Total Zero-Divisor Graph of A Field

Thassim Beevi Abdul Kader College for Women, Kilakarai

Abstract: Let F be a Field with z(F), its set of zero divisors. The total zero divisor graph of F, denoted Z(Γ(F)) is the undirected (simple) graph with vertices Z(F)=Z(F)-{0}, the set of non-zero, zero divisors of F, and for distinct x, y ∈ Z(F) the vertices x and y are adjacent if and only if x+y ∈ Z(F). In this paper, we study if Z(Γ(F)) is finite and every vertex of Z(Γ(F)) has a finite degree then F is finite and also prove that Z(Γ(F)) connected with diam≤3.

I. INTRODUCTION

In this paper, we study the total zero divisor graph is the (undirected) graph with vertices Z(F)=Z(F)-{0}. The set of non-zero zero divisor of F and for distinct x, y ∈ Z(F), the vertices x and y are adjacent if and only if x+y ∈ Z(F). It is denoted by Z(Γ(F)) and is the (induced) subgraph of total graph. We show that Z(Γ(F)) is finite then F is finite and not an integral domain, if every vertex of Z(Γ(F)) has finite degree then F is finite and also prove that Z(Γ(F)) connected with diam≤3. For some other recent papers on zero divisor graphs.

II. PRELIMINARIES

2.1Definition: The number of edges incident with a vertex Vis called the degree of V and it is denoted by d(V). The minimum and maximum degree of a vertex of a graph are respectively denoted by δ and Δ.

2.2 Definition: A graph G in which every vertex is adjacent to every other vertex is called a complete graph. Complete graph is represented as Kn where n is the number of vertices in Kn

2.3Definition: The chromatic number of a zero-divisor graph of a ring R, denoted by χ(Γ0(R)) is the minimal number of colors required to assign each vertex in a zero-divisor graph a color so that no two adjacent vertices are assigned the same color.

2.4Definition: A graph Γ0(R) is a k-colorable if Γ0(R) can be colored with less than or equal to k colors.

2.5Definition: A graph G is said to be a connected graph. If there is at least one path between every pair of vertices in G. Otherwise G is said to be a disconnected graph.

2.6Definition: Any two distinct vertices a and b in graph G, the distance between a and b, denoted by d(a, b) is the length of a shortest path connecting a and b.

2.7Definition: A ring R is called a coloring if χ(Γ0(R)) is finite.

2.8Definition: An element x∈R is said to be a zero divisor if there exists some element 0≠y∈R such that xy=0.

2.9 Presumption: χ(Γ0(R)) = 1 if and only if R = {0}.

2.10presumption: χ(Γ0(R)) = 2 if and only if R is an integral domain, R≅ Z4 or R≅ Z2[X]/(X)2.

2.11Definition: The chromatic number of a zero-divisor graph of a ring R is equal to the clique number of the ring. That is, χ(Γ0(R)) =cl(R).

III. MAIN RESULT

3.1Theorem: Let F be field then the total zero divisor graph if finite if and only if either or an integral domain. In particular if 1≤ Z(Γ(F))≤∞.Then F ia finite.

Proof: Let F be a field and Z(F) be the set of zero divisors in F and Let Z(Γ(F)) be the total zero divisor graph. Then all vertices of Z(Γ(F)) is non-zero, zero divisor of F. It is trivial that if F is finite then Z(Γ(F)) is also finite. Suppose that Z(Γ(F)) is finite and non-empty. This implies that Z(F) is finite, suppose these are two elements u, v∈F, u≠0, v≠0 suchthat u+v∈Z(F) Let I=Ann(Z), then u+v∈I Since u+v∈ Z(F) this implies that I⊆ Z(F) further I is finite and f(u+v)∈I for all f∈F if f(u+v)∈I then f∈Ann(u+v) suppose F is finite. Then there is an i∈I suchthat K=[{f∈F/f(u+v)=i}] is infinite. For any f, i∈K

f(u+v) = i, f(u+v) = i

(−i)u+v = 0

(−i)∈Ann(u+v) [since, K⊆Ann(u+v), K is f−i∈E⇒f−i∈Ann(u+v) infinite)

Where k⊆Ann(u+v)

Since f−i∈Z(F)

i.e. Ann(u+v)⊆ Z(F), is infinite, a contradiction therefore F must be finite.

3.2Theorem: Let f be a field with identity. Then S = F × Z2 is a field without identity, S = Z(S), and ΓF(S)≡ ΓF(F).
Proof: Clearly $S = Z(S)$ and $T$ has no identity. Define $\phi: F/\sim \rightarrow S/\sim$ by $\phi([u]) = [(u, 0)]$. It is easily verified that $Ann_E(u) = Ann_F(v)$ for $u, v \in F$ if and only if $Ann_E((u, 0)) = Ann_S((v, 0))$, and $[(u, 0)] = [(u, 1)]$ for every $u \in F$. Thus, $\phi$ is a well-defined bijection. Moreover, $\phi$ restricts to a graph isomorphism from $\Gamma_E(F)$ to $\Gamma_E(S)$ since $[(u, 0)] [(v, 0)] = [(0, 0)]$ if and only if $[u][v] = [0]$.

3.3 Theorem:
Let $F$ be a field such that $Z(F)$ is not an ideal of $F$ then $Z(F)$ is connected with $\text{diam} Z(F) = 2$.

Proof: Each $u \in Z(F)$ is adjacent to $0$. Thus, $u0v$ is a path in $Z(F)$ of length two between any two distinct $u, v \in Z(F)$. Moreover, there are non-adjacent $u, v \in Z(F)$ since $Z(F)$ is not an ideal of $F$.
So, $\text{diam} Z(F) = 2$.
Hence proved.

3.4 Theorem: Let $F$ be a field then $Z(F)$ is connected with $\text{diam} \leq 3$.

Proof:
Let $u, v$ be vertices in $Z((F))$.
There exists $u+z \in Z(F), v+w \in Z(F)$
If $u+v \in Z(F)$ then $uv$ is a path of length is perpendicular containing $u, v$.
If $u+v \in Z(F)$ and $w+z \in Z(F)$ then $u$ and $v$ are contained by a path $uv$ of length $\leq 3$.
If $u+v \in Z(F)$ and $w+z \in Z(F)$ then $u$ and $v$ are connected by a path $uv$ of length $= 2$.
Hence proved.

IV. Reference