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Tracking Control of Dynamic Nonlinear Systems via Improved Adaptive Fuzzy Control

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***Abstract*—In this paper, an Adaptive Fuzzy combined with Model Adaptive Reference System (MARS) is proposed to a single input-single output nonlinear system. The main goals of this paper are estimation and control of second order nonlinear system. Firstly, adaptive fuzzy system is applied to design the estimator and the controller. Next, a performance of system is improved by adjusting parameters of the controller via MARS. Finally, two examples are presented to illustrate of the proposed methods. By applying Lyapunov stability theory the adaptive law that is derived in this study is robust and convergent quickly. The simulation results and analysis show that the proposed method have better than Adaptive Fuzzy in the sense of robustness against disturbance.**

Keywords—Adaptive Fuzzy System; Model Reference Adaptive Systems (MARS); Coupled Tank Liquid Level; Robot Arm.

I. INTRODUCTION

The model of object is usually obtained by identification methods, then we use this model to design controller. If the characteristic of an object is not changing in all working process, the closed loop system with this controller is good for demanded designing. However, the parameters of object are unpredictably time variant in practice. Consequently, we need to have an online identification method based on adaptive algorithm which can be adapted by the changing parameters. To do this, the neural network is used [1, 2] to identify the dynamic systems with the back propagation algorithm and found on adaptive fuzzy control which is proposed by Li-Xin Wang [3, 4].

The nonlinear systems has attracted widespread attention in the recent decades [5, 6]. There are many controlled methods derived from linear controller as gain scheduling, Jacobi matrix … [7]. However, a question is that of when there exists a global change of coordinates, a diffeomorphism, in the state space that carries the nonlinear system into linear system. Krener [8] showed the importance of the Lie algebra of a vector fields associated with the system in studying such a question and gave an answer to this problem. D. Cheng [9] illustrated that we always find the coordinates transformation in order to transform a given nonlinear system with outputs into a controllable and observable linear one.

With the nonlinear system [5, 6, and 9], K. Khalil use the exact feedback linearization approach to define a control law with assumption that the modeled object is known clearly and states are measured. If we don't know exactly parameters of object, two trends will appear. The first trend consists in introducing a direct adaptive fuzzy control [3, 4] based on Lyapunov theory. The second trend, with which we are concerned, is carried out in controller design based on universal approximation [3, 10], the indirect adaptive fuzzy control. On the other hand, the controller in [3, 4, and 5] guarantees still the stable closed loop system, but it does not concentrate to performance of closed loop system, for example: the settling time, overshoot and error between the output signal and reference…Therefore, to improve performance of system, concretely, the redue error between reference trajectory and output trajectory, and rise the settling time of transient response.. Parameters of controller will be adjusted by adaptive law found on model adaptive reference system (MARS).

The remainder of this paper is organized as follows. In Section II, we provide the exact linearization of nonlinear. Section III estimation and control based on adaptive fuzzy mode control. Section IV then consider the improved adaptive fuzzy mode control. Simulation results are given in Section V and Section VI concludes this work.

II. EXACT LINEARIZATION OF NONLINEAR WITH OUTPUT

Consider the SISO nonlinear system given as:

$$\begin{cases} \dfrac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u & \boldsymbol{x} = (x_1\ x_2\ \dots\ x_n)^T \in \boldsymbol{R}^n \\ y = g(\boldsymbol{x}) & u, y, g(\boldsymbol{x}) \in \boldsymbol{R};\ \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{h}(\boldsymbol{x}) \in \boldsymbol{R}^n \end{cases} \quad (1)$$

We always find the coordinates transformation in order to transform the nonlinear system into linear system.

Theorem 1: Consider the SISO nonlinear system (1) with r is the characteristic number at $\boldsymbol{x}$. We assume $r=n$ in the $\mathcal{U}$ space, under the diffeomorphism:

$$\boldsymbol{z} = \boldsymbol{m}(\boldsymbol{x}) = (\boldsymbol{g}(\boldsymbol{x}),\ L_f \boldsymbol{g}(\boldsymbol{x}), \dots,\ L_f^{n-1}\boldsymbol{g}(\boldsymbol{x}))^T \quad (2)$$

the nonlinear system will be transformed into linear system:

$$\boldsymbol{z} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{b}w \tag{3}$$

where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \boldsymbol{b} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}; w = F(\boldsymbol{x}) + G(\boldsymbol{x}).u$$

Proof (see Appendix)

III. ESTIMATION AND CONTROL VIA ADAPTIVE FUZZY MODE CONTROL (AFC)

Without loss of generality, we consider the dynamic nonlinear has two state variables linearized by *Theorem 1* and given a structure as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = F(\boldsymbol{x}) + G(\boldsymbol{x})u \\ y = x_1 \end{cases} \tag{4}$$

where $\boldsymbol{x} = (x_1 \quad x_2)^T \in \boldsymbol{R}^2$; $u, y \in \boldsymbol{R}$; $F(\boldsymbol{x}) \in \boldsymbol{R}^2$; $G(\boldsymbol{x}) \in \boldsymbol{R}^2$ and $F(\boldsymbol{x}); G(\boldsymbol{x}) \in \boldsymbol{C}^\infty$ are smooth functions. The control target of system (4) is stable tracking a prior trajectory $y_m(t)$, called reference trajectory, as means:

$$\lim_{t \to \infty} e(t) = 0, \quad |e(t)| \text{ is finite} \tag{5}$$

where $e(t) = y_m(t) - y(t)$ is tracking error.

