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Topo-Geometric Model MZ: Feeded Objects

Andrianarizaka Marc Tiana,
Doctoral School in Engineering and Innovation Science and Technology (ED-STII)
Doctoral Research Team for Cognitive Science and Application (EAD-SCA)
University of Antananarivo, MADAGASCAR

Robinson Matio,
Doctoral School in Engineering and Innovation Science and Technology (ED-STII)
Doctoral Research Team for Cognitive Science and Application (EAD-SCA)
University of Antananarivo, MADAGASCAR

Andriamanohisoa Hery Zo,
Doctoral School in Engineering and Innovation Science and Technology (ED-STII)
Doctoral Research Team for Cognitive Science and Application (EAD-SCA)
University of Antananarivo, MADAGASCAR

Summary : **The Topo-geometric approach MZ requires notions of both vector geometry and affine geometry, and also uses topology concepts to transform the elementary notions of presenting a linear program problem. These different forms or presentation models of problems related to the Topo-geometric approach MZ are initially presented in this article and will be followed by elementary results demonstrated by the use of the supported objects of the supported model. Thus, this article is the precursor of the determination of redundant constraints by the Topo-geometric model MZ. The energized objects will thus constitute the language of the proofs of the subsequent propositions.**

Keywords : Linear programming, mathematical writing of the model, writing of the Topo-geometric model MZ, algorithm.

1 INTRODUCTION

This paper constitutes the basis of the Topo-geometric model MZ in search of redundant constraints in a problem of linear programming. This article follows the work on the Topo-Geometric MZ model published in MADA-ETI, ISSN 2220-0673, Vol.2, 2016, www.madarevues.gov.mg which develops the basic concept of Topo-geometric models, and presents the powered objects in search of the redundant constraints of a linear programming problem. All the mathematical object classes will first be presented from the simplest to the most complicated, to model the real concept allowing the decision making. Together with the presentation of each class, the existing operations and relationships will be analyzed; the representation of technical constraints by affine half-spaces is the most important part of this work.

2 THE MEAN OBJECTIVES

2.1 The scalars

A scalar is any real number. Symbolically, a scalar will be represented with a Latin or Greek or lowercase letter.
Examples: $x, y, z, ..., \alpha, ...$

2.2 Indexes

It is a positive or null integer that will be attached to an object of a model, as a unique identifier. Symbolically the indexes will be represented by i, j, k...

To represent the possible values of an index, the following notions will be used:
i = 1, 2...n meaning that the index i varies from 1 to n.

When an objective x of a model is an index, one writes:

$$\left(x_i\right)_{i=1,...,n}$$

which is a condensed representation of :

$$\begin{pmatrix} x_1 \\ x_2 \\ x_{i1} \end{pmatrix}:$$

An object can have two indexes as identifiers. This is the case with matrices, for example.
It is represented by

$$\left(a_{ij}\right)_{\substack{i=1,...,m \\ j=1,...,m}}$$

which designates the matrix :

$$\begin{pmatrix} a_{11}.....a_{in} \\ a_i \\ a_{m1}....a_{mn} \end{pmatrix}$$

To fully exploit the indexes, the symbol $\sum$ (summation) will also be used.
So, to represent

$$c_1x_1 + c_2x_2 + ... + c_nx_n ,$$

it will be simply written:

$$\sum_{j=1}^{n} c_j .x_j$$

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3 THE ELEMENTARY OBJECTS

3.1 Decision spaces

Take again the problems LPP in its known form which consists of finding the unknown-real $x_1, x_2, \ldots, x_n$, and maximize:

$$Z = c_1.x_1 + c_2 x_2 + \ldots + c_n x_n \qquad (3.01)$$

And which satisfy the following conditions :

$$\begin{cases} a_{11}x_1 + a_{i2}x_2 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2 \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_n \end{cases} \qquad (3.02)$$

i) $x_1 \ge 0, x_2 \ge 0 \ldots, x_n \ge 0$ (3.03)

where the coefficients:

$$(c_j)\ j = 1, \ldots, n$$

$$(a_{ij})\ i = 1, \ldots, n\ \ j = 1, \ldots, n$$

$$(b_i)\ i = 1, n$$

Each line of the conditions (3.02) is called a technical constraint.

Conditions (3.03) are called non-negativity constraints.

Definition 3.01: decision-action-point-vectors

We call decision-action any n-tuple :

$\begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix}$ Whose $x_1, x_2, \ldots, x_n$ are elements of $\mathbb{R}$

It will be noted later :

$$(x_j)\ j = 1, \ldots, n$$

Mathematically, we see that it is an element of $\mathbb{R}^n$ relative to a given base. It is also a point in the affine space $\mathbb{R}^n$ relative to a given landmark.

An action decision will be represented by an affine point. Which brings us to the decision-action definition?

Notation : an action decision is noted using a capital Greek letter: A, B, X, Y

Definition 3.02: decision marker - action - decision space

We call decision-action benchmark, the positive orthonormal geometric benchmark, noted :

$$(O, U_1, U_2, \ldots, U_n)$$

related to the aforementioned LPP problem. In this reference, the geometric origin O has a particular meaning : it represents the decision to do nothing, that is, if

$$O = \begin{pmatrix} x_{o1} \\ x_{o2} \\ x_{0n} \end{pmatrix}, \qquad x_{01} = x_{02} = x_{03} = \ldots = x_{0n} = 0$$

To each decision, variable x_j corresponds the unit vector U_j .

The affine space $\mathbb{R}^n$, equipped with the reference $(O, U_1, U_2, \ldots, U_n)$ is called the decision space relating to the LPP problem to be solved.

