} & \dots & \mathbf{Y}_{2k}^{(0)} & \dots & \mathbf{Y}_{2n}^{(0)} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ \mathbf{Y}_{k1}^{(0)} & \mathbf{Y}_{k2}^{(0)} & \dots & \mathbf{Y}_{kk}^{(0)} & \dots & \mathbf{Y}_{kn}^{(0)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{Y}_{n1}^{(0)} & \mathbf{Y}_{n2}^{(0)} & \dots & \mathbf{Y}_{nk}^{(0)} & \dots & \mathbf{Y}_{nn}^{(0)} \end{bmatrix}$$

is the admittance matrix for the sub-transient or transient zero-sequence network.

Then  

$$\begin{bmatrix} V_{1f}^{(0)} \\ V_{2f}^{(0)} \\ \vdots \\ V_{kf}^{(0)} \\ \vdots \\ V_{nf}^{(0)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & \dots & Y_{1k}^{(0)} & \dots & Y_{1n}^{(0)} \\ Y_{21}^{(0)} & Y_{22}^{(0)} & \dots & Y_{2k}^{(0)} & \dots & Y_{2n}^{(0)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_{k1}^{(0)} & Y_{k2}^{(0)} & \dots & Y_{kk}^{(0)} & \dots & Y_{kn}^{(0)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{n1}^{(0)} & Y_{n2}^{(0)} & \dots & Y_{nk}^{(0)} & \dots & Y_{nn}^{(0)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_{fa}^{(0)} \\ \vdots \\ 0 \end{bmatrix}$$

$$= Z^{(0)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_{fa}^{(0)} \\ \vdots \\ 0 \end{bmatrix}$$

Where,

Z <sup>(0)</sup> =	$Z_{11}^{(0)}$	$Z_{12}^{(0)}$		$Z_{1k}^{\left(0\right)}$		$Z_{1n}^{(0)}$
	${ m Z}_{21}^{(0)}$	${ m Z}_{22}^{(0)}$		$Z_{2k}^{\left(0\right)}$		$Z_{2n}^{(0)}$
	:	:		:		:
	$\mathrm{Z}_{\mathrm{kl}}^{(0)}$	$Z_{k2}^{\left(0\right)}$		$Z^{(0)}_{\ kk}$		$Z_{kn}^{(0)}$
	:	:	:	:	:	:
	$Z_{n1}^{(0)}$	$Z_{n2}^{\left(0\right)}$		$Z_{nk}^{\left(0\right)}$		$Z_{nn}^{(0)}$

	$Z_{11}^{(0)}$	${ m Z}_{12}^{(0)}$		$Z_{1k}^{\left(0\right)}$		$Z_{1n}^{(0)}$
	${ m Z}_{21}^{(0)}$	${ m Z}_{22}^{(0)}$		$\mathbf{Z}_{2k}^{(0)}$		$\mathbf{Z}_{2n}^{(0)}$
=	:	:		:		:
	$Z_{k1}^{(0)}$	$\mathbf{Z}_{k2}^{(0)}$	••	$Z^{(0)}_{\ kk}$		$Z_{kn}^{\left(0\right)}$
	:	:	:	:	:	:
	$Z_{n1}^{(0)}$	$\mathrm{Z}_{\mathrm{n2}}^{(0)}$		$\mathrm{Z}_{\mathrm{nk}}^{(0)}$		$Z_{nn}^{(0)}$

So

$$\begin{bmatrix} \mathbf{V}_{1f}^{(0)} \\ \mathbf{V}_{2f}^{(0)} \\ \vdots \\ \mathbf{V}_{kf}^{(0)} \\ \vdots \\ \mathbf{V}_{nf}^{(0)} \end{bmatrix} = \begin{bmatrix} -\mathbf{Z}_{1k}^{(0)} \mathbf{I}_{fa}^{(0)} \\ -\mathbf{Z}_{2k}^{(0)} \mathbf{I}_{fa}^{(0)} \\ \vdots \\ -\mathbf{Z}_{kk}^{(0)} \mathbf{I}_{fa}^{(0)} \\ \vdots \\ -\mathbf{Z}_{nk}^{(0)} \mathbf{I}_{fa}^{(0)} \end{bmatrix}$$