From (4), we express:

$$\begin{cases} \dot{y} = \dot{x}_1 = x_2 \\ \ddot{y} = \dot{x}_2 = F(\boldsymbol{x}) + G(\boldsymbol{x})u \\ y = x_1 \end{cases} \tag{6}$$

We assume that the parameters of system (6) are known exactly. Then, the state feedback controller shown as:

$$u = \frac{1}{G(\boldsymbol{x})}(\ddot{y}_m - F(\boldsymbol{x}) + k_p e + k_d \dot{e}) \tag{7}$$

where k_p and k_d are coefficients of Hurwitz polynomial:

$$h(s) = s^2 + k_d s + k_p \tag{8}$$

This controller will be made stable closed loop system as means (4).

In formula (7), the controller depends on functions $F(\boldsymbol{x}), G(\boldsymbol{x})$. If they are unknown, the controller cannot be implemented. Consequently, we need to estimate an online function $F(\boldsymbol{x}), G(\boldsymbol{x})$ based on adaptive fuzzy mode algorithm. In the [3], consider the MISO fuzzy logic control system has two inputs $\boldsymbol{x} = (x_1 \ x_2)^T \in \mathcal{U} \in \boldsymbol{R}^2$ and an output $\boldsymbol{b} \in \mathcal{U}$. This fuzzy logic system with the center-average defuzzifier, product inference, and singleton fuzzifier is of the following form:

$$b(\boldsymbol{x}) = \frac{\sum_{l=1}^{M} b^{-l} \left(\prod_{j=1}^{2} \mu_{A_j^l}(x_j) \right)}{\sum_{l=1}^{M} \left(\prod_{j=1}^{2} \mu_{A_j^l}(x_j) \right)} \tag{9}$$

where b^{-l} is the point at which output membership function μ_{B^l} achieves its maximum value and $\mu_{A_j^l}$ are input membership functions. If we fix the $\mu_{A_j^l}(x_j)$ and view the b^{-l} as adjustable parameters, then (5) can be written as:

$$b(\boldsymbol{x}) = \boldsymbol{\theta}^T \boldsymbol{\xi}(\boldsymbol{x}) \tag{10}$$

where $\boldsymbol{\theta}^T = (b^{-1}, \dots, b^{-M})$ is a parameter vector of the output membership functions, and $\boldsymbol{\xi}(\boldsymbol{x}) = (\xi^1(x), \dots, \xi^M(x))^T$ is a regressive vector with the regression $\xi^l(x)$ defined as:

$$\xi(\boldsymbol{x}) = \frac{\prod_{j=1}^{2} \mu_{A_{ij}^j}(x_j)}{\sum_{l=1}^{M} \left(\prod_{j=1}^{2} \mu_{A_{ij}^j}(x_j) \right)} \tag{11}$$

Assume that the functions $F(\boldsymbol{x})$ and $G(\boldsymbol{x})$ describing the system dynamics are unknown, we can be estimated the function $F(\boldsymbol{x})$ and $G(\boldsymbol{x})$ by fuzzy logic system with the adjusted parameters of output membership functions based on adaptive law.

$$\widehat{F}(\boldsymbol{x}) = \boldsymbol{\theta}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}); \ \widehat{G}(\boldsymbol{x}) = \boldsymbol{\theta}_G^T \boldsymbol{\xi}_G(\boldsymbol{x}) \tag{12}$$

where the parameter vector $\boldsymbol{\theta}_F^T; \boldsymbol{\theta}_G^T$ are updated online so that the approximate error between $F(\boldsymbol{x}); G(\boldsymbol{x})$ and $\widehat{F}(\boldsymbol{x}); \widehat{G}(\boldsymbol{x})$ is minimal. Define the optimal parameter vector as:

$$\begin{aligned} \boldsymbol{\theta}_F^* &= \arg\min_{\theta_F}(\sup_x | \boldsymbol{\theta}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) - F(\boldsymbol{x})) \\ \boldsymbol{\theta}_G^* &= \arg\min_{\theta_F}(\sup_x | \boldsymbol{\theta}_G^T \boldsymbol{\xi}_G(\boldsymbol{x}) - G(\boldsymbol{x})) \end{aligned} \tag{13}$$

If $\boldsymbol{\theta}_F \to \boldsymbol{\theta}_F^*; \boldsymbol{\theta}_G \to \boldsymbol{\theta}_G^*$ then $\widehat{F}(\boldsymbol{x}) \to F(\boldsymbol{x})$, $\widehat{G}(\boldsymbol{x}) \to G(\boldsymbol{x})$ that means the fuzzy system (9) can approximate smooth nonlinear functions with the arbitrary small error if the number of fuzzy rules is large enough [10]. The state feedback controller (7) can be expressed:

$$u^* = \frac{1}{\widehat{G}(\boldsymbol{x})}\left(-\widehat{F}(\boldsymbol{x}) + \ddot{y}_m + k_p e + k_d \dot{e}\right) \tag{14}$$

From (10), we see that if the parameters of systems have change by the time, the functions $F(\boldsymbol{x}), G(\boldsymbol{x})$ will change and the control signal u will be suitable adjusted for alterative parameters of object.

Theorem 2: Consider the nonlinear system (4), the state feedback controller (10) and $\widehat{F}(\boldsymbol{x}), \widehat{G}(\boldsymbol{x})$ in (12) are applied with the $\boldsymbol{\theta}_F^T$ and $\boldsymbol{\theta}_G^T$ updated by adaptive law:

$$\dot{\boldsymbol{\theta}}_F = -Q_F^{-1} \boldsymbol{\xi}_F(\boldsymbol{x}) s_f(e); \ \dot{\boldsymbol{\theta}}_G = -Q_G^{-1} \boldsymbol{\xi}_G(\boldsymbol{x}) s_f(e) u$$

The closed loop system will be asymptotic stable.

