Topological Characteristics of 9 Links, 2 D of Kinematic Chains Using Link Connectivity and Information Theory

Rajneesh Kumar Rai [1], Dr. Sunil Punjabi [2], Dr. Sunil Raj Madan [3]

[1]Head of Mechanical Engineering
Shri Dadaji Institute of Tech. & Science
[2]Associate Professor
Govt. Engg. College, Ujjain
[3]Principal
Mahakal Institute of Technology & Science, Ujjain
450001 Rehmapur, MP [India]

Abstract

It is possible to identify a kinematic chain by using sets of decimal numbers representing connectivity of individual link. This enables one to identify structurally similar and dissimilar links which decides isomorphism of kinematic chains. This will reveal about the behavior of the kinematic chains regarding power transmission capacity and power transmission efficiency.

Synthesis of kinematic chains has been studied in detail (1, 2, and 3). Almost all reported work deal with identification of distinct chain and isomorphism of kinematic chains and not much beyond. Besides providing an atlas of kinematic chains, this in itself does not provide any help to designer in selecting the best possible kinematic chain. In the present paper, a simple method based on connectivity and information theory is presented to detect isomorphism amongst the kinematic chains and possible rating of the numerous kinematic chains. The connectivity of vertices edge values and circuit value related to design invariants which in turn indicates the possible behavior of the k- chains in capacity of power transmission and power transmission efficiency. For a specified DOF, a number of kinematic chains can be formed with a given number of links and joints and kinematician/designer must be able to select the best kinematic chain from the view of power transmission capacity and power transmission efficiency. The proposed method is applied to 9- links, 2 DOF kinematic chains.

Key-Words: *Information theory, Kinematic chain, Isomorphism*

1. Introduction

Generation of kinematic chains has been reported by many investigators in last three decades (1-4). The problem of isomorphism identification among all the generalized kinematic chains also advised. Mruthuyunjaya and Raghvan (5) provide a matrix notation for the distinction of all possible mechanisms from kinematic chains. Yan and Hwang (6) presented a method based on linkage structural polynomials for detecting the isomorphism of planer linkage chains. Yan and Hwang (7) also introduced the idea of linkage code for identifying the topological structure of planer chains with simple joints. Jiger, C.H. and Lam, K.T.(8) proposed a method based on characteristic polynomials of contracted graphs and kinematic graphs, for the automatic identification of kinematic chains. Hwang et al (9) presented a method for the number synthesis of planer kinematic chains with simple joints, which consist of systematic generation of possible contracted link-link adjency. Almost all the reported work is based on identification of distinct chains and not much beyond. Besides providing an atlas of chains, these in itself, do not provide any help to the designer in selecting the best possible kinematic chain. Therefore, it is very much essential to provide more information to the designer regarding ability of kinematic chain which helps in making his choice. The structure of the kinematic chain alone is not capable to reveal its actual performance as dimensions of the links also influence it. However the structure can be used to reveal some characteristics like power transmission capacity, power transmission efficiency. In the present paper, a simple numerical method is proposed to relate the topology of a kinematic chain to its ability to generate corresponding power and efficiency. Also proposed method is to compare the kinematic chains and identification and isomorphism of kinematic chains.

2. Graphical Representation

For topographical study of kinematic chains, graph theory has been used extensively. In this case, each link of a kinematic chain is represented by a vertex and each joint by an edge of the graph.

3. Edge Value

An edge between the two vertices in a graph is signified by the number and type of other edge to which it is connected. Therefore contribution of every edge in a graph is quantified by a numerical value, which is the sum of numerical value of all other edges that are connected to it. This sum is called joint or edge value the edge value is related to vertex connectivity in the following manner.

The edge (ij) =

(Connectivity of vertex i)+ (Connectivity of vertex j) – 2 e_{ij} (1)

Where e_{ij} = 1, for turning pair.

