TO FIND A 2-TUPLE DOMINATING SET OF AN INDUCED SUBGRAPH OF A NON-SPLIT DOMINATING SET OF AN INTERVAL GRAPH USING AN ALGORITHM

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ABSTRACT:

In Graph Theory, a connected component of an undirected graph is a subgraph in which any two vertices are connected to each other by paths. For a graph G, if the subgraph G itself is a connected component then the graph G is called connected, else the graph G is called disconnected and each connected component subgraphs is called its components. A dominating set D a of graph $G(V,E)$ is a non-split dominating set, if the induced subgraph $< V-D >$ is connected. The non-split dominating number $\gamma_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set. The 2-tuple domination problem is to find a minimum size vertex subset such that every vertex in the graph is dominated by at least 2 vertices in the set. In this paper we discussed an algorithm to find a 2-tuple dominating set of an induced subgraph of a non-split dominating set of an interval graph.

Key Words: Interval family, Interval graph, connected graph, Dominating Set, Non-split dominating set, 2-tuple domination, design of an algorithm.

1.INTRODUCTION

An undirected graph $G=(V,E)$ is an interval graph(IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. Let $I = I_1, I_2, I_3, I_4, ...I_n$ be any interval family where each I_i is an interval on the real line and $I_i = a_i, b_i$ for $i = 1,2,3,4,..........n$. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i.Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and $2n$. Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contains both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected, and the list of sorted end point is given and the intervals in I are indexed by increasing right end points, that is $b_1 < b_2 < b_3 < .... < b_n$.

Let $G(V,E)$ be a graph. A set S is a dominating set of G if every vertex in $<V-D>$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set[7]. A dominating set D of G is connected dominating set, if the induced sub

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graph $<V-D>$ is connected. The connected domination number $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set. A dominating set D of a graph $G(V,E)$ is a non-split dominating set if the induced subgraph $<V-D>$ is connected. The non-split domination number $\gamma_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set[3].

In a graph G, a vertex is said to dominate itself and all of its neighbors. A dominating set of $G\ V,E$ is a subset D of V such that every vertex in V is dominated by at least one vertex in D. The domination number $\gamma\ G$ is the minimum size of a dominating set of G [7]. For a fixed positive integer m an m-tuple dominating set of $G\ V,E$ is a subset D of V such that every vertex in V is dominated by at least m vertices of D. As introduced by Harary and Haynes[6], a m-tuple dominating set D is a set $D\subseteq V$ for which $|N[v]\cap D|\geq m$ for every $v\in V$, where $N[v]=\{v\}\cup\{u\in V:(u,v)\in E\}$ is the closed neighborhood of the vertex v [1,2]. A double dominating set D is said to be minimal if there does not exist any $D'\subset D$ such that D' is a double dominating set of G. A double dominating set D, denoted by $\gamma_{x2}\ G$, is said to be minimum, if it is minimal as well as it gives double domination number. Let $G\ V,E\ ,V=\ 1,2,...,N\ ,|V|=n, |E|=m$ be a connected interval graph in which the vertices are given in the sorted order of the right end points of the interval representation of the graph. Intervals are labeled according to increasing order of their endpoints. This labeling is referred to as an interval graph ordering. The purpose of this paper is to find the 2-tuple dominating set of an induced subgraph a non-split dominating set by introducing the Algorithm 2DIG [4,5].

2. MAIN THEOREMS

2.1 Theorem

If i and j are any two intervals in I such that $i\in D, j\neq 1$ and j is contained in i, if there is at least one interval to the left of j that intersect j and there is at at least one interval $k\neq i$ to the right of j that intersect j. Then non-split domination occurs in G and also the cardinality of the 2-tuple dominating set of the connected induced sub graph $G_1\ =\ <V-D>$ is $|D_1|=n-1$.