Proof:

The second derivative of the output error is expressed:

$$\begin{aligned} \ddot{y} &= F(\boldsymbol{x}) + G(\boldsymbol{x})u \\ &= -\left(\widehat{F}(\boldsymbol{x}) - F(\boldsymbol{x})\right) - \left(\widehat{G}(\boldsymbol{x}) - G(\boldsymbol{x})\right)u + \widehat{F}(\boldsymbol{x}) + \widehat{G}(\boldsymbol{x})u \\ \ddot{y} &= \ddot{y}_m + k_p e + k_d \dot{e} - \left(\widehat{F}(\boldsymbol{x}) - F(\boldsymbol{x})\right) - \left(\widehat{G}(\boldsymbol{x}) - G(\boldsymbol{x})\right)u \\ \ddot{e} + k_p e + k_d \dot{e} &= \left(\boldsymbol{\theta}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) - \boldsymbol{\theta}_F^{*T} \boldsymbol{\xi}_F(\boldsymbol{x})\right) + \left(\boldsymbol{\theta}_G^T \boldsymbol{\xi}_G(\boldsymbol{x}) - \boldsymbol{\theta}_G^{*T} \boldsymbol{\xi}_G(\boldsymbol{x})\right)u \\ s_f &= \left(\boldsymbol{\theta}_F^T - \boldsymbol{\theta}_F^{*T}\right)\boldsymbol{\xi}_F(\boldsymbol{x}) + \left(\boldsymbol{\theta}_G^T - \boldsymbol{\theta}_G^{*T}\right)\boldsymbol{\xi}_G(\boldsymbol{x})u \\ &= \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{\xi}_G(\boldsymbol{x})u \end{aligned}$$

$\tilde{\theta}_F^T = \left(\theta_F^T - \theta_F^{*T}\right)$, $\tilde{\theta}_G^T = \left(\theta_G^T - \theta_G^{*T}\right)$ are the parameter error

Consider the Lyapunov candidate function:

$$V = \frac{1}{2}s_f^2 + \frac{1}{2}\tilde{\theta}_F^T \boldsymbol{Q}_F \tilde{\theta}_F^T + \frac{1}{2}\tilde{\theta}_G^T \boldsymbol{Q}_G \tilde{\theta}_G^T \quad ; \boldsymbol{Q}_F > 0; \boldsymbol{Q}_G > 0$$

where $\boldsymbol{Q}_i \in \boldsymbol{R}^{d\times d} (d = \dim\theta_i)$ is a positive definite matrix. Take the derivative of V with respect to time and notice that $\dot{\tilde{\boldsymbol{\theta}}}_i = \dot{\boldsymbol{\theta}}_i$, we have:

$$\dot{V} = s_f \dot{s}_f + \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G$$
$$\dot{V} = s_f\left(\tilde{\boldsymbol{\theta}}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{\xi}_G(\boldsymbol{x})u\right) + \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G$$
$$\dot{V} = \tilde{\boldsymbol{\theta}}_F^T\left(s_f \boldsymbol{\xi}_F(\boldsymbol{x}) + \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F\right) + \tilde{\boldsymbol{\theta}}_G^T\left(s_f \boldsymbol{\xi}_G(\boldsymbol{x})u + \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G\right) \le 0 \quad \square$$

Choose the parameters update law as follow:

$$\dot{\boldsymbol{\theta}}_F = -\boldsymbol{Q}_F^{-1} s_f \boldsymbol{\xi}_F(\boldsymbol{x});\ \dot{\boldsymbol{\theta}}_G = -\boldsymbol{Q}_G^{-1} s_f \boldsymbol{\xi}_G(\boldsymbol{x})u$$

with the error surface is defined:

$$s_f(e) = k_1 e + k_2 \frac{de}{dt} \qquad (15)$$

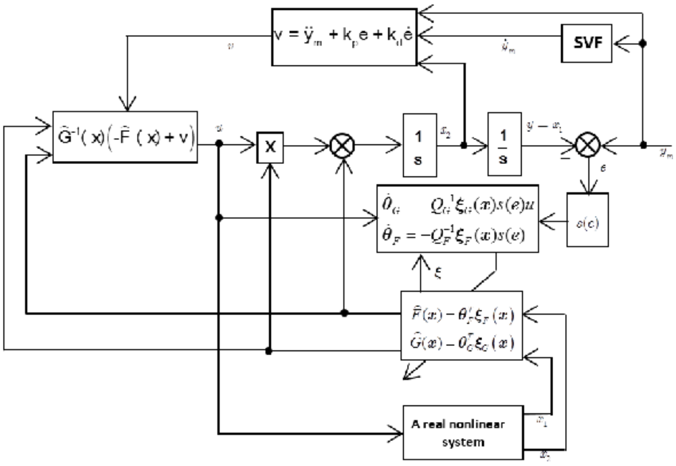

Fig. 1. The overall scheme of identification and control via adaptive fuzzy system (AFC)

where SVF is state variable filter. The derivative of the error can be created by using SVF. The parameters of this state variable filter are chosen in such as the parameters of the reference model.

The following steps to design the estimation and control via adaptive fuzzy system:

Step1: The regressive vector $\xi^l(x)$ as:

$$\xi(\boldsymbol{x}) = \frac{\prod_{j=1}^{2}\mu_{A_{ij}^l}(x_j)}{\sum_{l=1}^{M}\left(\prod_{j=1}^{2}\mu_{A_{ij}^l}(x_j)\right)} \qquad (16)$$

Step 2: The adaptive law:

$$\dot{\boldsymbol{\theta}}_F = -\boldsymbol{Q}_F^{-1} s_f \boldsymbol{\xi}_F(\boldsymbol{x});\ \dot{\boldsymbol{\theta}}_G = -\boldsymbol{Q}_G^{-1} s_f \boldsymbol{\xi}_G(\boldsymbol{x})u \qquad (17)$$

Step 3: Estimation the function:

$$\widehat{F}(\boldsymbol{x}) = \boldsymbol{\theta}_F^T \boldsymbol{\xi}_F\left(\boldsymbol{x}\right);\ \widehat{G}(\boldsymbol{x}) = \boldsymbol{\theta}_G^T \boldsymbol{\xi}_G\left(\boldsymbol{x}\right)$$

