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# Tracking Control of Piezoelectric Actuators using PID based Sliding Mode

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***Abstract*—Sliding mode control (SMC) is a nonlinear control technique that has excellent robustness to model uncertainties and disturbances. SMC can act as an ideal scheme for process control applications. The existing SMC, however, has the drawback in chattering control. To eliminate chattering, a new PID based sliding mode control (PIDSMC) is proposed. Three components of the proposed control system include a compensation for process nonlinearity, a linear feedback of state tracking errors, and a PID control of sliding surface function. The studies shows the PEA can be modeled as a linear dynamic system with matched uncertainties. This paper presents the development of proportional-integral-derivative-based (PID) sliding mode, in which the switching function is replaced by a PID regulator.**

***Index terms-PID control; Sliding mode control; Piezoelectric devices.***

## I. INTRODUCTION

Piezoelectric actuators (PEAs) have been widely used in the fields of micro- and nano-positioning due to their high displacement resolution (sub nanometer) and large actuating force (typically a few hundreds of N). The nonlinear effects of PEAs due to hysteresis and creep [1][3], and the distributed nature of their vibration dynamics [5]have shown to be able to degrade the closed loop tracking control performance and may lead to system instability[2].For improvement, the development of model-based controllers for compensation have drawn considerable attention.

PEAs utilize the inverse piezoelectric effect of piezoelectric materials to generate forces and displacements.By applying electric charges on a piezoelectric material ( by means of an electric voltage), a stress can be generated, resulting in an actuating force. This actuating force alongwith the external force acts on a piezoelectric material causes deformation of the piezoelectric material , i.e. generating a displacement.The models of PEA can be classified into two categories: phenomenon-based models and physics-based models.Thephenomenon-based models of PEA are dependent on the experimental results alone, in which nonlinear and linear effects are lumped. For example, the vibration dynamics and the hysteresis are combined to form a rate-dependent hysteresis model for PEAs. The physics-based models of PEA is related to the decoupling of linear and nonlinear effects by means individual sub-models of PEAs.

The PEA is modeled as a cascade of a nonlinear sub-model for the rate-independent hysteresis and a linear sub-model for the vibration dynamics. The sliding mode control is used as the closed-loop control schemes for PEAs. The merit of the sliding-mode based techniques is that they are insensitive to uncertainties in the input channels of the plant.These uncertainties are referred as matched uncertainties or matched unknown inputs. The PEA can be modeled as a linear dynamic system with a matched unknown input to account for hysteresis, creep, and model error. Many advanced feedback controllers were developed for PEA tracking control with the idea of minimizing or rejecting the effect of thedisturbances on the PEA output displacement.

The robust controllers were designed to minimize the effects of the disturbances based on a cost function. If the disturbances is treated as an unknown input applied to the PEA through the same channel as the known input, the effect of thedisturbances on the PEA performance can becompletely rejected by the use of sliding-mode-based (SM-based) controllers [6]. On the basis of SM control [8], various control techniques have been reported for tracking control of PEAs. The control actions generated by SM control and its modified forms are discontinuous due to the use of the sign function. Such discontinuity of the control action induces the chattering problem, which is highly undesirable[4].To remove the chattering problem, a modified sliding modecontrol with boundary layer (SMCBL) were developed [5]. Here, the discontinuous control actionare replaced with a continuous approximation of the switching function with a boundary layer. The system states cannot converge to the sliding surface hence causing a steady state error [3].One solution to overcome the steady-state error is to replace the discontinuous control action in the SM control with a continuous one that can be determined by a proportional-integral-derivative (PID) regulator. This results in the PID-based sliding mode (PIDSM)control [4].The integral component in the PIDSM control can eliminate the steady state error. In the proportional-integral derivative-based sliding mode controller (PIDSMC) the switching control term in the conventional sliding mode control is replaced with a PID regulator for tracking control of PEAs. The PIDSMC can reduce tracking error, chattering, and can eliminate the steady-state error [8]. One of the advantage of the PIDSM control is that the bounds on the matched uncertainties, as required by other control schemes (such as SM control and SMCBL) are not required. The PID regulator can generate


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theoretically infinite control signal to force the switching function to zero.