In a similar way, positive-sequence voltage at each bus contributed by equivalent current source as is determined by

$Y_{11}^{(1)}$	$Y_{12}^{(1)}$	••	$\mathbf{Y}_{1k}^{(1)}$		$Y_{1n}^{(1)}$	$\begin{bmatrix} \Delta V_{1f}^{(1)} \\ \Delta V_{2f}^{(1)} \\ \vdots \end{bmatrix}$	$\begin{bmatrix} 0 \end{bmatrix}$
$Y_{21}^{(1)}$	$Y_{22}^{(1)}$		$Y_{2k}^{\left( l\right) }$		$\mathbf{Y}_{2n}^{(1)}$	$\Delta V_{2f}^{(1)}$	0
:	:	••	:		:	:	:
$Y_{k1}^{(1)}$	$Y_{k2}^{\left( l\right) }$	••	$Y_{kk}^{(l)}$		$\mathbf{Y}_{kn}^{(1)}$	$\Delta V^{(1)}_{ m kf}$ :	$=$ $\left  \begin{array}{c} \cdot \\ -\mathbf{I}_{\mathrm{fa}}^{(1)} \end{array} \right $
:	:	:	:	:	:	:	
$Y_{n1}^{(1)}$	$Y_{n2}^{\left( 1\right) }$		$Y_{nk}^{\left( 1\right) }$		$\mathbf{Y}_{nn}^{(1)}$	$\Delta V_{nf}^{(1)}$	

This gives

$$\begin{bmatrix} \Delta \mathbf{V}_{1f}^{(1)} \\ \Delta \mathbf{V}_{2f}^{(1)} \\ \vdots \\ \Delta \mathbf{V}_{kf}^{(1)} \\ \vdots \\ \Delta \mathbf{V}_{nf}^{(1)} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & \dots & Z_{1k}^{(1)} & \dots & Z_{1n}^{(1)} \\ Z_{21}^{(1)} & Z_{22}^{(1)} & \dots & Z_{2k}^{(1)} & \dots & Z_{2n}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ Z_{k1}^{(1)} & Z_{k2}^{(1)} & \dots & Z_{kk}^{(1)} & \dots & Z_{kn}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1}^{(1)} & Z_{n2}^{(1)} & \dots & Z_{nk}^{(1)} & \dots & Z_{nn}^{(1)} \end{bmatrix}$$

If pre-fault current is ignored, then the pre-fault voltage at each bus is the same and equal to that at fault bus k before fault occurs, which is assumed to be  $V_{\rm f}$ . So the positive sequence voltage at each bus when fault occurs can be written as follows.



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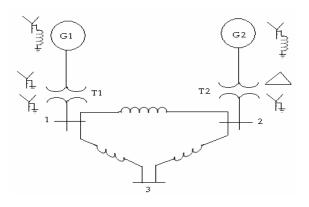


Fig. 4. Single line diagram 1

### IV. MATHEMATICAL ANALYSIS

#### A. Sequence impedance networks

Firstly let us obtain the sequence impedance networks. From the data given in table 4.1 the following positive, negative and zero sequence impedance networks are obtained in fig.4.6, 4.7 and 4.8 respectively.

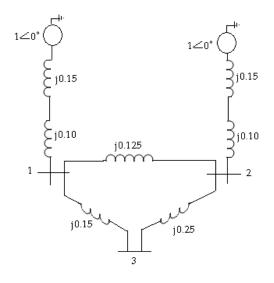
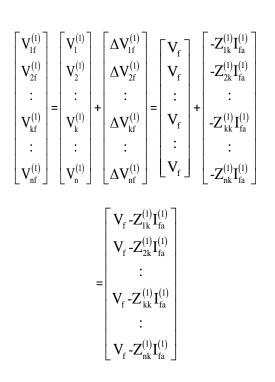