Proof:-

Let $I=\{i_1,i_2,.....,i_n\}$ be the given interval family and G is an interval graph corresponding to I. Suppose there is at least one interval $k\neq i$ to the right of j that intersect j. Then it is obvious that j is adjacent to k in $<V-D>$, so that there will not be any disconnection in $<V-D>$, then G is a connected graph. Again we have to show that the cardinality of 2-tuple dominating set of an induced subgraph of non-split dominating set is $|D_1|=n-1$, where $n=7$. Infact the number of intervals $I=\{1,2,.....,10\}$. Next we will find the 2-tuple dominating set of an induced subgraph $<V-D>$ by using an Algorithm 2DIG as follows:

2.2 An algorithm for finding 2-tuple domination:-

In a connected induced subgraph $<V-D>$ of G the vertices are ordered by an interval graph ordering. First of all we treat none of a vertex of $V(G)$ is a member of dominating set D_1. Then insert vertices one by one by testing their consistency. If a vertex v is dominated by at least two vertices then leave it, otherwise take the highest numbered adjacent vertex (vertices) from $N[v]$ as member(s) of dominating set D_1.

Let us associate a new term $M_i(v)$ for a vertex $v \in V$, for all $i = 0,1,2,.....,k(k = |N(v)|)$ to each adjacent vertices of v in order to an interval graph ordering of intervals as follows,

$$M_i(v) = \min \; N[v] - \bigcup_{j=0}^{i-1} M_i(v)$$

$$\text{With } M_0(v) = \min \; N(v)$$

Basically, $M_0(v) = L(v)$, the lowest numbered adjacent vertex of v. In connection with the name of $L(v)$ we call this $M_i(v)$ as the p - th numbered adjacent vertex of v. Let $u, v \in V$. If for some i $(i = 0,1,2,.....,|N(v)|)$, $|N(v)| - i = p$ such that $u = M_i(v)$, then u is called the p - th numbered adjacent vertex of v.

2.3 An algorithm 2DIG:

Input: An interval graph $G = (V, E)$ with IG ordering vertex set $V = \{1,2,3,.....,n\}$

Output: 2-tuple dominating set D_1 and 2-tuple domination number $\gamma_{X2}(G)(=|D_1|)$

Step 1: Set $f(j) = 0, \forall j = 1,2,.....,n;$ // Assume that no vertices are the members of D_1.//

Step 2: Set $i = 1, D_1 = \phi$ and $L = \phi$.

Step 2.1: Compute $W_i(f) = \sum_{v \in N[i]} f(v)$;

Step 2.2: If $W_i(f) = 0$ then // At least the vertex i not adjacent to any of the vertices of D_1//

Set $f(M_m(i)) = 1$ and $f(M_{m-1}(i)) = 1;$ where m is the ending neighbourhood of i in the table of p - th numbered adjacent vertices.

$D_1 = D_1 \cup \{M_m(i)\} \cup \{M_{m-1}(i)\}$ and $L = L \cup \{i\};$

Else if $W_i(f) = 1$ then // At least the vertex i is connected to one of the vertex of D_1.//

If $f(M_m(i)) = 1$ then set $f(M_{m-1}(i)) = 1;$ $D_1 = D_1 \cup \{M_{m-1}(i)\};$

Else

Set $f(M_m(i)) = 1;$

$D_1 = D_1 \cup \{M_m(i)\};$

End if;

$L = L \cup \{i\};$

Else

Go to Step 2.3;
End if;

Step 2.3: Calculate $i = i+1$ and go to Step 2.1 and continue until $i > n$; end 2DIG.

An algorithm 2DIG gives the set D_1 which is the minimum 2-tuple dominating set and $|D_1|$, the 2-tuple domination number of the interval graph $G = (V, E)$. Here we denote the Set L as the set of leading vertices corresponding to the 2-tuple dominating set D_1.

Now we will find the 2-tuple dominating set of an interval graph as follows an illustration,

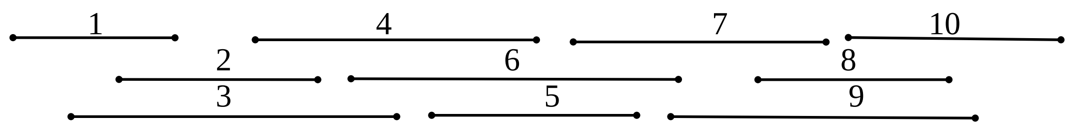

Fig.1: Interval family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4}, nbd [3] = {1,2,3,4,6}, nbd [4] = {2,3,4,5,6},
nbd [5] = {4,5,6,7}, nbd [6] = {3,4,5,6,7,9}, nbd [7] = {5,6,7,8,9}, nbd [8] = {7,8,9,10},
nbd [9] = {6,7,8,9,10}, nbd [10] = {8,9,10}
We may constructed an interval graph G of I interms of neighbours, from G the dominating set DS = {3,7,10}