Most of the sliding-mode-based schemes for the PEA control need the feedback of its states to close the system[9]. All of the system states are not measurable, so state estimators or observers are required. Different methods to estimate the states can be classified into two categories: nonmodel based and model based. Nonmodel-based filters/differentiators, such as the *α–β* filterand the sliding mode differentiator can generate large phase lags if the desired level of noise suppression is enforced or can generate excessive chattering in noisy systems. On comparison to non model-based filters/differentiators, model-based observers, e.g., the Luenberger observer, can generate more accurate estimations. Presence of uncertainties can degrade the performance of model-based observers. So the system uncertainties can be treated as a lumped unknown input to the system model. A type of model-based observers called as unknown input observers (UIOs) can be used to estimate the system states. According to the observer matching condition the rank of the product of the output matrix and the unknown input matrix in the state space model of the system must be equal to that of the unknown input matrix [10].

The objective of this paper are to develop a PID based sliding mode controller(PIDSMC). The organization of the rest of this paper can be summarized as follows. The modelling of PEAare explained in Section II. The PIDSMC for PEA-driven system is developed in Section III. Simulation results are developed in Section IV.

## II. PEA MODEL

The modeling and control of PEAs has proven to be a challenging task. Fig.1 shows the schematic of a PEA with the end-effector is connected to the base through flexure hinges anddriven by a piezoelectric element under an input voltage of $u(t)$, $f_e(t)$ is the external load applied to the end-effector and $y(t)$ the displacement of the end-effector or the system output.The PEA is represented as a physics-based model. The linear and nonlinear effects of the PEA are decoupled by means of individual sub-models that are connected in cascade.

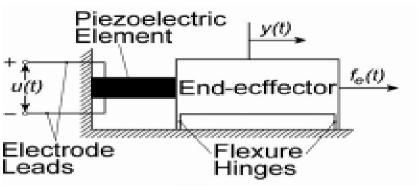


Fig.1.Schematic of a PEA

The block diagram of PEA model is shown in Fig. 2. In this, the blocks of H, V and $F_c$ represents the PEA hysteresis, vibration dynamics and creep respectively; *f(t)* and $y_0(t)$ represents represent the internal actuation force and the output displacement of the end-effector without creep, respectively [2].

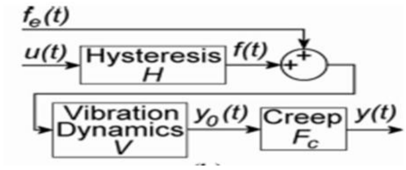


Fig.2.Physics-based model of PEA

The block $H$ represents the nonlinear hysteretic relationship between the input voltage, *u(t)* and the internal actuating force, *f(t)*. These hysteresis is the dominant form of PEA nonlinearity [4] and can be represented by means of rate-independent hysteresis models, such as the classical Preisach hysteresis model [1][5],or the differential-equation-based model [5]. For SM-based controllers, the effect of hysteresis is treated as a matched unknown input to the block $V$ , hence the accurate representation of hysteresis is not needed. The block $V$ represents the vibration dynamicsrelating the internal actuating force, *f(t)*and the external force, $f_e(t)$, to the end-effector displacement without considering the creep.The vibration dynamics of a PEA can be approximated by a linear combination of several second-order systems or one second-order system if the mass of the end-effector driven by the piezoelectric element is much larger than that of the piezoelectric element itself. The block $F_c$in Fig.2represents the creep, which can be either linear [1]or nonlinear [7]. Here, a linear sub-model is assumed to be used for $F_c$and then the blocks of $F_c$ and $V$ are swapped withoutchanging the output displacement, *y(t)*. If the second-order system is used for the block $V$ and the approximation error, along with the effects of $f_e(t)$, $H$ , and $F_c$ , are lumped together as the matched unknown input to the block $V$ , one has

$$\ddot{y}(t)+2\zeta\omega_n\dot{y}(t)+\omega_n^2 y(t)=\omega_n^2 K_1[f(t)+f_e(t)+\varepsilon_0(t)] \quad (1)$$

where $\xi$, $\omega_n$,and $K_1$are the damping ratio, the natural frequency, and the steady state gain of thesecond-order system, respectively. The input to $V$ is represented by $Ku(t)+K\varepsilon(t)$ , where $K$ is a known nominal gain and $\varepsilon(t)$ is an unknown input added to $u(t)$. The output induced by $\varepsilon(t)$ accounts for effects such as hysteresis, creep, the external loads, and the error induced by approximating $V$ witha second-order system . The state space representation for the PEA is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\xi\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ K\omega_n^2\end{bmatrix}(u+\varepsilon)$$

$$\dot{X}=AX+Bu+B\varepsilon$$

$$y=[1 \quad 0][x_1 \quad x_2]^T=CX \quad (2)$$

This equation is referred to as the nominal model of the PEA if $\varepsilon(t)$=0. The states of the PEA model are $x_1$and $x_2$. These states represents the displacement and the velocity of the end-effector.The model parameters were identified and given by $\xi$ = 0.82, $\omega_n$= 5450 rad/s, and $K$ = 0.142$\mu$m.