4. Identification of Chains and Link

The term "Kinematics Structure" of a mechanism implies information in regards to:-

(1) Type of links and information as to which link is connected to which other link and to which type of a joint, and

(2) Which of the link is a frame link.

It follows therefore that the problem of indentifying kinematics chain and mechanism can be solved by involving a scheme to indentify each link as a component of chain. This is done by proposing a link-link type connectivity matrix which is capable of storing all relevant structural information about an individual link. A square matrix, as shown below, each diagonal element a_{ij} denotes degree of link i , is used. The off diagonal elements a_{ij} = 0, if links i and j are not directly connected. The element a_{ij} equals the numeral which is representative of the degree of link j. Consider figure 1 given in table 1, the link- link type connectivity matrix will be:

3	0	3	3	2	0	0	0	0
0	3	3	3	0	2	0	0	0
3	3	3	0	0	0	2	0	0
3	3	0	3	0	0	0	2	0
3	0	0	0	2	2	0	0	0
0	3	0	0	2	2	0	0	0
0	0	3	0	0	0	2	0	2
0	0	0	3	0	0	0	2	2
0	0	0	0	0	0	2	2	2

Each link of a kinematic chain can be represented by number in decimal form, whose integer portion represents degree of the concerned link, while the numeral in k_{th} place after decimal point indicates number of links of degree k each, connected to link under consideration.

Therefore the representative decimal number or connectivity number for link – 1 represented by first row of link-link connectivity matrix is given as under:

$$N_1 = 3 + 2\,(10)^{-3} + 1\,(10)^{-2}$$
$$= 3.012$$

Similarly the representative decimal numbers for remaining links are as under,

N_2 = 3.012, N_3 = 3.012, N_4 = 3.012, N_5 = 2.011, N_6 = 2.011, N_7 = 2.011, N_8 = 2.011, N_9 = 2.02.

Therefore, the kinematic chain is identified by the set,

/ 4 (3.012) / 4 (2.011) / 2.02/ (3)

Total connectivity value of the chain is 22.112

5. Power Transmission Capacity Calculation

The present paper takes into consideration the structure of kinematic chain and not the dimensions, strength etc. It can thus be generalized saying that chain with great connectivity value can transmit more power However , the problem arises when compression has to be made between the kinematic chains with the same number of elements and DOF , which have equal connectivity. Therefore, a numerical measure becomes necessary to compare such kinematic chains,

Let the total connectivity of the chain is D and d_i be the connectivity of link i. Then the ratio of the design parameters of link i to the total design parameters of the chain will be d_i/D.Hence,

$$\sum_{i=1}^{n} (d_i/D) = 1 \qquad (4)$$

Where n is the total number of links

A mathematical expression that can be considered for estimating the power transmitting capacity of a kinematic chain must satisfy the following requirements:

(1) The quantum of power transmitted is maximum when all elements have equal design parameters (i.e. $d_1=d_2=d_3$ --------$=d_n$)
(2) No single element of the kinematic chain can transmit the entire power.
(3) The quantum of power transmitted by a fixed link is zero.
A mathematical equation that satisfies the above requirements is expressed as

$$P_i \log_{10} P_i$$

Where $Pi = d_i/D$ (5)

It is chosen in such a way that $1\log_{10} 1 = 0$, and $0 \log_{10} 0 = 1$, in order to satisfy the above mentioned requirements, especially the last two conditions.Then power transmission capacity "P" of the kinematic chain is expressed as

$$P = \Sigma P_i \log_{10} P_i \quad (6)$$

Where $P_i = d_i/D$

d_i = connectivity of link "i"

D = Total connectivity of the kinematic chain.

To understand the concept, we take example of fig1.Connectivity of link 1, 2, 3,&4 is 3.012 whereas connectivity of link 5,6,7,8 is 2.011 and connectivity of link 9 is 2.02. Total connectivity of the chain is 22.112. Therefore power transmission capacity is :

$$P = - \Sigma\, 4\,(3.012/22.112) \log_{10} (3.012/22.112) + 4\,(2.011/22.112) \log_{10} (2.011/22.112) + (2.02/22.112) \log_{10} (2.02/22.112)$$

$$= 0.944755553$$

The power transmission capacities of other kinematic chains are shown in the table1.