Step 4: Define the error surface:

$$s_f(e) = k_1 e + k_2 \frac{de}{dt}$$

Step 5: The state feedback controller:

$$u^* = \frac{1}{\widehat{G}(\boldsymbol{x})}\left(-\widehat{F}(\boldsymbol{x}) + \ddot{y}_m + k_p e + k_d \dot{e}\right)$$

IV. IMPROVED ADAPTIVE FUZZY MODE CONTROL(IAFC)

In formula (14), two parameters $k_p; k_d$ of the state feedback controller are chosen in (8) that they still guarantee the stable closed loop system. However, it does not concentrate to performance of closed loop system, for example: the settling time, overshoot and error between the output signal and reference…Therefore, to improve performance of system, concretely, the decrease error between reference trajectory and output trajectory, and rise the settling time, two parameters $k_p; k_d$ will be adjusted by adaptive law based on model adaptive reference system

The controller (14) can be rewritten as:

$$u^* = \frac{1}{\widehat{G}(\boldsymbol{x})}\left(-\widehat{F}(\boldsymbol{x}) + \ddot{y}_m + \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e\right) \qquad (18)$$

where $\boldsymbol{\theta}_k^T = (k_p\ \ k_d)$; $\boldsymbol{\xi}_e = (e\ \ \dot{e})^T$. The $\boldsymbol{\theta}_k^T$ will be updated by the adaptive law based on reference system in *Theorem 3*.

Theorem 3: Consider the nonlinear system (1), the state feedback controller (18) and $\widehat{F}(\boldsymbol{x}), \widehat{G}(\boldsymbol{x})$ in (12) are applied with the $\boldsymbol{\theta}_a^T$; $\boldsymbol{\theta}_b^T$ and $\boldsymbol{\theta}_k^T$ updated by adaptive law:

$$\dot{\boldsymbol{\theta}}_F = -Q_F^{-1}\boldsymbol{\xi}_F(\boldsymbol{x})\varepsilon;\ \dot{\boldsymbol{\theta}}_G = -Q_G^{-1}\boldsymbol{\xi}_G(\boldsymbol{x})\varepsilon u;\ \dot{\boldsymbol{\theta}}_k = Q_k^{-1}\boldsymbol{\xi}_e\varepsilon$$

The closed loop system will be asymptotic stable.

Proof:

The second derivative of the output error between output and reference is expressed:

$$\ddot{y} = F(\boldsymbol{x}) + G(\boldsymbol{x})u$$
$$= -\left(\widehat{F}(\boldsymbol{x}) - F(\boldsymbol{x})\right) - \left(\widehat{G}(\boldsymbol{x}) - G(\boldsymbol{x})\right)u + \widehat{F}(\boldsymbol{x}) + \widehat{G}(\boldsymbol{x})u$$
$$\ddot{y} = \ddot{y}_m + \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e - \left(\widehat{F}(\boldsymbol{x}) - F(\boldsymbol{x})\right) - \left(\widehat{G}(\boldsymbol{x}) - G(\boldsymbol{x})\right)u$$
$$\dot{x}_{2m} - \dot{x}_2 = \left(\boldsymbol{\theta}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) - \boldsymbol{\theta}_F^{*T}\boldsymbol{\xi}_F(\boldsymbol{x})\right) + \left(\boldsymbol{\theta}_G^T \boldsymbol{\xi}_G(\boldsymbol{x}) - \boldsymbol{\theta}_G^{*T}\boldsymbol{\xi}_G(\boldsymbol{x})\right)u - \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e$$
$$\dot{\varepsilon} = \left(\boldsymbol{\theta}_F^T - \boldsymbol{\theta}_F^{*T}\right)\boldsymbol{\xi}_F(\boldsymbol{x}) + \left(\boldsymbol{\theta}_G^T - \boldsymbol{\theta}_G^{*T}\right)\boldsymbol{\xi}_G(\boldsymbol{x})u - \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e$$
$$= \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{\xi}_G(\boldsymbol{x})u - \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e$$

$\tilde{\boldsymbol{\theta}}_F^T = \left(\boldsymbol{\theta}_F^T - \boldsymbol{\theta}_F^{*T}\right)$, $\tilde{\boldsymbol{\theta}}_G^T = \left(\boldsymbol{\theta}_G^T - \boldsymbol{\theta}_G^{*T}\right)$ are the parameter error

Consider the Lyapunov candidate function:

$$V = \frac{1}{2}\varepsilon^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_F^T \boldsymbol{Q}_F \tilde{\boldsymbol{\theta}}_F^T + \frac{1}{2}\tilde{\boldsymbol{\theta}}_G^T \boldsymbol{Q}_G \tilde{\boldsymbol{\theta}}_G^T + \frac{1}{2}\boldsymbol{\theta}_k^T \boldsymbol{Q}_k \boldsymbol{\theta}_k$$
$$\boldsymbol{Q}_F > 0; \boldsymbol{Q}_G > 0; \boldsymbol{Q}_k > 0$$

where $\boldsymbol{Q}_i \in \boldsymbol{R}^{d\times d} (d = \dim\theta_i)$ is a positive definite matrix. Take the derivative of V with respect to time and notice that $\dot{\tilde{\theta}}_i = \dot{\boldsymbol{\theta}}_i$, we have:

$$\dot{V} = \varepsilon\dot{\varepsilon} + \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G + \boldsymbol{\theta}_k^T \boldsymbol{Q}_k \dot{\boldsymbol{\theta}}_k$$
$$\dot{V} = \varepsilon\left(\tilde{\boldsymbol{\theta}}_F^T \boldsymbol{\xi}_F(\boldsymbol{x}) + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{\xi}_G(\boldsymbol{x})u - \boldsymbol{\theta}_k^T \boldsymbol{\xi}_e\right) + \tilde{\boldsymbol{\theta}}_F^T \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F + \tilde{\boldsymbol{\theta}}_G^T \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G +$$
$$+ \boldsymbol{\theta}_k^T \boldsymbol{Q}_k \dot{\boldsymbol{\theta}}_k$$
$$\dot{V} = \tilde{\boldsymbol{\theta}}_F^T\left(\varepsilon\boldsymbol{\xi}_F(\boldsymbol{x}) + \boldsymbol{Q}_F \dot{\boldsymbol{\theta}}_F\right) + \tilde{\boldsymbol{\theta}}_G^T\left(\varepsilon\boldsymbol{\xi}_G(\boldsymbol{x})u + \boldsymbol{Q}_G \dot{\boldsymbol{\theta}}_G\right) +$$
$$+ \boldsymbol{\theta}_k^T(\boldsymbol{Q}_k \dot{\boldsymbol{\theta}}_k - \varepsilon\boldsymbol{\xi}_e) \le 0 \qquad \square$$

Choose the parameters update law as follow

$$\varepsilon\boldsymbol{\xi}_F(\boldsymbol{x}) + \boldsymbol{Q}_F\dot{\boldsymbol{\theta}}_F = 0 \Rightarrow \dot{\boldsymbol{\theta}}_F = -\boldsymbol{Q}_F^{-1}\varepsilon\boldsymbol{\xi}_F(\boldsymbol{x})$$
$$\varepsilon\boldsymbol{\xi}_G(\boldsymbol{x})u + \boldsymbol{Q}_G\dot{\boldsymbol{\theta}}_G = 0 \Rightarrow \dot{\boldsymbol{\theta}}_G = -\boldsymbol{Q}_G^{-1}\varepsilon\boldsymbol{\xi}_G(\boldsymbol{x})u$$
$$\boldsymbol{Q}_k\dot{\boldsymbol{\theta}}_k - \varepsilon\boldsymbol{\xi}_e = 0 \Rightarrow \dot{\boldsymbol{\theta}}_k = \boldsymbol{Q}_k^{-1}\varepsilon\boldsymbol{\xi}_e$$

where a weighting matrix $\boldsymbol{Q}_k$ given as:

$$\boldsymbol{Q}_k = \begin{pmatrix} q_{k11} & 0 \\ 0 & q_{k22} \end{pmatrix}; \dot{\boldsymbol{\theta}}_k = Q_k^{-1}\boldsymbol{\xi}_e\varepsilon \Leftrightarrow \begin{pmatrix} \dot{k}_p \\ \dot{k}_d \end{pmatrix} = \begin{pmatrix} 1/q_{k11} & 0 \\ 0 & 1/q_{k22} \end{pmatrix}\begin{pmatrix} e \\ \dot{e} \end{pmatrix}\varepsilon$$

$$k_d(t) = k_d(0) + \frac{1}{q_{k22}}\int(\dot{y}_m - \dot{y})\varepsilon dt$$

$$k_p(t) = k_p(0) + \frac{1}{q_{k11}}\int(y_m - y)\varepsilon dt; \quad \varepsilon = \dot{y}_m - \dot{y} \tag{19}$$

and $k_p(0)$; $k_d(0)$ are chosen in(8) to guarantee the stability of the closed loop system.

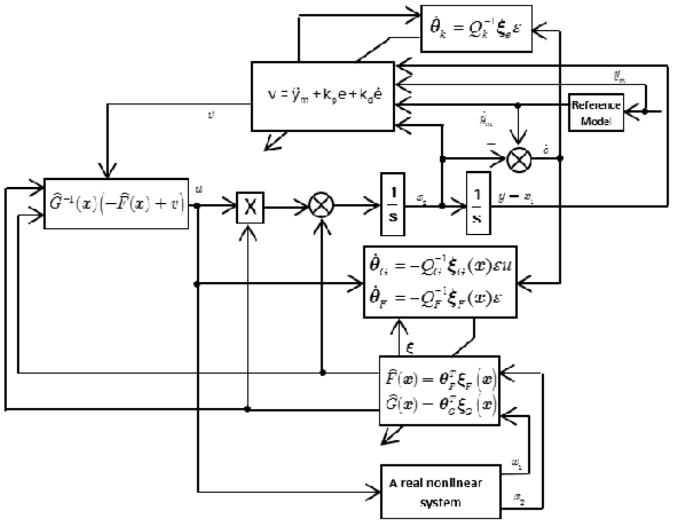

Fig. 2. The overall scheme of identification and control via improved adaptive fuzzy system (IAFC)

Reference Model: Explicit output reference, derivative output, and acceleration profile set point signals are created using the reference model, which is described by the transfer function:

$$H_{ref} = \frac{w_n^2}{s^2 + 2zw_n s + w_n^2} \tag{20}$$

The parameters of the reference model are chosen such as the higher order dynamics of the system will not be excited [11].The following steps to design the identification and control based on improved adaptive fuzzy system:

From *Step1* to *Step 3* is same the following steps in Section 3 noticed that we replace s_f with ε .

Step 4: The parameters of controller are updated by adaptive law as:

$$\dot{\boldsymbol{\theta}}_k = Q_k^{-1}\boldsymbol{\xi}_e\varepsilon; \ \varepsilon = \dot{y}_m - \dot{y} \tag{21}$$

Step 5: The state feedback controller:

$$u^* = \frac{1}{\widehat{G}(\boldsymbol{x})}\left(-\widehat{F}(\boldsymbol{x}) + \ddot{y}_m + \boldsymbol{\theta}_k^T\boldsymbol{\xi}_e\right)$$

V. ILLUSTRATIVE EXAMPLE

In this section, we present two examples of tracking reference set point with a 1 DOF robot arm and a coupled tank liquid level system. The simulation illustrates the convergence of error under our proposed AFC and IAFC approach.