## III. PID-BASED SLIDING MODEL CONTROL (PIDSMC)


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The PIDSM controller is modified from a SM controller by replacing the switching control action with a continuous one generated by a PID regulator. The PIDSM tracking controller developed for the PEA acts as a state tracking controller that forces the state vector (consisting of the output displacement and velocity) of the PEA to track the desired or reference state vector. The development of such a controller is shown as follows.

$$W = [w_1 \quad w_2]^T = \begin{bmatrix} w & \dot{w} \end{bmatrix}^T \tag{3}$$

where *w* be the desired output displacement with the second-order derivative existing and *W* be the desired or reference state vector. The state tracking error vector is the difference between the state vector of the PEA ,*X*, and and the desired state vector, *W*.

$$e = X - W \tag{4}$$

$$e = [e_1 \quad e_2]^T = \begin{bmatrix} x_1 - w & x_2 - \dot{w} \end{bmatrix}^T \tag{5}$$

The error system can be defined as

$$\dot{e} = \dot{X} - \dot{W}$$

$$= Ae + AW + Bu + B\varepsilon - \dot{W} \tag{6}$$

The objective of control is to find the control action u(t). Thus the state vector e of the error system in Eq.(6) can be brought to the origin,i.e. $e_1 = 0$ and $e_2 = 0$.The terms of $AW$ and $-\dot{W}$ in Eq. (6)are known, hence their effects one can be compensated by a part of *u*,

$$u = \left(B^T B\right)^{-1} B^T (\dot{W} - AW) + u_{SM} \tag{7}$$

$u_{SM}$ is the control action generated by a PIDSM regulator. On substitution of Eq.(7) into (6) and making use of Eq.(2) yields

$$\dot{e} = Ae + Bu_{SM} + B\varepsilon$$

$$\begin{bmatrix} e_2 \\ A_{21}e_1 + A_{22}e_2 + B_2 u_{SM} + B_2\varepsilon \end{bmatrix} \tag{8}$$

where $A_{21} = -\omega_n^2$, $A_{22}$= -2 $\xi\omega_n$ and$B_2 = K\omega_n^2$. Inorder to determine $u_{SM}$ the sliding surface should be defined. The switching function is made to be zero. This switching function is a linear combination of the states of the error system.

$$q = [m \quad 1][e_1 \quad e_2]^T \tag{9}$$

where m is the parameter to characterize the sliding surface. It meets the following equation

$$me_1 + e_2 = me + \dot{e} = 0 \tag{10}$$

Hence the motion of the system Eq.(8) on the sliding surface Eq.(10) is known as the sliding motion.*m* >0 is needed to ensure the stability of the sliding motion.

$$\dot{q} = me_2 + A_{21}e_1 + A_{22}e_2 + B_2 u_{SM} + B_2\varepsilon \tag{11}$$

In this equation $B_2\varepsilon$ is the lumped uncertainty, and the other terms of $me_2$, $A_{21}e_1$ and $A_{22}e_2$ are estimated with the state estimator.$u_{SM}$can be divided into two parts,i.e.

$$u_{SM} = -B_2^{-1}(me_2 + A_{21}e_1 + A_{22}e_2) + u_3 \tag{12}$$

where $-B_2^{-1}(me_2 + A_{21}e_1 + A_{22}e_2)$ is the control part that compensates for the measurable terms and maintains the states of the system. $u_3$compensates for the lumped uncertainty, $B_2\varepsilon$. In sliding mode control,

$$u_3 = -\eta \,\mathrm{sgn}(q) \tag{13}$$

where η is a positive number larger or equal to the bound of ε. The discontinuity of $u_3$ causes chattering in the control of PEAs. To eliminate chattering and steady state error problems in the PEA displacement one of the solution is to use PIDSMC, in which $u_3$ takes the form of a PID component as,

$$u_3 = -(Pq + I\int_0^t q\,dt + D_c\,\dot{q}) \tag{14}$$

where $P \geq 0$ , $I \geq 0$ , and $D_c \geq 0$ are the PID parameters.

## IV.SIMULATIONS

Tracking control of PEAs is a challenging task. A PID controller was simulated for PEA tracking.

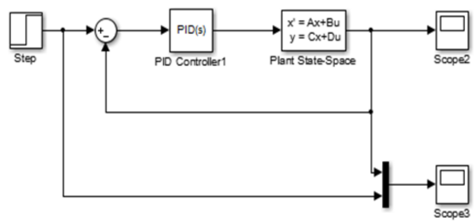


Fig.3. Simulation using a PID controller


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The step input given is $5*10^{-6}$m. The result for this simulation is obtained in Fig.4.