6. Transmission Efficiency

Using equation (1), the numerical value of edge value of all links of a kinematic chain can be determined. Let Ji be the edge value of the graph or kinematic chain with i^{th} edge, then the edge value of the graph or kinematic chain can be represented as:

$$J = \sum_{i=1}^{k} J_i \quad (7)$$

Where k is the total number of edges in the graph.
The ratio of edge value of the edge i to the edge value of the graph is Ji/J

$$\text{Hence } \sum_{i=1}^{k} J_i/J = 1 \quad (8)$$

A mathematical expression that can be considered for estimating the transmission efficiency of a kinematic chain must satisfy the following requirements:
(1) Energy transmitted is maximum when all the edges/joints are of equal value
(2) No single joint of a kinematic chain will transmit the entire value.
(3) Fixed elements, if any, in a joint will not contribute to the edge value.

A mathematical equation that satisfies the above requirements is expressed as:

$$(- Ji / J) \log10 (Ji /J) \quad (9)$$

The expression has all the traits of earlier expression (8).Therefore the energy transfer E through all the joints of a kinematic chain can be expressed as :

$$E = -\Sigma (J_i / J) \log_{10} (J_i /J) \quad (10)$$

Where
J_i= Edge value of i^{th} edge (link)
J= Edge value of the graph (Kinematic chain)

The expression (10) assumes a maximum or ideal value (6) E_m when $J_1 = J_2$ =----J_k, which satisfies the first of the above three requirements. Equation (8) and (10) are analogous, once again to the equation of entropy in the information theory. The transmission efficiency is defined as the ratio of the actual energy transfer (E) to the maximum energy transfer (Em).
Therefore, the transmission efficiency (T_e) of a kinematic chain can be expressed as,

$$T_e = E/E_m \quad (11)$$

When $Em = - 11\ (1/11 \log_{10} 1/11)$
$= 1.041392685$

Consider the graph of kinematic chain of figure 1, as shown, the edge connectivity of edge 'a'

$= N_1 + N_3 -2 e_{ij}$ (According to equation 1)
$=3.012 + 3.012 – 2$
$= 4.024$

Similarly , edge connectivity of other edges b,c,d,e,f,g,h,i,j and k for the kinematic chain given by figure 1 are 4.024,4.024,3.023,2.021,2.031,3.023,3.023,4.0424,2.022 and 3.023 .Therefore the total edge connectivity of the kinematic chain or graph is:

a + b + c + d + e + f + g + h + i + j + k
= 34.264

Therefore, $E = -\Sigma (J_i / J) \log_{10} (J_i /J)$

$$= -\Sigma [(4\times4.024/34.262) \log_{10} (4.024/34.262) + (4 \times3.023/34.262) \log_{10} (3.023/34.262) + (2.021/34.262) \log_{10} (2.021/34.2625) + (2.031/34.262) \log_{10} (2.031/34.262)+(2.022/34.262)\log_{10} (2.022/34.262)]$$

$$= 1.027335901$$

Therefore, transmission efficiency

T_e = 0.986501937

Now, transmission efficiency of other kinematic chains is shown in the table1.

7. Chain Isomorphism by using the concept of link-loop scheme

The chain isomorphism checking can also be done by the concept of link-loop scheme. While the identification code based on connectivity scheme usually detect isomorphism among the kinematic chains, but in some cases it is desirable to have an additional check. The additional check is based on link-loop scheme. This can be explained as below:

For example, the chain number 14, of figure 14, consists of 4 loops marked 1, 2, 3 and 4, the last peripheral loop shown by dotted lines. The codes and the corresponding values can be determined for each loop as follows:

Loop1: The links 1, 4, 5, 9, 6 and 2 participate in forming loop1. The decimal numbers of each of the links have been determined as 3.021, 2.002, 3.021, 2.002, 3.021 and 3.021. Therefore the loop value can be taken as sum of all the decimal numbers. Thus the loop value is 16.088

Loop2: The links which participate in loop 2 are 5, 9, 6, 8, 7. By considering the decimal numbers of each link, the loop value is 12.066

Loop3: The links which participate in loop3 are 1, 5, 7, 8, 6, 2 and 3. By considering the decimal numbers of each link, the loop value is 18.108

Loop4: The links which participate in loop 4 are 1, 4, 2 and 3. By considering the decimal numbers of each link, the loop value is 10.046

After determining all the loop values, one can find link - loop identification code for figure 14 as:

/ 16.088 / 12.066 / 18.108 / 10.046 /

Hence overall identification code of chain shown in figure 1 is:

/ 4 (3.012) / 4 (2.011) / 2.02 16.088 / 12.066 / 18.108 / 10.046 /

Similarly for chain number 15 shown in figure 15, the overall identification code is:

/2(3.021)/3.012/3.011/2 (2.002)/2.02/2(2.011) / 2(15.087)/10.046/13.067/

Thus we see that chain14 and chain15 resembles isomorphism based on link-link identification code. But when we look at link-loop identification code then they resemble non- isomorphism.

8. Conclusion

(1) In this paper a very simple quantitative approach based on connectivity of chain is presented to detect isomorphism amongst the kinematic chains/mechanisms. If there is one to one equivalence of identification codes (decimal numbers) of links of chains, then the chains are isomorphic otherwise not. This is shown in the table1 by their identification codes of chain1, chain2, etc. The method is also very convenient as it tends itself to be suitable for comparison by inspection.

(2) Every kinematic chain can be represented by a graph. The constituent of the graph will be vertices, edges, etc. Power transmission capacity will be indicated by vertices and their connectivity and edges and their values will indicate transmission efficiency.

(3) Although according to link-link identification code, chain 14 and chain 15 are isomorphic but according to link- loop scheme, they are not isomorphic as their identification codes based on link- loop scheme are different.

(4) In each of kinematic chain, vertices and their connectivity will indicate the capacity for power transmission. Edges and their values reveal the efficiency of transmission. The above facts are analogous to two kinematic chains developing different power with different efficiency.

(5) Suggested numerical measures have strong resemblance to properties and the equations commonly used in information theory. Hence there is every possibility that various other properties developed in information theory may be used with proper interpretation for the study of the linkage, kinematic chains.