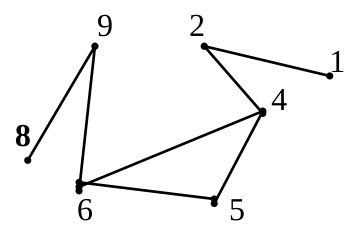

Fig.2: Vertex induced subgraph $<V-D>$- Connected graph from G

nbd [1] = {1,2}, nbd [2] = {1,2,4}, nbd [4] = {2,4,5,6},
nbd [5] = {4,5,6}, nbd [6] = {4,5,6,9}, nbd [8] = {8,9}, nbd [9] = {6,8,9}

To find the minimum 2-tuple dominating set, we have to compute all p- th numbered adjacent vertices.

M_i v \ v	1	2	4	5	6	8	9
M_0 v	1	1	2	4	4	8	6
M_1 v	2	2	4	5	5	9	8
M_2 v	-	4	5	6	6	-	9
M_3 v	-	-	6	-	9	-	-

First set $f(j)=0, \forall j \in V$. In Step 2, set $i=1, D_1=\phi$ and $L=\phi$, that is initially D_1 and L are empty. Step 2 repeats for n times. Here $n=7$, the number of vertices in the induced sub graph $G_1 = <V-D>$.

Now we will illustrate the iterations in the following way:

Iteration (1) :

For the first iteration $i=1$

$$N[1] = \{1,2\}$$

$$w_1(f) = f(N[1])$$

$$w_1(f) = f(1)+f(2) = 0$$

The first condition of if-end if is satisfied. Since $w_1(f)=0$, we find $M_1(1)=2, M_0(1)=1$

Then set $f(1)=1, f(2)=1$

Also set

$$D_1=\phi \cup \{1,2\} \Rightarrow D_1=\{1,2\}, L=\{1\} \quad i=i+1=2$$

Iteration (2):

For the second iteration $i=2$

$$N[2] = \{1,2,4,\}$$

$$w_2(f) = f(N[2])$$

$$w_1(f) = f(1)+f(2)+f(4) = 1+1+0=2$$

That is the vertex 2 is dominated by two vertices 1 and 2 of D_1. So, in this iteration D_1 could not be calculated. Hence D_1 and L remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

$$N[4] = \{2,4,5,6\}$$

$$w_4(f) = f(N[4])$$

$$w_4(f) = f(2)+f(4)+f(5)+f(6)=1+0+0+0=1$$

Here the domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now, we check either $f(M_3(4))=1$ or not. We see that $f(M_3(4))=f(6)=0$ and hence set $f(6)=1$. Update D_1 by $D_1 \cup \{6\}=\{1,2,6\}$ and L by $L \cup \{3\}=\{1,3\}$. The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i=4$

$$N[5] = \{4,5,6\}$$

$$w_5(f) = f(N[5])$$

$$w_5(f) = f(4)+f(5)+f(6)=0+0+1=1$$

Here also the domination criteria is not satisfied. Now, we check either $f(M_2(5))=1$ or not. We see that

$f(M_2(5)) = f(6) = 1$ and hence set

$f(M_1(5)) = 1,$

i.e., $f(5) = 1, D_1 = D_1 \cup \{5\} = \{1,2,5,6\}, L = L \cup \{4\} = \{1,3,4\}$.

The iteration number i is being increased to 5.

Iteration (5):

For the fifth iteration $i = 5$

$N\ 6\ =\ 4,5,6,9$

$W_6\ f\ = f(N[6])$

$W_6\ f\ = f\ 4\ + f\ 5\ + f\ 6\ + f\ 9\ = 0+1+1+0 = 2$

In this iteration D_1 and L remain unchanged. The iteration number i is being increased to 6.

Iteration (6):

For the sixth iteration $i = 6$

$N\ 8\ =\ 8,9$

$W_8(f) = f(N[8])$

$W_8(f) = f\ 8\ + f\ 9\ = 0+0 = 0$

The first condition of if-end if is satisfied.