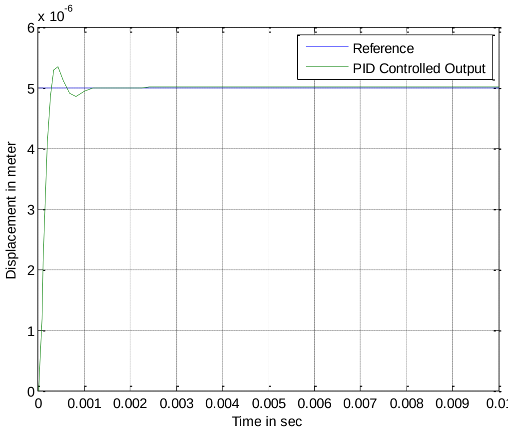


Fig.4. Result for tracking control using PID control

The actuator can generate motion in a range of 12 µm with a resolution of 0.7 nm.The displacements presented below hasa sampling interval $\Delta T$ = 0.00005 s and in a time period of 8seconds.The parameters $\xi = 0.82$, $\omega_n$ = 5450 rad/s, and K = 0.142 µm.

The nominal gain $K$was calculated by using K=$y_{max}$/ $u_{max}$ ,where $y_{max}$is the measured PEA displacement upon the application of the voltage with a maximum allowable amplitude $u_{max}$ . This leads to $K$ = 0.1347µm V.The PIDSM controller parameters were $m$ = 2200, $P$ =0.004, $I$ =0.06, $D_c = 1\times10^{-8}$ .

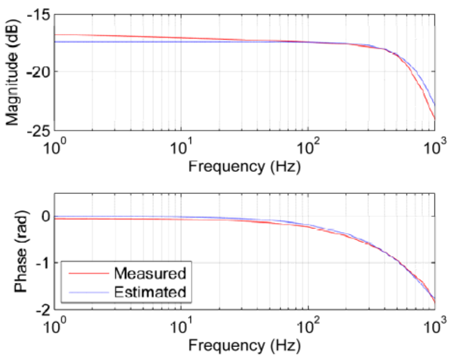


Fig. 5. Comparison of PEA frequency responses

The above Fig.5 illustrates the effectiveness of PEA model, the frequency responses of the measurements and model predictions. A simulation to examine the sliding mode behavior of the PIDSM controller without the presence of any state estimators can be performed.

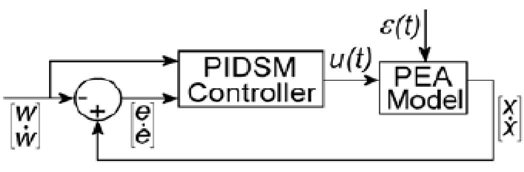


Fig.6 Block diagram of the system for simulation

For PEA modeling various parameters were identified. The initial conditions of $X_0 = [x_{10} \quad x_{20}]^T = [5 \quad 0]^T$ with ε(t)=0 can be used for the PEA model.

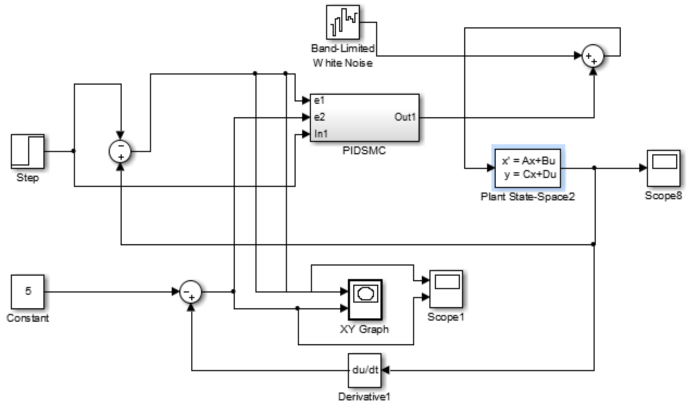


Fig.7. Simulation block for[5 0] initial conditions

The simulation was performed for a period of 1second.The PID values are given for the control purpose.