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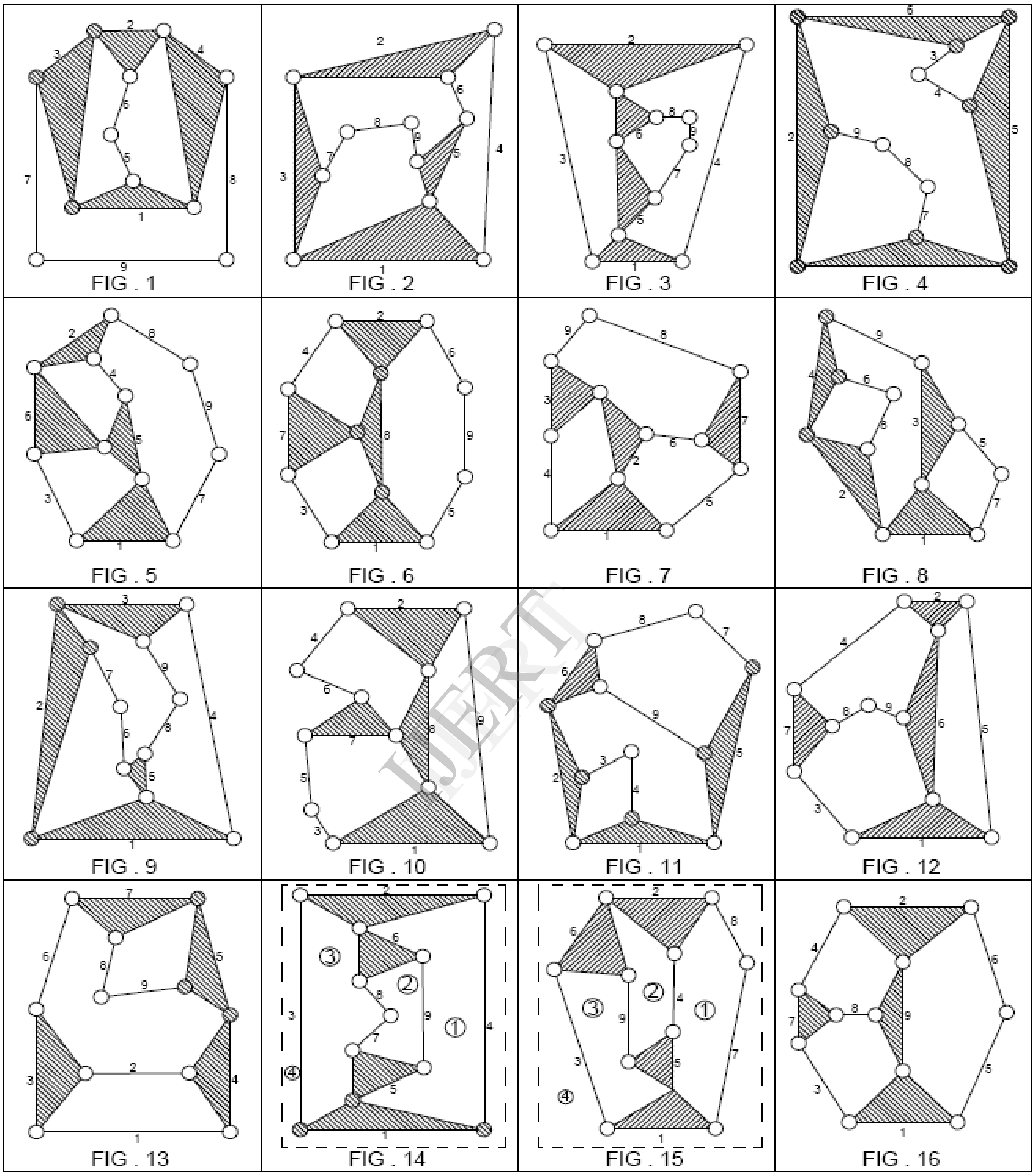
Table 1: Rating of 9 links, 2 dof kinematic change