A. 1 DOF Robot Arm

Consider the 1 DOF robot arm [1] has the dynamic equation shown as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\left(\dfrac{g}{l}\sin(x_1) + \dfrac{d}{ml^2}x_2\right) + \dfrac{1}{ml^2}u \\ y = x_1 \end{cases} \tag{22}$$

where $\boldsymbol{x} = (x_1 \ x_2)^T = (\theta \ \dot{\theta})^T$; θ is the angle of robot arm and $\dot{\theta}$ is the velocity of robot arm, $l = 0.85m$ is length of arm. $d = 0.85kg/m^2$ is the coefficient of friction, $m = 0.9kg$ is weight of robot arm, and $g = 9.8m/s^2$ is the gravity acceleration .

we have $F(\boldsymbol{x}) = -\left(\dfrac{g}{l}\sin(x_1) + \dfrac{d}{ml^2}x_2\right)$ and $G(\boldsymbol{x}) = -\dfrac{1}{ml^2}$.

We will identify and design the controller following steps as: Choose the inputs membership function of $\widehat{F}(\boldsymbol{x})$; $\widehat{G}(\boldsymbol{x})$ have shaped in Gaussian over the interval [-1 1]:

$$\mu_{A_j^l}(x_j) = \exp\left(-\left(x_i - c_j^l\right)^2 / 2\left(\delta_j^l\right)^2\right) \tag{23}$$

and $\widehat{F}(\boldsymbol{x})$; $\widehat{G}(\boldsymbol{x})$ are identified in *Theorm 2* for AFC and *Theorm 3* for IAFC:

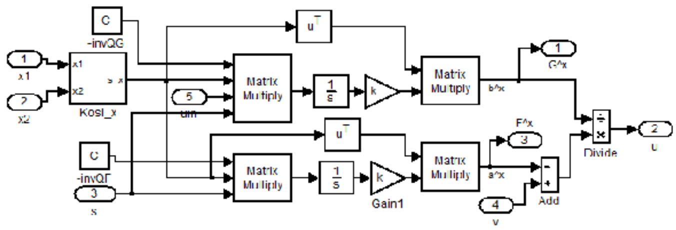

Fig. 3. Identification nonlinear system via adaptive fuzzy system

We choose the fix coefficients $k_p = 1$; $k_d = 5$ of AFC and initial parameters $k_p(0) = 1$; $k_d(0) = 5$ of IAFC. The results of simulation with the two algorithms AFC and IAFC contain two parts: the estimation the smooth function $\widehat{F}(\boldsymbol{x})$; $\widehat{G}(\boldsymbol{x})$ and error between output and reference trajectory:

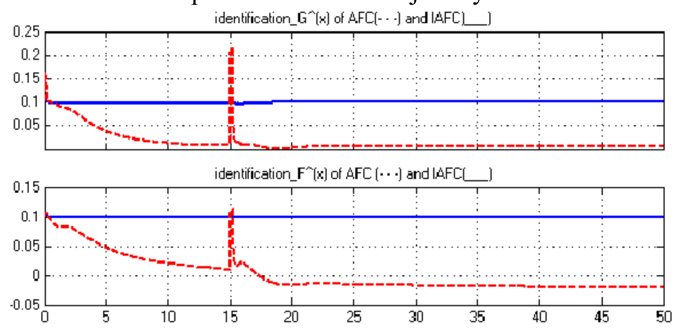

Fig. 4. Estimation two nonlinear functions of systems

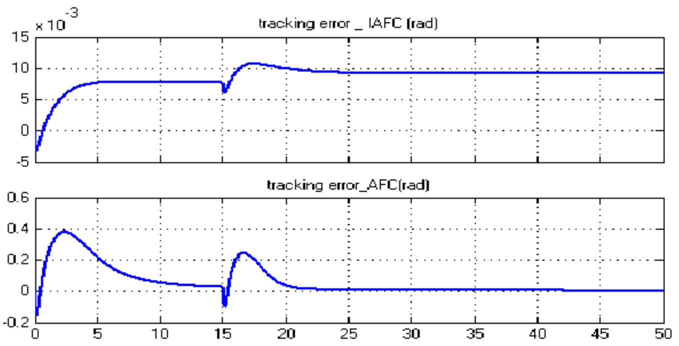

Fig. 5. Tracking error with IAFC (1st line) and with AFC (2nd line)

Comparison of angle control simulation between AFC and IAFC is present. With $\hat{F}(\boldsymbol{x}); \hat{G}(\boldsymbol{x})$ both AFC and IAFC are able to identify (see Fig.4). Response of both AFC and IAFC are almost same. However, it is clearly that the tracking errors for the AFC due to reference trajectory are larger than those by IAFC (see Fig.5), concretely, the tracking errors for IAFC is about 100 times as small as those for AFC. The adaptive gains $\boldsymbol{\theta}_k^T = (k_p \; k_d)$ of the IAFC automatically reach stationary values (see Fig.6).