Despite this striking resemblance to Euclidean affine geometry, a limiting aspect will be presented from the start.

3.2 Postulate

In the topo-geometric theory MZ, the orthonormal affine frame of the decision space action of the problem LPP to be solved is unique.

This assumption means that :

- The concept of basic change does not exist.
- The concept of change of origin does not exist.

However, following a presolved analysis, it may be possible that n changes, which completely modifies the LPP problem.

4 OPERATIONS ON ACTION-DECISIONS

Given an LPP problem, according to definition 3.01, a decision is represented by a point in the decision-action space and / or by a vector of the vector space $\mathbb{R}^n$

This dual nature (both affine and vectorial) of decision-actions is very important.

It allows to express :

The creation of other stock-decisions based on existing stock-decisions

The search for other decision-actions according to a given direction.

4.1 Amplifier of an action decision

Let the decision-action space relate to the reference.

Let X_0 be an action decision and α a given positive real number.

We call amplifier of X_0 using α, the generation of an action decision X such that

$$X = \alpha X_0$$

Mathematically, it is therefore the multiplier of vector X_0 by the scalar α

4.2 Addition of two decision-actions

Let X_0 and X_1 be two decision-actions in a LPP problem.

The addition of X_0 and X_1 is called the creation of a new action decision X such that

$$X = X_0 + X_1$$

<u>Note 4.01</u>

The combination of 2.3.1 and 2.3.2 allows us to model the conic combination concept of two or more action-decisions.

Let the decision-actions $X_1, X_2 \ldots X_k$.

The conic combination of these decisions is the new decision X defined by:

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_k X_k$$

$\alpha_1, \alpha_2 \ldots, \alpha_k$ are positive real numbers or zero.

Note 4.02

From the previous remark, it is clear that the decision satisfies the constraint (2.03) if it is a conic combination of $U_1, U_2 \dots U_n$, that is to say:

$X = \alpha_1 U_1 + \alpha_2 U_2 + \dots + \alpha_n U_n$ with $\alpha_1 \geq 0, \alpha_2 \geq 0 \dots \alpha_n \geq 0$

Let's show that it is reciprocal and true.

Let X be an action decision satisfying the constraint (2.03)

$X = (x_{ji})$ with $x_{ji} \geq 0$

but

$(xj) = \sum_{j=1}^{n} x_j U_j$ with $x_j \geq 0$

4.3 Search axe defined by decision-action Xo and direction the vector decision-action V

Let X_0 and V be action-decisions.

We call search axis starting from X_o and direction U the set of decision actions, noted $\mathbb{R}(X_0, V)$ defined by

$\mathbb{R}(X_0, V) = \{X \in \mathbb{R}^n / X = X_0 + \alpha V$, with $\alpha \in \mathbb{R}$ and $\alpha \geq 0\}$

Mathematically, it is the affine ray derived from X_0 and the vector V.

Conceptually, the research idea is justified by the fact that in practice, any solver is based on a search algorithm starting from an initial point.

Thus, from a given decision-action X_0, and a given search direction V, one can search the points in the universe of decisions that will satisfy the constraints (3.02) and (3.03).

This research idea is not confined to a single research direction. It is also possible to adopt simultaneously two search directions V_1 and V_2. Hence the concept of "research plan ", Starting from X_0 and direction V_1 and V_2.

4.4 Plan of research starting from a decision-action X_0 and directions V_1 and V_2

Let X_0 be a decision-action point and V_1 and V_2 two decision-action vectors of it.

We call search plane starting from X_0 and directions V_1 and V_2, the set noted $P(X_0, V_1, V_2)$ defined as follows :

$P(X_0, V_1, V_2,) = \{X \in \mathbb{R}^n / X = X_0 + \alpha_1 V_1 + \alpha_2 V_2$, where $\alpha_1 \in \mathbb{R}^+$ and $\alpha_2 \in \mathbb{R}^+\}$

By generalizing, we have :

4.5 Research of X_0 direction and $V_1, V_2 \dots V_k$, vectors decisions

Let X_0 be a decision-action point associated with a LPP.

Let $V_1, V_2 \dots V_k$, k decision- action vectors of the same problem LPP.

We call vertex cone X_0 and directions $V_1, V_2, \dots, V_k$, the action decision set denoted:

$C(X_0, V_1, V_2 \dots V_k)$ defined as follows:

$C(X_0, V1, V_2 \dots V_k)$.

It should be noted at once that research axes and research plans are only special cases of research axes.

5 THE TECHNICAL CONSTRAINTS

In practice, any action-decision always has an impact. In the field of linear programming, two categories of impacts can be distinguished :

- Impact-performance.
- Resource impacts.

In this study, we only deal with the impact-resources.

Definition 5.01 resource impact

Let X be an action decision point of a given LPP problem. Let V be a null vector of $\mathbb{R}^n$.We call resource impact according to V, the scalar product of X and V in $\mathbb{R}^n$ noted:

$\Im(X, V) = \langle X, V \rangle$

Where $\langle X, V \rangle$ denotes the scalar product U with V.

The vector V is called unit consumption vector V of the given resource.

Definition 5.02 constraint resources

In our theory, we assume that a constraint that is logical or material may be associated with a resource that is always limited in the associated LPP problem.

Let $A = (a_j)_{j=1\dots n}$ be a vector V of consumption of a given resource.