S.N.	Identification code for chains and links	Power Transmission Capicity	Transmission Efficiency
01	/4(3.012)/4(2.011)/2.02/	0.945446	0.986501
02	/2(3.021)/3.012/3.011/2(2.002)/2.02/2(2.011)/	0.945305	0.990234
03	/3.022/2(3.021)/3.012/2(2.002)/(2.011)/2.02/	0.945266	0.937116
04	/4(3.012)/4(2.011)/2.01/	0.945403	0.987279
05	/2(3.021)/2(2.002)/2(3.012)/2.02/	0.945302	0.990178
06	/3(3.021)/2(2.002)/3.003/2(2.011)/2.02/	0.945301	0.990229
07	/2(3.021)/3.013/3(2.002)/3.03/2(2.011)/	0.945154	0.994202
08	/3.031/3.021/2(3.012)/4(2.011)/2.002/	0.945266	0.99075
09	/2(3.102)/2(3.021)/2.002/4(2.011)/	0.944623	0.989152
10	/3(3.021)/3.003/2.002/4(2.001)/	0.945302	0.990153
11	/2(3.021)/2(3.012)/4(2.011)/2.002/	0.945303	0.990090
12	/2(3.03)/0.3.021/3.012/2(2.011)/3(2.002)/	0.945125	0.989115
13	/3.03/2(3.021)/3.012/3(2.002)/2(2.011)/	0.945574	0.994149
14	/4(3.021)/2(2.011)/3(2.002)/	0.945159	0.994145
15	/4(3.021)/2(2.011)/3(2.002)/	0.945159	0.994145
16	/3.02/2(3.021)/3.012/2(2.011)/3(2.002)/	0.944964	0.991376
17	/2(3.021)/2(3.012)/4(2.011)/2.002/	0.945303	0.990090
18	/2(3.03)/2(3.021)/5(2.002)/	0.945013	0.998328
19	/3.03/2(3.021)/4(2.002)/2.002/	0.945013	0.998322
20	/4.031/3.021/3.011/2(2.0101)/2(2.011)/	0.886605	0.914543
21	/4.031/3.03/3.0201/2(2.0011)/2.002/2.011/2.0101/	0.886639	0.905924
22	/2(3.021)/3(2.0011)/2.0101/2.011/2.02/	0.895736	0.988488
23	/4.022/2(3.0201)/2(2.011)/2(2.0101)/2.002/2.02/	0.938601	0.978795
24	/4.031/3.03/3.0201/2.02/2(2.0101)/2(2.002)/	0.9383991	0.989247
25	/4.031/3.021/3.011/2(2.0101)/2(2.011)/2.02/2.0011/	0.938543	0.981733
26	/4.022/2(3.0201)/2(2.011)/2(2.0101)/2.02/2.002/	0.938601	0.978843
27	/4.031/3.021/3.0111/3(2.011)/2(2.0101)/2.0011/	0.938466	0.981623
28	/3.031/3.03/4.04/4(2.0011)/2(2.011)/	0.938193	0.992397
29	/3.03/3.0201/3(2.0101)/2.011/2.0011/2.002/	0.938395	0.985518
30	/4.022/2(3.0201)/4(2.011)/2(2.0101)/	0.938602	0.978699
31	/4.031/3.021/3.0111/2(2.011)/3(2.0101)/2.002/	0.843911	0.981547
32	/4.03/3.03/3.021/2(2.011)/2(2.0011)/2(2.0101)/	0.938401	0.992281
33	/4.031/3.03/3.0201/3(3.011)/2(2.0011)/2.0101/	0.844013	0.982033
34	/4.04/3.021/2(2.0011)/2(2.0101)/2(2.011)/2.021/	0.938440	0.947698
35	/4.04/2(3.021)/2(2.0011)/2(2.0101)/2(2.011)/	0.938341	0.984865
36	/2(4.04)/4(2.0101)/2.02/2(2.0002)/	0.931532	0.943568
37	/2(4.04)/6(2.0101)/2.002/	0.931542	0.984430
38	/4.0201/4.0301/3.03/4(2.0101)/2.02/2.011/	0.932977	0.975770
39	/5.05/3.03/3(2.01001)/2(2.00101)/2.02/2.011/	0.926538	0.960322
40	/5.044/3.02001/2.02/2(2.011)/4(2.01001)/	0.926743	0.964273