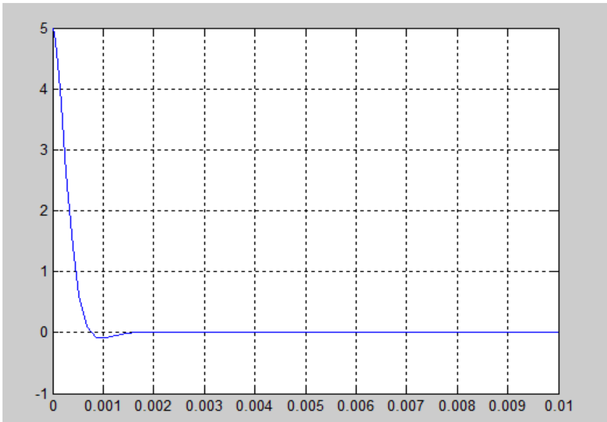

Fig. 8.Result for tracking of[ 5 0] initial conditions

For hysteresis compensation the value of $m$ was set to $m$ = 22000. Different values were tuned for the best performance when tracking $w(t) = 5\sin(2\pi \cdot 10t) + 5$ with $P$ =0.0004, $I$ =1, $D_c = 1\times10^{-8}$ .

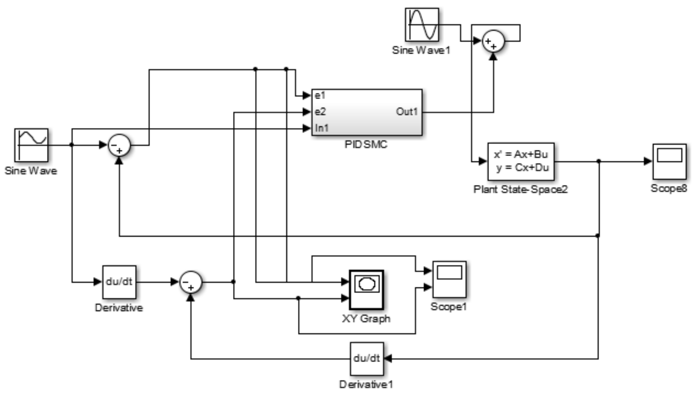


Fig.9. Simulation for hysteresis compensation


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The simulation is done to track a series of sine waves with a frequency of 150Hz.

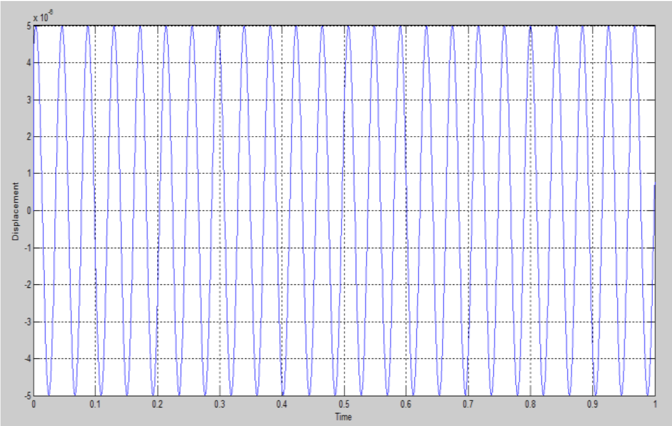


Fig.10. Results of the measured displacements

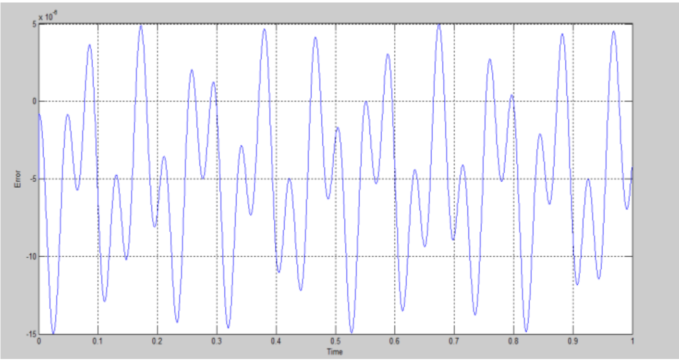


Fig. 11. Tracking Error using PIDSMC

The results of simulation to track a series of sine wave is represented in Fig.10 and Fig.11. The measured displacement tries to track the desired displacement with the tracking error as shown.

## V. CONCLUSION

Tracking control of PEAs has been proven to be a challenging task, due to the involvement of the PEA nonlinear properties such as hysteresis, creep and dynamics. A number of control schemes based on the state feedback have been developed for improved performance. This paper presents the design and development of PIDSMC for the PEA tracking control.

The existing SM-based control methods shows promising for use in the PEA tracking control due to their capability of rejecting matched nonlinearities. The problems of chattering and steady state error occurs using ideal SM control thusdegrading the tracking performances. So a PIDSMCis used for better PEA tracking.

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