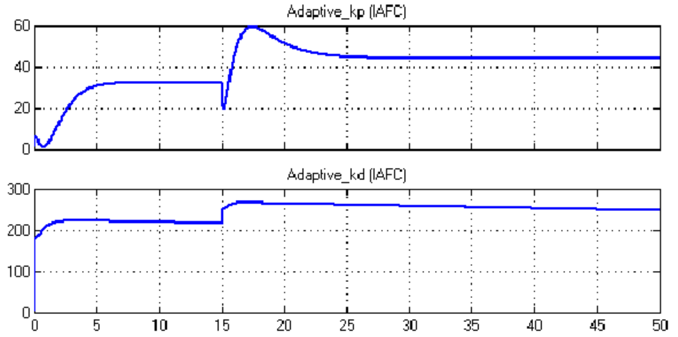

Fig. 6. Adaptive gains of the IAFC

B. *Coupled Tank Liquid Level System*

In [12], the modeled equation of coupled tank liquid level system and parameters given as:

$$x = (x_1 \; x_2)^T = (h_1 \; h_2)^T$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dfrac{-b_{12}a_{12}\sqrt{2g\left|x_1 - x_2\right|}}{A_1} \\ \dfrac{1}{A_2}\left(b_{12}a_{12}\sqrt{2g\left|x_1 - x_2\right|} - b_2 a_2\sqrt{2gx_2}\right) \end{pmatrix} + \begin{pmatrix} \dfrac{k}{A_1} \\ 0 \end{pmatrix} u(t) \tag{24}$$

$$y = x_2$$

$$\begin{cases} \dfrac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u \\ y = g(\boldsymbol{x}) \end{cases}$$

TABLE I. THE PARAMETERS OF COUPLED LIQUID LEVEL SYSTEM

Parameter	Value
A_1, A_2 the cross-sectional area of tank 1 and tank 2	100cm^2
Q_{in} the flow rate of liquid into tank 1	u(cm^3/sec)
Q_{out} the flow rate of liquid into tank 2	0.8cm^3/sec
h_1, h_2 the height of liquid in tank1 and tank 2	cm
b_2 the cross sectional area of outlet pipe in tank 2	0.5cm^2
a_{12} the cross-sectional area interaction pipe between tank 1and tank 2	0.5cm^2
b_{12} the value ratio of interaction pipe between tank1 and tank 2	1.5
g acceleration of gravity	981cm^2/sec
a_2 the value ratio of outlet pipe of tank 2	1.5

We use the Theorem 1 in order to convert (24) into (4) where:

$$F(\boldsymbol{x}) = L_f^2 g(\boldsymbol{x}); G(\boldsymbol{x}) = L_h L_f g(\boldsymbol{x}) \tag{25}$$

Assuming that the function $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ describing the system dynamics are unknow, so that we employ adapative fuzzy system to online identify $F(\boldsymbol{x})$ and $G(\boldsymbol{x})$. The control target guarantee for liquid level of Tank 2 at the setpoint. Opened valve ratio of pump 1 is adjusted by control law, and pump 2 open arbitrary.

Fistly, we will estiamte the $\hat{F}(\boldsymbol{x}); \hat{G}(\boldsymbol{x})$ by the adaptive fuzzy system. The inputs membership function are shaped in Gaussian over the interval [0 100] as (19) and $\boldsymbol{\theta}^T$ is parameter vector of output membership function which is updated by adaptive law in *Thoerm 2* for AFC and *Theorm 3* for IAFC as Fig. 3. We choose the fix coefficients $k_p = 1; \; k_d = 5$ of AFC and initial parameters $k_p(0) = 1; \; k_d(0) = 5$ of IAFC given as:

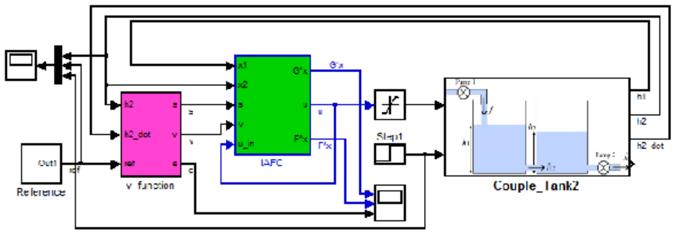

Fig. 7. Simulation control structure with MATLAB

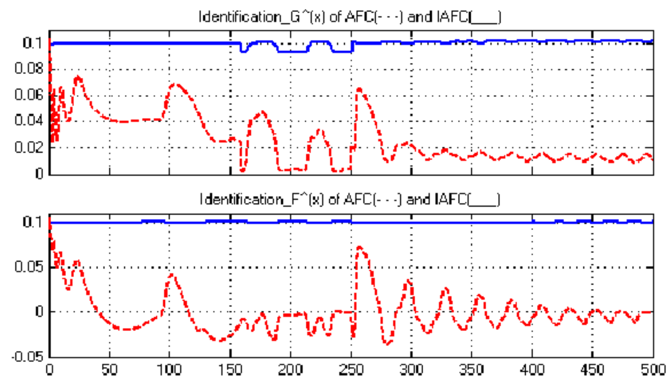

Fig. 8. Estimation two nonlinear functions of systems

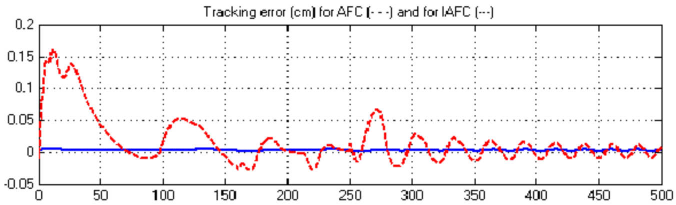

Fig. 9. Tracking error with AFC (- - -) and with IAFC (___)