Figures of 9 links, 2dof Kinematic Chains



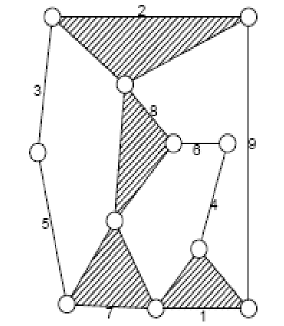


FIG . 17

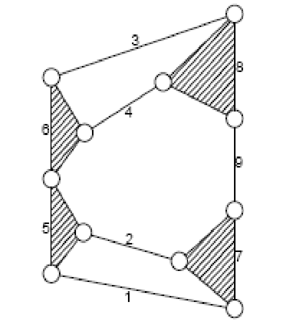


FIG . 18

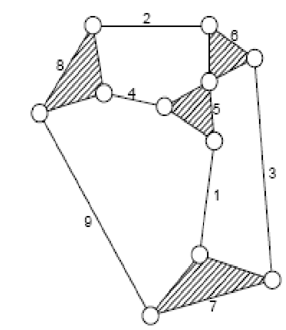


FIG . 19

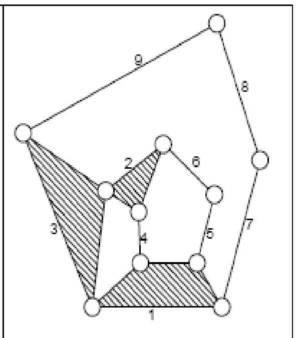


FIG . 20

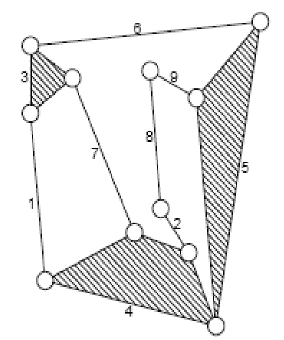


FIG . 21

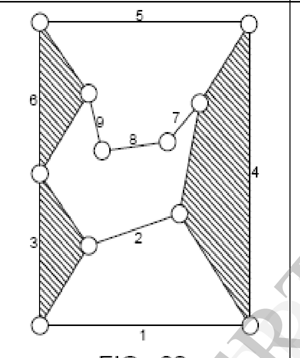


FIG . 22

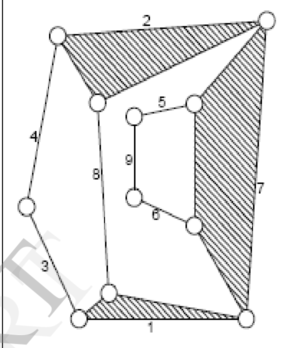


FIG . 23

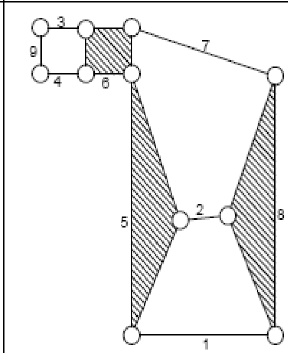


FIG . 24

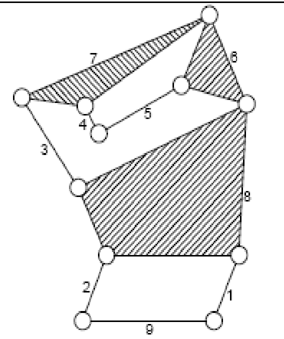


FIG . 25

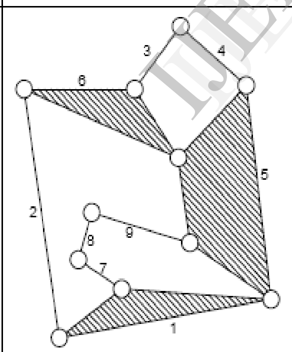
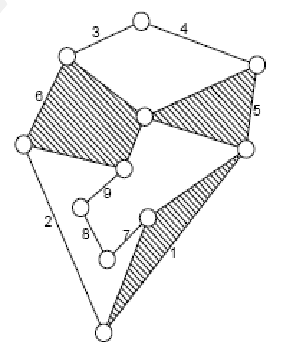


FIG . 26



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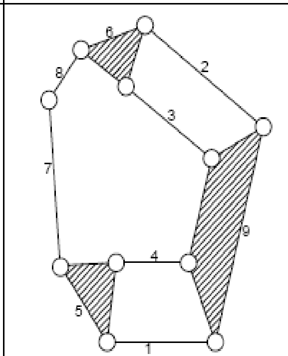

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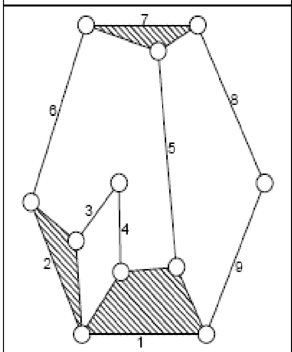