Fig. 5. Positive Sequence impedance network



In a similar way, the negative-sequence voltage at each bus can be computed by

$$Z^{(2)} = \begin{bmatrix} Z_{11}^{(2)} \\ V_{2f}^{(2)} \\ \vdots \\ V_{kf}^{(2)} \\ \vdots \\ V_{nf}^{(2)} \end{bmatrix} = \begin{bmatrix} -Z_{1k}^{(2)} I_{fa}^{(2)} \\ -Z_{2k}^{(2)} I_{fa}^{(2)} \\ \vdots \\ -Z_{kk}^{(2)} I_{fa}^{(2)} \end{bmatrix}$$

$$Z^{(2)} = \begin{bmatrix} Z_{11}^{(2)} & Z_{12}^{(2)} & \dots & Z_{1k}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & \dots & Z_{2k}^{(2)} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ Z_{k1}^{(2)} & Z_{k2}^{(2)} & \dots & Z_{kk}^{(2)} & \dots & Z_{kn}^{(2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1}^{(2)} & Z_{n2}^{(2)} & \dots & Z_{nk}^{(2)} & \dots & Z_{kn}^{(2)} \end{bmatrix}$$

TABLEI. POWER SYSTEM NETWORK PARAMETERS

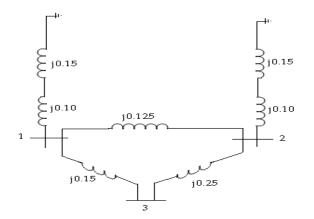


Fig. 6. Negative Sequence impedance network

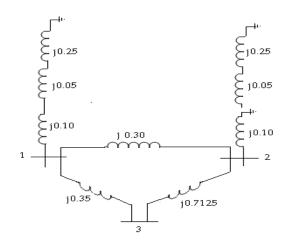


Fig. 7. Zero Sequence impedance network

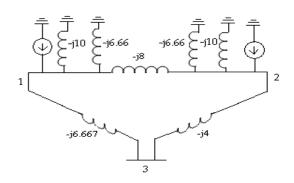


Fig. 8. Positive Sequence admittance network

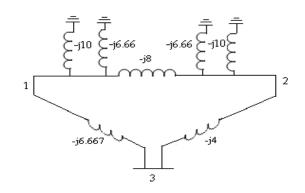
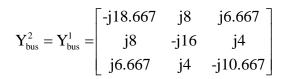


Fig. 9. Positive Sequence admittance network



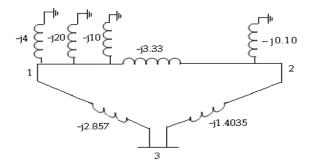


Fig. 10. Zero Sequence admittance network

	-j8.690	j3.3333	j2.8571	
$Y_{bus}^0 =$	j3.3333	-j14.7368	j1.4035	
	j2.8571	j1.4035	-j4.2606	

## B. IMPEDANCE MATRICES

The impedance matrices are obtained from the admittance matrices.

$$Z^{1}_{bus} = Z^{2}_{bus} = \begin{bmatrix} j0.1450 & j0.1050 & j0.1300 \\ j0.1050 & j0.1450 & j0.1200 \\ j0.1300 & j0.1200 & j0.2200 \end{bmatrix}$$

$$Z_{bus}^{0} = \begin{bmatrix} j0.1820 & j0.0545 & j0.1400 \\ j0.0545 & j0.0864 & j0.0650 \\ j0.1400 & j0.0650 & j0.3500 \end{bmatrix}$$

## C. Single Line To Ground Fault At Bus 3 Through A Fault Impedance Of J0.1

When single line to ground fault occurs, the sequence components of fault current at bus three are given by

$$I_{f3}^{1} = I_{f3}^{2} = I_{f3}^{0} = \frac{V_{k}(0)}{Z_{kk}^{1} + Z_{kk}^{2} + Z_{kk}^{0} + 3Z_{f}}$$
$$= \frac{V_{3}(0)}{Z_{33}^{1} + Z_{33}^{2} + Z_{33}^{0} + 3Z_{f}}$$
$$= \frac{1 \angle 0^{0}}{j0.22 + j0.22 + j0.35 + 3(j0.1)}$$
$$= \frac{1 \angle 0^{0}}{j1.09}$$
$$= -j0.9174 \text{ p.u.}$$

The fault current is

$$\begin{bmatrix} \mathbf{I}_{f3}^{a} \\ \mathbf{I}_{f3}^{b} \\ \mathbf{I}_{f3}^{c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{f3}^{0} \\ \mathbf{I}_{f3}^{0} \\ \mathbf{I}_{f3}^{0} \end{bmatrix}$$
$$= \begin{bmatrix} 3\mathbf{I}_{f3}^{0} \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 3(-j0.9174) \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} -j2.7522 \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 2.7523 \angle -90^{0} \\ 0 \end{bmatrix}$$