Let b be the limit value of the resource, the zone of respect of the consumption of the resource limited by b is the set noted ZR (A, b) defined by :

$ZR(A, b) = \{X \in \mathbb{R}^n / \langle A, X \rangle \leq b\}$ (5.01)

In affine geometry, this set is none other than the negative half-space delimited by the noted affine hyperplane. HP (A, b) defined by:

$HP(A, b) = \{X \in \mathbb{R}^n / \langle A, X \rangle = b\}$ (5.02)

NB: The vector A is not an action-decision it is linked to a given resource and allows to calculate the impact of an action X decision on this resource

6 The constraints of non-negativity

The non-negativity constraints translate the fact that the elementary decisions x_i, where i = 1... n must be non-negatives, is :

$x_i \geq 0 \quad \forall i = 1, 2 \dots n$

Let us note immediately that the relations :

$x_i \geq 0 \quad \forall i = 1, 2 \dots n$

can be written:

$x_i \leq 0 \quad \forall i = 1, 2 \dots n$

Expression equivalent to:

$\langle -U_i, X \rangle \leq 0$

Therefore, a non-negativity constraint can also be considered as a resource constraint delimited by $ZR = (-U_i, 0)$ called non-negativity zone ZNN_i.

However the combination of all non-negativity constraints has a particular geometric meaning that we call " non-negativity cone ".

6.1 non-negativity cone

We call non-negativity cone noted C_0^+ the cone of search from point-decision O and direction of all decision vectors :

$$U_i \ ,i=1,2,....,n$$

Geometrically we have:

$$C_0^+ = \left\{ X \in \mathbb{R}^n \ / \ X = \sum_{i=1}^{n} \alpha_i U_i \quad where\ \alpha_i \geq 0 \right\} \qquad (6.01)$$

$$= C\hat{o}ne(O,U_1,U_2,...,U_n)$$

Note 6.01:

It is obvious that $C_o^+ = \cap_{i=1}^{i} ZNN_i$

6.2 Non-negativity face

It is to be recalled that the geometric topo reference of the LPP problem is immutable (O, U_0, U_2..., U_n).

This means that we cannot change the place of a vector U_i of this reference, for all i, for all

i = 1, 2..., n.

We call the non-negativity face, denoted by FNN_i, the decision point set defined as follows:

$$FNN_i = \begin{cases} P(O,U_1,Un) \ for\ i=1 \\ P(O,U_{i-1},U_i) \ for\ i=2,3,...,n \end{cases}$$

6.3 Non-negativity axes

For all i = 1, 2... n, we call non-negativity axis, ANN_i , the set of decision point defined as follows :

ANN_i =R (O, U_i) i = 1.2... n (6.02)

Note that ANN_i is none other than the search axis starting from O and with the direction U_i.

Recall the concept of redundant constraints.

7 REDUNDANT CONSTRAINTS

A redundant constraint is a constraint which can be deleted from a system of linear constraints without changing the feasible region or acceptable solution area.

If we look at the next system of *m* and n constraints of linear inequality, no negative (m ≥ n), we can adopt the matrix writing:

$$AX \leq B, \quad X \geq 0, \qquad (7.01)$$

But

$$A \in R^{m\,x\,n}, \quad b \in R^{m}, X \in R^{n},$$

And $0 \in R^{n}$.

Let

$$AT_i \leq b_i$$

be the *i* th constraint of system (7.01) and let

$$S = \{X \in R^{n} / AT_i\, X \leq bi,\ X \geq 0\}$$

The acceptable solution area associated with the system (7 .01).

Let

$$S_k = \{X \in R^{n} / AT_i\, X \leq b, X \geq 0, i \neq k\}$$

The acceptable solution area associated to the constraint :

$$A_i X \leq b_i, \quad i = 1, 2, \ ..., m, i \neq k$$

of the system .

The k^{th} constraint :

$$A_k X \leq b_k \qquad (1 \leq k \leq m)$$

is redundant for the system (7.01) if and only if

S = S_k.

Definition 7.01

Redondant constraints can be classified as weak and strong redundant constraints. 1.3.2

Low redundancy constraints

The constraint $AT_i\ X \leq b_i$ is weakly redundant if she is redundant and

$AT_i\ X = b_i$ for all X ∈ S.

8 RELATIONSHIP BETWEEN MZ OBJECTS

The relationships described in this section are the basic ones. More complex relationships will be discussed in the next chapter.

Moreover, the combinations between point-decisions and point-vectors have already been seen. Therefore, only the following points will be dealt with:

- The relation between a decision-point and a constraint-technical zone.

- The relation between a research axis and a zone of technical constraints.

- The relation between a research plan and a zone of technical constraints.

8.1 Relationship between a decision--point and a technical constraint zone

Since a point-decision is represented with a point, and a zone of technical constraints is represented by a half-space, a set of decision-point, the essential same basic relation between one of these two object classes, is the ensemblist relation of belonging.

Let X_0 be a decision point and ZCT (A, b) a constraint zone. The relationship

$$X_0 \in ZCT(A,b) \quad means \quad < A, X_0 > \leq b$$

Since in the LPP problem model, the constraints are indexed, i.e. numbered from 1 to n, the corresponding technical constraints areas will also be noted. :

ZCT_i i = 1,2... n

This makes it possible to represent the system of techniques as follows:

Let X_0 be a decision-point satisfying all the technical constraints of the problem.

We have :

$$< A_i, X_0 > \leq b_i \quad for \quad i = 1,2,...,n$$

This can be written :

$X_0 \in ZCT_i$ For everything i = 1, 2... n

From where,

$$X_0 \in \cap_{i=1}^{m}\ ZCT_i \qquad (8.01)$$

Relationship that is a basis for redundancy and infeasibility analysis.
Moreover, when X_0 does not belong to a ZCT_i, we also say that X_0 is exterior to ZCT_i as the outer term, and its inner opposite has a topological connotation, the topological definitions of these terms will be recalled.