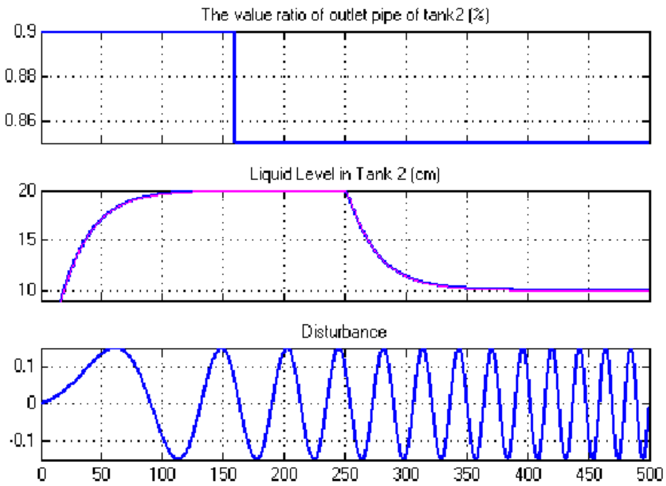

Fig. 10. Tracking output with both of AFC and IAFC

In this coupled tank liquid level system, we consider that this system is affected by disturbace $d(t)$ which has changing frequency by the time as Fig.10. With $\widehat{F}(\boldsymbol{x})$; $\widehat{G}(\boldsymbol{x})$ both AFC and IAFC could be evaluated when there is disturbance effect on the system (see Fig.8). Comparison of liquid level control simulation between AFC and IAFC is present. Response of both AFC and IAFC are almost same (see Fig.10). However, it is obviously that the tracking errors for the IAFC due to reference liquid level are less than those by AFC (see Fig.9). The adaptive $\boldsymbol{\theta}_k^T = (k_p \; k_d)$ of the IAFC automatically alter when disturbance inluence on system (see Fig.11).

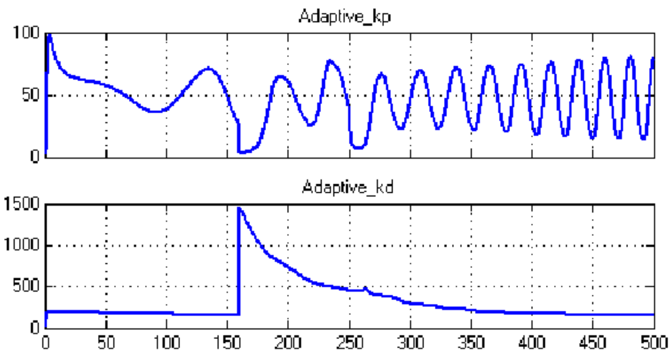

Fig. 11. Adaptive gians of the IAFC

VI. CONCLUSION

In the paper we have presented adaptive fuzzy system combined with MARS, improved adaptive fuzzy system, offers a potential on deliver more accurate and high overall performance in the presence of all the preceding issues. We investigate the effect of the identifier and controller from the simulation results. Compare to the case with AFC and IAFC in two illustrated examples (1DOF robot arm and coupled tank liquid level), for instance, can do the following (see Fig.4, Fig.5, Fig.8, and Fig.9): (a) Improve the transient behavior of the system; (b) Decrease the sensitivity to plant parameter changes; (c) Eliminate steady-state errors; and (d) Decrease the influence of load disturbances and measurement noise. Strong properties achieved via the proposed method confirm that improved adaptive fuzzy system is an attractive approach for controlling single input-single output nonlinear systems.

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APPENDIX

The relative order of single input-single output (SISO) system is defined:

$$L_h L_f^k \boldsymbol{g}(\boldsymbol{x}) = \begin{cases} = 0, \; 0 \le k \le r-2 \\ \ne 0, k = r-2 \end{cases}$$

$$\frac{dy}{dt} = \frac{\partial g(\boldsymbol{x})}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial g(\boldsymbol{x})}{\partial \boldsymbol{x}} \big(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u\big) = L_f g(\boldsymbol{x}) + \underbrace{L_h L_f^0 g(\boldsymbol{x})}_{=0} u$$

$$\frac{dy^2}{dt} = \frac{\partial L_f g(\boldsymbol{x})}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f g(\boldsymbol{x})}{\partial \boldsymbol{x}} \big(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u\big) = L_f^2 g(\boldsymbol{x}) + \underbrace{L_h L_f^1 g(\boldsymbol{x})}_{=0} u$$

$$\vdots$$

$$\frac{dy^{r-1}}{dt} = \frac{\partial L_f^{r-2} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f^{r-2} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \big(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u\big) = L_f^{r-1} g(\boldsymbol{x}) + \underbrace{L_h L_f^{r-2} g(\boldsymbol{x})}_{=0} u$$

$$\frac{dy^{r}}{dt} = \frac{\partial L_f^{r-1} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f^{r-1} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \big(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}(\boldsymbol{x}).u\big) = L_f^{r} g(\boldsymbol{x}) + \underbrace{L_h L_f^{r-1} g(\boldsymbol{x})}_{\ne 0} u$$

with $n = r$, we chose the coordinate transformation as:

$$\boldsymbol{z} = \boldsymbol{m}(\boldsymbol{x}) = \big(m_1(\boldsymbol{x}), m_2(\boldsymbol{x}), \ldots, m_n(\boldsymbol{x})\big)^T ; m_k(\boldsymbol{x}) = L_f^{k-1} \boldsymbol{g}(\boldsymbol{x})$$

$$\frac{dz_1}{dt} = \frac{\partial m_1}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial g(\boldsymbol{x})}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{dt} = L_f g(\boldsymbol{x}) = z_2$$

$$\frac{dz_2}{dt} = \frac{\partial m_2}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f g(\boldsymbol{x})}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{dt} = L_f^2 g(\boldsymbol{x}) = z_3$$

$$\vdots$$

$$\frac{dz_{n-1}}{dt} = \frac{\partial m_{n-1}}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f^{r-2} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{dt} = L_f^{r-1} g(\boldsymbol{x}) = z_n$$

$$\frac{dz_{n}}{dt} = \frac{\partial m_{n}}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial L_f^{r-1} g(\boldsymbol{x})}{\partial \boldsymbol{x}} \frac{d\boldsymbol{x}}{dt} = L_f^{r} g(\boldsymbol{x}) + L_h L_f^{r-1} g(\boldsymbol{x}) u = \underbrace{L_f^r g(\boldsymbol{m}^{-1}(\boldsymbol{z}))}_{F(\boldsymbol{x})} + \underbrace{L_h L_f^{r-1} g(\boldsymbol{m}^{-1}(\boldsymbol{z}))}_{G(\boldsymbol{x})} u$$

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