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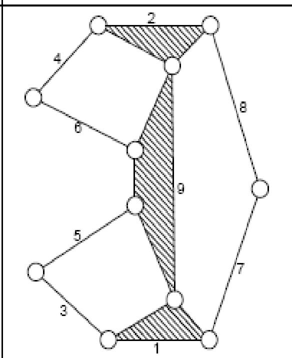

FIG . 30

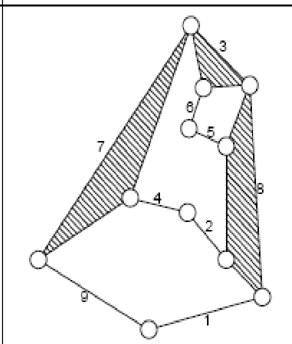

FIG . 31

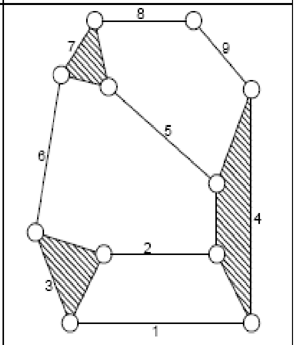

FIG . 32

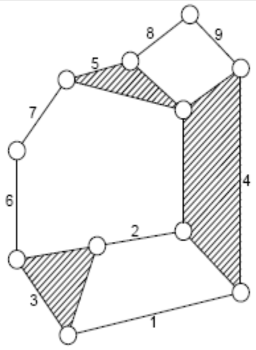

FIG . 33

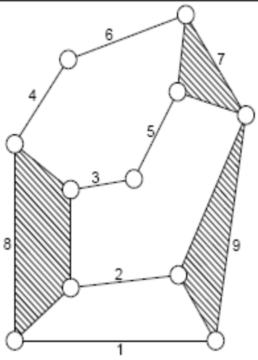

FIG . 34

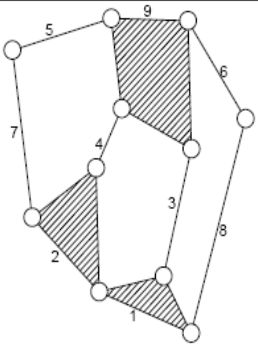

FIG . 35

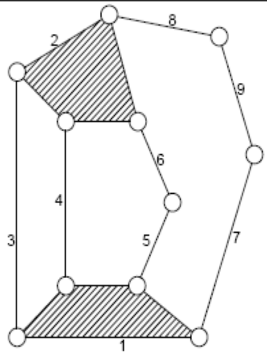

FIG . 36

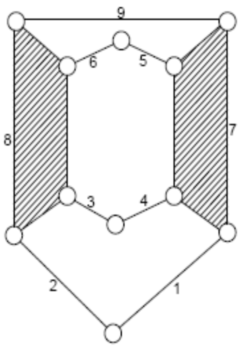

FIG . 37

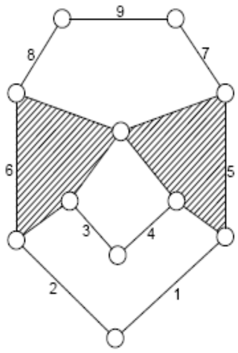

FIG . 38

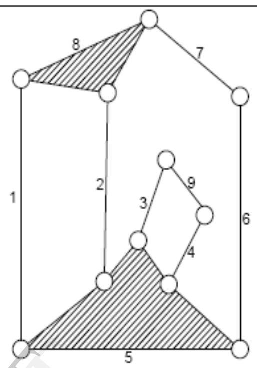

FIG . 39

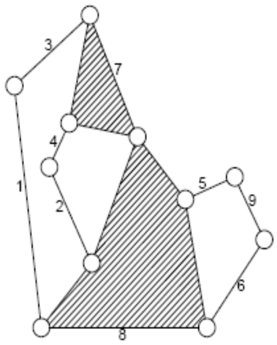

FIG . 40