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Total Zero-Divisor Graph of A Field

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***Abstract*: Let F be a Field with z(F), its set of zero divisors. The total zero divisor graph of F, denoted Z(Γ(F)) is the undirected (simple) graph with vertices $Z(F)^*$=Z(F)-{0}, the set of non-zero, zero divisors of F, and for distinct x, y ∈ $z(F)^*$ the vertices x and y are adjacent if and only if x+ y ∈z(F). In this paper, we study if Z(Γ(F)) is finite and every vertex of Z(Γ(F)) has a finite degree then F is finite and also prove that Z(Γ(F)) connected with diam≤3.**

I. INTRODUCTION

In this paper, we study the total zero divisor graph is the (undirected) graph with vertices $Z(F)^*$=Z(F)- {0}. The set of non-zero zero divisor of F and for distinct x, y∈ $z(F)^*$, the vertices x and y are adjacent if and only if x+ y ∈z(F). It is denoted by Z(Γ(F)) and is the (induced) subgraph of total graph. We show that Z(Γ(F)) is finite then F is finite and not an integral domain, if every vertex of Z(Γ(F)) has finite degree then F is finite and also prove that Z(Γ(F)) is connected with diam≤3. For some other recent papers on zero divisor graphs.

II. PRELIMINARIES

2.1Definition:

The number of edges incident with a vertex Vis called the degree of V and it is denoted by d(V). The minimum and maximum degree of a vertex of a graph are respectively denoted by δ and Δ.

2.2 Definition:

A graph G in which every vertex is adjacent to every other vertex is called a complete graph. Complete graph is represented as K_n where n is the number of vertices in K_n

2.3Definition:

The chromatic number of a zero-divisor graph of a ring R, denoted by $\chi(\Gamma_0(R))$ is the minimal number of colors required to assign each vertex in a zero-divisor graph a color so that no two adjacent vertices are assigned the same color.

2.4Definiton:

A graph $\Gamma_0(R)$ is a k-colorable if $\Gamma_0(R)$ can be colored with less than or equal to k colors.

2.5Defnition:

A graph G is said to be a connected graph. If there is at least one path between every pair of vertices in G. otherwise G is said to be a disconnected graph.

2.6Definition:

Any two distinct vertices a and b in graph G, the distance between a and b, denoted by d (a, b) is the length of a shortest path connecting a and b.

2.7Definition:

A ring R is called a coloring if $\chi(\Gamma_0(R))$ is finite.

2.8Definition:

An element x∈R is said to be a zero divisor if there exists some element 0≠y∈R such that xy=0.

2.9 Presumption:

$\chi\big(\Gamma_0(R)\big) = 1$ if and only if R = {0}.

2.10presumption:

$\chi(\Gamma_0(R))$ = 2 if and only if R is an integral domain, R≅ $\mathbb{Z}_4$ or R ≅ $\mathbb{Z}_2[X]/(X)^2$.

2.11Definition:

The chromatic number of a zero-divisor graph of a ring R is equal to the clique number of the ring. That is, $\chi\big(\Gamma_0(R)\big)$ =cl(R).

III. MAIN RESULT

3.1Theorem:

Let F be field then the total zero divisor graph if finite if and only if either or an integral domain. In particular if 1≤ Z(Γ(F))≤ ∞.Then F ia finite.

Proof;

Let F be a field and Z(F) be the set of zero divisors in F and Let Z(Γ(F)) be the total zero divisor graph. Then all vertices of Z(Γ(F)) is non-zero, zero divisor of F.

It is trivial that if F is finite then Z(Γ(F)) is also finite.

Suppose that Z(Γ(F) is finite and non-empty. This implies that Z(F) is finite, suppose these are two elements u, v∈F, u≠0,v≠0.suchthat u+v∈Z(F)

Let I=Ann(Z), then u+v∈I

Since u+v∈ $Z(F)$this implies that I⊆ $Z(F)$ further I is finite and f(u+v) ∈I for all f∈F.[∵u+v∈ I,f∈F⟹f(u+v) ∈I]suppose F is finite.Then there is an i∈I suchthat K={f∈F/f(u+v)=i}is infinite.

For any f, t∈K

F(u+v) = i, t(u+v) = i

(f−t) (u+v) =0

(f−t) ∈Ann(u+v) {since, K⊆Ann(u+v), K is f−t∈J⇒f-tVAnn(u+v) infinite)

Where k⊆Ann(u+v)

Since f-t∈Z(F)

i.e. Ann(u+v) ⊆Z(F), is infinite, a contradiction therefore F must be finite.

3.2Theorem:

Let f be a field with identity. Then S = F × $\mathbb{Z}_2^0$ is a field without identity, S = Z(S), and Γ_E(S)≅ Γ_E(F).

Proof: Clearly S = Z(S) and T has no identity. Define ϕ: F/∼ ⟶ S /∼ by ϕ([u]) = [(u, 0)]. It is easily verified that Ann_F(u) = Ann_F (v) for u, v ∈ F if and only if Ann_S ((u, 0)) = Ann_S ((v, 0)), and [(u, 0)] = [(u, 1)] for every u ∈ F. Thus, ϕ is a well-defined bijection. Moreover, ϕ restricts to a graph isomorphism from Γ_E(F) to Γ_E(S) since [(u, 0)] [(v, 0)] = [(0, 0)] if and only if [u][v] = [0].

3.3Theorem:

Let F be a field such that Z(F) Is not an ideal of F then Z(F) is connected with diamZ(F)=2

Proof: Each u∈ $Z(F)^*$ is adjacent to 0. Thus, u0v is a path in Z(F) of length two between any two distinct u, v∈ $Z(F)^*$ Moreover, there are non-adjacent u, v∈ $Z(F)^*$ since Z(F) is not an ideal of F.

So, diamZ(F)=2.

Hence proved

3.4Theorem: Let F be a field then Z(F) is connected with diam≤3

Proof:

Let u, v be vertices in Z((F)),

There exists u+z ∈Z(F), v+W∈Z(F)

If u+v ∈Z(F) then uv is a path of length is perpendicular containing u, v.

If u+v∈ Z(F) and w+z ∈Z(F) then u and v are contained by a path uwv of length≤3

If u+v ∈Z(F) and w+z∈ Z(F) then u and v are connected by a path uv of length=2.

Hence proved.

IV. REFERENCE

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