8.1.1 Projection of a point on a hyperplane
Let ZCT_i be the area of technical constraints delimited by the HP hyperplane (A_i, b_i) and let X_0 be any decision-point.
We call projection of X_0 on the hyperplane HP (A_i, b_i), the point noted X'_0 such that :

X_0 belongs to HP (A_i, b_i) and is collinear to the vector A_i.
X'_0 is the intersection of HP (A_i, b_i) with the straight line passing through X_0 and whose direction vector is A_i.

Proof :
♣
Mathematically this right is defined by :

$$X \in \mathbb{R}^n \quad with \quad X = X_0 + \alpha A_{i,} \qquad \alpha \in \mathbb{R}$$

The intersection is written,

$$X'_0 = X_0 + \alpha A_i$$
$$< A_i, X'_0 > = b_i$$
$$< A_i, X_0 + \alpha A_i > = b_i$$
$$< A_i, X_0 > + \alpha < A_i, A_i > = b_i$$

$$\Rightarrow \alpha < A_i, A_i > = b_i - < A_i, X_0 >$$
$$\Rightarrow \alpha = \frac{b_i - < A_i, X_0 >}{< A_i, A_i >}$$

Finally, we have :

$$X_0 = X_0 + \left[\frac{b_i - < A_i, X_0 >}{< A_i, A_i >}\right] A_i \qquad (8.02)$$

♦

Moreover, the distance between X_0 and X'_0 denoted d (X_0, X'_0) is equal to :

$$d(X_0, X'_0) = \left|\frac{bi - < A_i, X_0 >}{< A_i, A_i >}\right| \sqrt{< A_i, A_i >} \qquad (8.03)$$
$$d(X_0, X'_0) = \left|\frac{b_i - < A_i, X_0 >}{\sqrt{< A_i, A_i >}}\right|$$

This distance is also called the distance from the point Xo to the hyperplane HP (A_i, b_i).

8.1.2 Orientation of the vector Ai with respect to ZCT_i
Proposition 8.01
The vector is always oriented from inside to outside of ZCT_i.
Proof:
♣
Let X_0 a ZCT_i point that does not belong to the hyperplane $HP(A_i, b_i)$:

$$X_0 \in ZCT_i \quad \text{and} \quad X_0 \notin HP(A_i, b_i)$$

Is X_0' the projection of X_0 on $HP(A_i, b_i)$
We have :

$$X_0' = X_0 + \left(\frac{b_i - < A_i, X_0 >}{< A_i, A_i >}\right) A_i$$

$$\overrightarrow{X_0 X_0'} \left[\frac{b_i - < A_i, X_0 >}{< A_i, A_i >}\right] A_i$$

The product of $\overrightarrow{X_0 X_0'}$ with A_i is :

$$< \overrightarrow{X_0 X_0'}, A_i > = \left(\frac{b_i - < A_i, X_0 >}{< A_i, A_i >}\right) < A_i, A_i >$$

$$< \overrightarrow{X_0 X_0'}, A_i > = b_i - < A_i, X_0 >$$

but $X_0 \in ZCTi$
which means :

$$< A_i, X_0 > \leq b_i \quad \text{and} \quad X_0 \notin HP(A_i, b_i)$$

So $< A_i, X_0 > \leq b_i$

Therefore we have: $< A_i, X_0 > < b_i$

From where $b_i - < A_i, X_0 > > 0$

Finally we get the following relation:

$$< \overrightarrow{X_0 X_0'}, A_i > > 0$$

♦ 8.1.3

Direction of a decision vector with respect to ZCT_i
Let V be a decision vector, and let ZCT_i be a technical constraint zone V parallel to HP (A_i; b_i), that is to say at the ZCT_i boundary.

8.1.3.1 External orientation
We say that V has an external orientation with respect to ZCT_i, if <V, A_i> is strictly positive.

8.1.3.2 Parallel orientation
We say that V has a parallel orientation with respect to ZCT_i, if <V, A_i> is equal to 0.

Note 8.01:
The qualification "parallel" refers to the fact that V is parallel to HP (A_i ; b_i), that is to say at the ZCT_i border.

8.1.3.3 Internal orientation
We say that V has an internal orientation with respect to ZCT_i, if <V, A_i> is strictly negative.

8.1.4Characterization of an Inner Point of ZCT_i
Proposition 8.02
X_0 is an inner point of ZCT_i if and only if :

$$< A_i, X_0 > < b_i$$

Proof:
♣
a) Let X_0 be a point inside ZCT_i.
According to 2.7.1.3.1, there exists a strictly positive reality r such that the ball whose center is X_0 and with a radius r is entirely contained in ZCT_i.

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Logically, this means that if X_0' designates the projection of X_0 on $HP(A_i, b_i)$, then the distance between X_0 and X_0 ' is equal to :

$$\frac{|b_i - < A_i,\ X_0 >|}{\sqrt{< A_i,\ A_i >}}$$

And

$$r \le \frac{b_i < A_i,\ X_0 >}{\sqrt{< A_i,\ A_i >}}$$ but r is strictly positive,

$$\frac{b_i - < A_i, X_0 >}{\sqrt{< A_i, A_i >}}$$ is strictly positive from where

$$b_i - < A_i,\ X_0 > \text{ is also strictly positive}.$$

b) Reciprocally, X_0 is a point of ZCT_i such that $< A_i,\ X_0 > < b_i$

Let us show that this is an inside point of ZCT_i.
Let X_0' be the projection of X_0 on the hyperplane. We saw that :
The distance between X_0 and X_0' is equal to:

$$\frac{b_i - < A_i, X_0 >}{\sqrt{< A_i, A_i >}}$$

Let

$$r = \frac{1}{2}\ \frac{b_i - < A_i, X_0 >}{< A_i, A_i >}$$

We have $r > 0$;

and it is obvious that the ball whose center is X_0 and radius r is entirely included in ZCT_i, which means that X_0 is an internal point of ZCT_i .