The symmetrical components of voltages during fault at buses 1, 2 and 3

# At bus 1

$$\begin{bmatrix} V_{f1}^{0} \\ V_{f1}^{1} \\ V_{f2}^{2} \end{bmatrix} = \begin{bmatrix} 0-Z_{13}^{0}I_{3}^{0} \\ V_{1}^{1}(0)-Z_{13}^{1}I_{3}^{1} \\ 0-Z_{13}^{2}I_{3}^{2} \end{bmatrix} = \begin{bmatrix} 0-j0.14(-j0.9174) \\ 1-j0.13(-j0.9174) \\ 0-j0.13(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.1284 \\ 0.8807 \\ -0.1193 \end{bmatrix}$$

## At bus2

$$\begin{bmatrix} V_{12}^{0} \\ V_{12}^{1} \\ V_{22}^{1} \end{bmatrix} = \begin{bmatrix} 0 - Z_{23}^{0} I_{3}^{0} \\ V_{2}^{1}(0) - Z_{23}^{1} I_{3}^{1} \\ 0 - Z_{23}^{2} I_{3}^{2} \end{bmatrix} = \begin{bmatrix} 0 - j0.065(-j0.9174) \\ 1 - j0.120(-j0.9174) \\ 0 - j0.120(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.0596 \\ 0.8899 \\ -0.1101 \end{bmatrix}$$

# At bus3

$$\begin{bmatrix} V_{r_3}^0 \\ V_{r_3}^1 \\ V_{r_3}^2 \end{bmatrix} = \begin{bmatrix} 0-Z_{23}^0 I_3^0 \\ V_3^1(0)-Z_{33}^1 I_3^1 \\ 0-Z_{33}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0-j0.35(-j0.9174) \\ 1-j0.22(-j0.9174) \\ 0-j0.22(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.3211 \\ 0.7982 \\ -0.2018 \end{bmatrix}$$

# The voltages during fault are

At bus 1

$$\begin{bmatrix} \mathbf{V}_{f1}^{a} \\ \mathbf{V}_{f1}^{b} \\ \mathbf{V}_{f1}^{c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{f1}^{0} \\ \mathbf{V}_{f1}^{1} \\ \mathbf{V}_{f1}^{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} -0.1284 \\ 0.8807 \\ -0.1193 \end{bmatrix}$$
$$= \begin{bmatrix} 0.633 \angle 0^{0} \\ 1.0046 \angle -120.45^{0} \\ 1.0046 \angle 120.45^{0} \end{bmatrix}$$

At bus 2

$$\begin{bmatrix} \mathbf{V}_{r2}^{a} \\ \mathbf{V}_{r2}^{b} \\ \mathbf{V}_{r2}^{c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{r2}^{0} \\ \mathbf{V}_{r2}^{1} \\ \mathbf{V}_{r2}^{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} -0.0596 \\ 0.8899 \\ -0.1101 \end{bmatrix}$$
$$= \begin{bmatrix} 0.7207 \angle 0^{0} \\ 0.9757 \angle -117.43^{0} \\ 0.9757 \angle +117.43^{0} \end{bmatrix}$$

At bus 3

$$\begin{bmatrix} V_{f3}^{a} \\ V_{f3}^{b} \\ V_{f3}^{c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} V_{f3}^{0} \\ V_{f3}^{1} \\ V_{f3}^{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} -0.3211 \\ 0.7982 \\ -0.2018 \end{bmatrix}$$

$$\begin{bmatrix} 0.2752 \angle 0^{0} \\ 1.0647 \angle -125.56^{0} \\ 1.0647 \angle +125.56^{0} \end{bmatrix}$$

## v. CONCLUSION

This paper presents a method to tackle typical unsymmetrical faults. It is found that the bus impedance matrix method involves comparatively less computations than the thevenin's equivalent method. The proposed approach has another advantage over traditional method that it is more intuitive when matrix approach is adopted to tackle a fault problem.

### VI. REFERENCES

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