♦

8.1.4.1 S border point

A point $X_0 \in \mathbb{R}^n$ is said to be a border point of S if :
X_0 is an element of Set ;
Whatever the positive real number r is, the open ball of center X_0 and radius r contains both elements of S other than X_0 and elements of $\overline{S}$ (the complement of S in Euclidean affine space $\mathbb{R}^n$).

8.1.5 S border

Let S be a non-empty set in the Euclidean affine space with the orthonormal coordinate system
$(O, U_1, U_2, ..., U_n)$. We call the boundary of S the set noted Fr (S) containing all the boundary points of S.

Proposition 8.03

We characterize the border points of the ZCT_i by:

$$Fr(ZCT_i) = HP(A_i; b_i)$$

Proof:
♣
Let's first show the first inclusion :

$$Fr(ZCT_i) \subset HP(A_i, b_i)$$

Let X_o be an element of Fr (ZCT_i), that means that X_0 is also an element of ZCT_i so,

$$< A_i,\ X_0 > \le b_i$$

If $Xo \notin HP(A_i, b_i)$, this means that $< A_i,\ X_0 > \ne b_i$

From where,

$< A_i, X_0 > < b_i$, meaning that X_0 is an inner point of ZCT_i, which contradicts the fact that X_0 is a border point of ZCT_i.

Reciprocal :

$$HP(A_i, b_i) \subset Fr(ZCT_i)$$

Let X_0 be a point of HP (A_i,bi) which means that :

$$< A_i,\ X_0 > = b_i$$

Let r be any strictly positive real number and X_1 the point defined by :

$$X_1 = X_0 + \frac{r}{2} . \frac{A_i}{\sqrt{< A_i, A_i >}}$$

Note that the distance between X_0 and X_1 is equal to $\frac{r}{2}$, which is strictly less than r.
Moreover, according to Proposition 8.01, A_i is always oriented towards the outside of ZCT_i so X_1 do not belong to ZCT_i.
So X_1 belongs to the open ball of center X_0 and radius r.
Finally, let us show that the open ball of center X_0 and radius r contains points of ZCT_i other than X_0.
Let X_2 the point defined by :

$$X_2 = X_0 - \frac{r}{2} . \frac{A_i}{\sqrt{< A_i, A_i >}}$$

We obtain:

$$< A_i, X_2 > = < A_i, X_0 - \frac{r}{2} . \frac{A_i}{\sqrt{< A_i, A_i}} >$$

$$= < A_i, X_0 > - \frac{r}{2} . \frac{1}{\sqrt{< A_i, A_i >}} < A_i, A_i >$$

$$< A_i, X_2 > = < A_i, X_0 > - \frac{r}{2}$$

But

$$< A_i,\ X_0 > = b_i \quad \Rightarrow \quad < A_i,\ X_2 > = b_i - \frac{r}{2}$$

As

$$r > 0 \quad \Rightarrow \quad \frac{r}{2} > 0$$

From where

$$b_i - \frac{r}{2} < b_i$$

We can write :

$$< A_i,\ X_2 > < b_i$$

Which means that X_2 belongs to ZCT$_i$.

In summary, every strictly positive real number in the open ball of center X_0 and radius r contains both elements of ZCT$_i$ other than X_0 and elements of ZCT$_i$.

This means that X_0 is a frontier point of ZCT$_i$.

Finally, in combination with the two way, we have :

$$Fr(ZCT_i) \subset HP(A_i, b_i)$$

and

$$Fr(ZCT_i) \supset HP(A_i, b_i)$$

So we have

$$Fr(ZCT_i) = HP(A_i,\ b_i)$$

Proposition 8.04

The zone of technical constraints ZCT$_i$ is topologically closed.

Proof :

♣

Let X_0 be a point not belonging to ZCT$_i$ (where ZCT$_i$ denotes the complement of ZCT$_i$ in the affine space).

As

$$X_0 \notin ZCT_i$$

So

$$< A_i, X_0 > > b_i$$

Let X_0' be the projection of X_0 on $HP(A_i, b_i)$. We showed that :

The distance between X_o' and X_0 is equal to :

$$\frac{|b_i - < A_i \ \ X_0 >|}{\sqrt{A_i, A_i}}.$$

but

$$< A_i, X_0 > > b_i \Rightarrow b_i - < A_i, X_0 > \ < 0$$

so

$$|b_i - < A_i,\ X_0 >| = -(b_i - < A_i,\ X_0 >)$$
$$= < A_i,\ X_0 > - b_i$$

The distance between X_0 and the border $HP(A_i, b_i)$ is equal to :

$$\frac{< A_i,\ X_0 > - b_i}{\sqrt{< A_i,\ A_i >}}$$

Let :

$$r = \frac{1}{2}.\frac{< A_i,\ X_0 > - b_i}{\sqrt{< A_i,\ A_i >}} \text{ be}$$

It is easy to show that the open ball of center X_0 and radius r is entirely contained in $\overline{ZCT_i}$.

Therefore $\overline{ZCT_i}$ is open.

So ZCT$_i$ is closed.

In summary of Proposition 8.03 and Proposition 8.04, each zone of ZCT$_i$ technical constraints is closed and their boundary is none other than the hyperplane $HP(A_i, b_i)$.

8.2 Relationship between a search axis R (X_0, V) and a technical constraint zone ZCT$_i$

Consider an area of ZCT$_i$ technical constraints. Let also be the radius R(X_0, V) coming from X_0 and direction vector V, representing a search axis. This relationship is based on the existence and uniqueness of the solution of the equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle,\ \ \alpha \text{ is the unknown.}$$

Proof:

♣

Since the two objects are sets of decision points, the main combination that can be imagined between them is the intersection. In addition, since the area of technical constraint is closed and the hyperplane HP (A_i, b_i) is closed, we will be much more interested in the intersection of the radius R (X_0, V) with this boundary HP (A_i, b_i). Indeed, if X_0 is outside of ZCT$_i$, then such an intersection gives us the point of entry from the outside to the inside of ZCT$_i$ along the radius. On the other hand, if X_0 is inside ZCT$_i$, it gives the exit point of ZCT$_i$.

Let I denote this point of intersection : as I belongs to R (X_0, V)

We have $I = X_0 + \alpha V \quad avec \quad \alpha > 0$

Since I also belongs to HP (A_i, b_i), we can write :

$$\langle A_i,\ I\rangle = b_i$$

By combining these two relationships, we have

$$\langle A_i,\ X_0 + \alpha V\rangle = b_i$$

By developing, we get

$$\langle A_i,\ X_0\ \rangle + \alpha\langle A_i, V\rangle = b_i$$

which is an equation where α is the unknown.

This equation gives :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle \qquad (8.04)$$

The existence and uniqueness of α depend on the values of b i - (A_i, X o) and (A_i,V), hence X_0 and V.

8.2.1 Where X_0 is outside ZCT$_i$ and where V is not facing inwards from ZCT$_i$

The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has no solution.

Proof:

♣

This case is mathematically translated by :

$$\begin{cases}\langle A_i,\ X_0\rangle \ \rangle \ b_i \\ \langle A_i,\ V\rangle \geq 0\end{cases} \text{ and } \langle A_i,\ X_0\rangle \ \rangle \ b_i$$

leads that,

$$b_i - \langle A_i,\ X_0\rangle \ \langle \ 0$$

Also as

$$\alpha \geq 0 \quad et \quad \langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle \ \langle \ O$$

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which is impossible.
In this case I do not exist.

8.2.2 Where X_0 is outside ZCT_i and where V is inward ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has a unique solution.

$$\alpha = \frac{b_i - \langle A_i,\ X_0\rangle}{\langle A_i,\ V\rangle} \rangle 0$$

Proof:
♣
We have :

$$b_i - \langle A_i, X_0\rangle \langle 0 \quad et \quad \langle A_i, V\rangle \langle 0$$

which gives

$$\alpha = \frac{b_i - \langle A_i,\ X_0\rangle}{\langle A_i,\ V\rangle} \rangle 0$$

which is in addition a unique value.

In this case we say that we have a single point of entry starting from X_0, and moving along the axis of R (X_0, V).

8.2.3 Where X_0 is on ZCT_i and where V is outside ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

admits a null solution.

Proof:
♣
This case results in :

$$b_i - \langle A_i,\ X_0\rangle = 0 \ and\ \langle A_i,\ V\rangle \rangle 0$$

The only solution available is :

$$\alpha = 0$$

Meaning that,

$$I = X_0$$

♦

8.2.4 Where X_0 is on the ZCT_i border and where V is parallel oriented to HP (A_i, b_i)

The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has an infinity of solutions.

Proof :
♣
Mathematically we translate this case by :

$$b_i - \langle A_i,\ X_0\rangle = 0 \ \ and\ \ \langle A_i,\ V\rangle = 0$$

This gives us infinity of solutions. In fact, R(X_0, V) is included in HP (A_i ; b_i), and the intersection is none other than R (X_0, V).

8.2.5 Where X_0 is on the ZCT_i border and where V is inward ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has a unique solution.

Proof :
♣
In this case

$$b_i - \langle A_i, X_0\rangle = 0 \quad et \quad \langle A_i, V\rangle \langle 0$$

which leads to

$$\alpha = 0$$

That is to say $I = X_0$, unique solution.

♦

8.2.6 Where X_0 is inside ZCT_i and where V is outside ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has a unique solution

$$\alpha = \frac{b_i - \langle A_i, X_0\rangle}{\langle A_i, V\rangle}$$

Proof :
♣
This case results in $\langle A_i, X_0\rangle \langle b_i \ \ et\ \langle A_i, V\rangle \rangle 0$:
Which leads to:

$$b_i - \langle A_i,\ X_0\rangle \rangle 0$$

Hence the unique solution

$$\alpha = \frac{b_i - \langle A_i,\ X_0\rangle}{\langle A_i,\ V\rangle} \quad (2.12)$$

♦

8.2.7 Case where Xo is inside ZCT_i and where V is parallel oriented to ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has no solution.

Proof :
♣
As in the previous case, we have :

$$b_i - \langle A_i,\ X_0\rangle \rangle 0 \text{ but } \langle A_i, V\rangle = 0$$

Which leads to an impossibility, meaning that $R(X_0, V)$ will never intercept the border of ZCT_i.

♦

8.2.8 Where X_0 is inside ZCT_i and where V is inward ZCT_i
The equation :

$$\alpha\langle A_i,\ V\rangle = b_i - \langle A_i,\ X_0\rangle$$

has no solution.

Proof :

♣

As recently,

$b_i - \langle A_i,\ X_0 \rangle \rangle 0\ and\ \langle A_i,\ V \rangle \rangle 0$

In this case, the equation

$$\alpha \langle A_i,\ V \rangle = b_i - \langle A_i,\ X_0 \rangle$$

has no positive solution, meaning that $R(X_0, V)$ will never intercept the border of ZCT_i.

♦

8.2. 9 Algorithm for determining the intersection of a search axis R (X_0, V) with the technical constraint zone ZCT_i

The previous eight cases can be summarized in the following algorithm :

- Case where there is no intersection.

No intersection :

- Case with

$$\langle A_i,\ X_0 \rangle \rangle b_i \quad and \quad \langle A_i,\ V \rangle \langle 0$$

We have a single point of intersection

$$\left\{ X_0 + \frac{b_i - \langle A_i, X_0 \rangle}{\langle A_i, V \rangle} V \right\}$$

- If $\langle A_i, X_0 \rangle = b_i$ and $\langle A_i, V \rangle \rangle 0$.

There is unique solution :

$$\{X_0\}$$

-If $\langle A_i, X_0 \rangle = b_i$ and $\langle A_i, V \rangle \langle 0$.

There is unique solution:

$$\{ X_0 + \frac{b_i - \langle A_i, X_0 \rangle}{A_i, V} V \}$$

-If $\langle A_i, X_0 \rangle \langle b_i$ and $\langle A_i, V \rangle \geq 0$

No solution : $\varnothing$

8.3 Relationship between a search-plane C(X_0, V_1, V_2) with linearly independent to V_1 and V_2 and a of ZCT_i technical constraints zone

As for section 8.2, these two objects are sets of decision points, so this section will describe their intersection.

Let I be such a point. It is thus of the form :

$I = X_0 + \alpha_1 V_1 + \alpha_2 V_2$ with α_1 *and* α_2 are positives.

More like I belongs to the border of ZCT_i , we can write :

$$\langle A_i,\ I \rangle = b_i$$

Which leads to :

$$\langle A_i,\ X_0 + \alpha_1 V_1 + \alpha_2 V_2 \rangle = b_i$$

From where

$$\langle A_i, X_0 \rangle + \alpha_1 (A_i, V_1)\rangle + \alpha_2 (A_i, V_2)\rangle = b_i$$

Let

$$\alpha_1 \langle A_i, V_1 \rangle + \alpha_2 \langle A_i, V_2 \rangle = b_i - \langle A_i, X_0 \rangle$$

which is a linear equation with two unknowns variables, α_1 *et* α_2 positive.

As for 2.6.2, the existence and uniqueness of the solutions to this problem depend on the three objects X_0, V_1 *et* V_2 .

- For X_0, there are three possible situations : to be outside of ZCT_i, or be on the border of ZCT_i or be inside of ZCT_i.

- For V_1 there are three possibilities : be outward facing from ZCT_i or be oriented parallel to ZCT_i or be oriented towards the inside of ZCT_i

- For V_2 the possibilities are the same as for V_1.

All in all, we have $3 \times 3 \times 3$, that is to say, 27 possible cases to be made.

8.4 Search along an axis R (X_0, V) in the ZNN non negativity zone

The ZNN non-negativity zone has been defined as

$$ZNN = \bigcap_{j=1}^{n} ZNN_i$$

where

$$ZNN_i = ZR(-U_i,\ \varnothing) \qquad (8.05)$$

Conceptually, this means that we assimilate ZNN_i to a resource constraint zone, and if a X_0 point belongs to ZNN_i this means that X_0 satisfies the constraint associated with ZNN_i . It is therefore a stage situation.

The concept of research implies that there is a situation or state of deposition, and that from this situation, one move to another situation or state. And we have already seen that this research, when it is linear, can be modeled by the concept of axis of research starting from a given point and moving according to a given vector $R(X_0, V)$

In this section, the starting point of the search is the point X_0, which is supposed to be located in the ZNN non-negativity zone. Since the new decisions found must have remained in ZNN, it is logical to study the conditions under which this search leads us out of ZNN. And as in the case of ZCT_i technical constraint zones, we will need to define the boundary concept in ZNN.

For ZNN_i, there is no problem.

Indeed,

$$Fr(ZNN_i) = \mathrm{HP}(-U_i,\ 0)$$

given that ZNN_i is only half affine delimited by the hyperplane $\mathrm{HP}(-U_i, O)$.

And as HP ($-U_i$, 0) is included in ZNN_i, we know that ZNN_i is a closed area.

8.4.1 Frontier of the ZNN_i non-negativity zone

Proposition 8.05

$$Fr(ZNN) = \bigcup_{j=1}^{n} \left(HP(-U_j, 0) \cap C_0^+ \right)$$

Proof :

♣

It suffices to show that for every j, $HP\left(-U_j,0\right)\cap C_0^+$ is a function of ZNN.

Step 1:

Let us show that every X point of $HP\left(-U_j,0\right)\cap C_0^+$ is a border point of ZNN.

Let r be any positive real number. This is to show that the open ball of center X and radius r denoted _______B (X, r), contains both an element of ZNN other than X and an element of ZNN other than X is:

$X = (x_i)_{i=1\ldots,n}$

The fact that X ∈ HP (-U_j, o) ∩ C_o means that

$x_i \geq 0$, ∀ i = 1... n and that $x_j \geq 0$

Let us take the point X' = $(x'_i)_{i=1\ldots n}$ defined as follows :

$$x'_i = \begin{cases} x_i & for \quad all \quad i \neq j \\ -\frac{1}{2} & if \quad i = j \end{cases}$$

Similarly let us take the point:

X'' = $(x''_i)_{i=1\ldots n}$ defines as follows :

$$x''_i = \begin{cases} x_i & for\,all \quad i \neq j \\ +\frac{1}{2} & if \quad i = j \end{cases}$$

It is obvious that :

X' ∈ ZNN and has X'' ∈ ZNN

Moreover, it is also obvious that :

X' ∈ B (X, r) and X'' ∈ B (X, r)

This shows that the open ball B (x, r) contains both the point X 'which is the element outside ZNN and the point X'' which is the element in ZNN, and it is obvious that X' is different from X and that X'' is different from X.

Therefore X is a border point of ZNN.

2nd step:

Let us show that HP (-U_j, 0) ∩ C_0^+

And in summary, HP (-U_j, 0) is indeed a ZNN border and their meeting is also a border of ZNN.

In addition, let X be a point of ZNN such that :

$$X \notin \bigcup_{j=1}^{n} HP(-Uj,\,0) \cap C_0^+$$

It means that :

$$X \notin \bigcup_{j=1}^{n} HP(-Uj,\;0) \cap C_0^+)$$

Given that $C^+{}_0$ = ZNN, it means that

$$X \notin \bigcup_{j=1}^{n} HP(-U_j,\;0) \cap C_0^+$$

Let X^j be the projection of X on HP (-U_j, 0) and d_j the distance between X and X_j.

As

$$X^j \neq X \qquad \forall_j = 1,\ldots,n$$

Then $d_j \neq$ o $\quad \forall_j = 1,\ldots,n$

Let

$$r = \frac{1}{2}\min(dj) \qquad \forall j = 1,\ldots,n$$

It is obvious that r> 0 and that the open ball B (X, r) is included in ZNN.

There fore,

$$\bigcup_{j=1}^{n} HP(-U_j,\;0) \cap C_0^+) = Fr(ZNN)$$

<u>Note</u> 8.02

$HP\left(-U_j,0\right)\cap C_0^+$ is the cone

$$C\left(O,U_1,U_2,\ldots,U_{j-1},U_{j+1},\ldots,U_n\right)$$

We will call it " conic wall of non-negativity j Noted $MCNN_j$.

Proof :

♣

$$MCNN_j = HP\left(-U_j,0\right)\cap C_0^+$$

$$et\;ZNN = \bigcup_{j=1}^{n} MCNN_j$$

In other words, the ZNN border is none other than the conic wall meeting of non-negativity.

Note further that

$$MCNN_j \subset HP\left(U_j,0\right)$$

So

$$MCNN_j \subset Fr\left(ZNN_j\right)$$

♦

8.4.2 Exit point of ZNN following R (X_0, V)

Proposition 8.06

We assume that X_0 is in ZNN. Since ZNN is closed, the exit point is the point of contact or intersection between $R\left(X_0,V\right)$ and $Fr\left(ZNN_j\right)$.

Proof

♣

Let Fj be the point of $R\left(X_0,V\right)$ et $Fr\left(ZNN_j\right)$ which is none other than $HP\left(-U_j,0\right)$,

$I_j \in R\left(X_0,V\right)$, so I_j is the form

$$I_j = X_0 + \alpha_j V \quad \text{with } \alpha_j \geq 0$$

Similarly

$$I \in HP(-U_j, 0) \ ,$$

So we check the equation :

$$< -U_j, I >= 0 \quad ou \quad < U_j, I_j >= 0$$

From where, $< U_j, X_0, \alpha_j V > = 0$

$$< U_j, X_0 > + \alpha_j < U_j, V > = 0$$

Finally, we obtain the equation:

$$\alpha_j < U_j, V > = - < U_j, X_0 >$$

with the condition $\alpha_j \geq 0$.

Based on the results of section 5.2, we can say that the research axis $R(X_0, V)$ leads us outside of ZNNj only if V is oriented outside ZNN_j,

that is, if

$$< -U_j, v > \ > 0 \ ,$$

in which case,

$$\alpha_j = \frac{- < U_j, X_0 >}{< U_j, V >} \qquad (8.06)$$

Let $X_0 = (X_0^i) i = 1, \ldots, n$

$V = (v_i) i = 1, \ldots, n$ and $< U_j, X_0 >= x_i^0$

$< U_i, V >= v_j$

The expression of α_j becomes :

$$\alpha_j = \frac{-x_i^o}{v_j} \qquad (8.07)$$

Which is good non-negative because $x_i^o \geq 0$ *and* $v_j < 0$.

♦

8.4.3 ZNN exit point following R (X_0, V)

We assume that X_0 is in ZNN. Still based on previous results, $R(X_0, V)$ leads us out of at least one ZNN_j, that is, there exists a j such that $v_j < 0$ noting :

$$V = (v_j) \ j = 1, \ldots, n \ .$$

Let IN (V) denote the set of indices j such that $v_j < 0$.

$$IN(V) \ = \{ j \in [1, n] \quad tel \quad que \quad v_j < 0 \}$$

Hence the proposal :

Proposition 8.07

The research axis is in the non-negativity zone ZNN only if IN (V) is non-empty. In this case, the exit point I is given by :

$$I = X_0 + \alpha V \ ,$$

Or

$$\alpha = \min_{j \in In(V)} \left\{ \frac{-x_i^o}{v_j} \right\} \qquad (8.08)$$

Proof:

♣

Given that $R(X_0, V)$ is a totally ordered set, the exit point of I such that verify:

$$I = \min_{j \in IN(V)} (I_j) \qquad where \quad I_j = X_0 (X_i^0) V \qquad (8.09)$$

The Proof is immediate. The output point I is none other than the first smallest I_j.

♦

9 CONCLUSION

Through this article, we presented all the art mathematical object classes required for topo-geometric modeling MZ and the mean objectives associated with Topo-geometric definitions, as well as the conventional operations and properties of constraints LPP defining hyperplanes, are mentioned.

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