Since $w_8\ f\ = 0,$ we find $M_1\ 8\ = 9, M_0\ 8\ = 8$.

Then set $f\ 8\ = 1, f\ 9\ = 1$

$D_1 = \{1,2,5,6,8,9\}, L = \{1,3,4,6\}$

Iteration (7):

For the seventh iteration $i = 7$

$N\ 9\ =\ 6,8,9$

$W_9\ f\ = f(N[9])$

$W_9\ f\ = f\ 6\ + f\ 8\ + f\ 9\ = 1+0+1 = 2$

In this iteration D_1 and L remain unchanged.

$\therefore D_1 =\ 1,2,5,6,8,9\ , L = \{1,3,4,6\}$

$|D_1| =$ Cardinality of D_1

$= 6$.

2.4 Theorem

If i and j are any two intervals in I such that $i \in D_1, j = 1$ and j is contained in i, if there one more interval other than i that intersect j, the induced subgraph $<V-D>$ is connected, then the cardinality of 2-tuple dominating set of $<V-D>$ is $|D_1| = n-3$

Proof:-

Let $I = \{i_1, i_2, ..., i_n\}$ be the given interval family and G is an interval graph of I. Let $j = 1$ be the interval contained in i, where $i \in D$. Suppose k is an interval, $k \neq i$ and k intersect j. Since $i \in D$, the induced sub graph $<V-D>$ does not contain i. Further in $<V-D>$, the vertex j is adjacent to the vertex k and hence the graph G will not be

disconnected in $<V-D>$. Let G_1 be an induced sub graph $<V-D>$ of G. Since $\{i_1, i_2, ..., i_n\} \in G$ and $\{i_1, i_3, i_4, i_5, i_6, i_8, i_9, i_{10}\} \in G_1$. Now we have to find the 2-tuple dominating set from the induced sub graph G_1. In this connection to find the 2-tuple dominating set we used an algorithm 2DIG as discussed in the Theorem 1.
Now we will find the 2-tuple dominating set as follows an interval family,

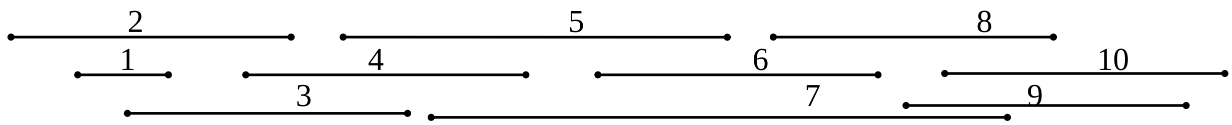

Fig.1: Interval Family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4}, nbd [3] = {1,2,3,4,5}, nbd [4] = {2,3,4,5,7},
nbd [5] = {3,4,5,6,7}, nbd [6] = {5,6,7,8}, nbd [7] = {4,5,6,7,8,9,10}, nbd [8] = {6,7,8,9,10},
nbd [9] = {7,8,9,10},nbd [10] = {7,8,9,10}
Dominating set of an interval graph of G is DS = {2,7}

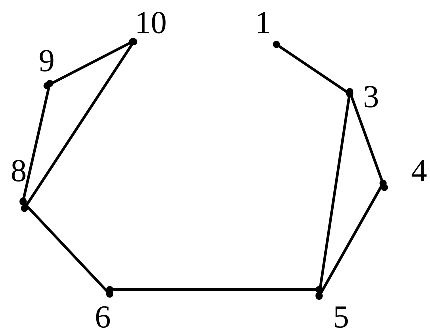

Fig.2: Vertex induced graph $<V-D>$**- Connected graph**

nbd [1] ={1,3}, nbd [3] ={1,3,4,5}, nbd [4] ={3,4,5}, nbd [5] ={3,4,5,6}, nbd [6] ={5,6,8},
nbd [8] ={6,8,9}, nbd [9] ={8,9}, nbd [10] ={8,9,10}

To find the minimum 2-tuple dominating set, we have to compute all p - th numbered adjacent vertices as follows.

$M_i\ v \setminus v$	1	3	4	5	6	8	9	10
$M_0\ v$	1	1	3	3	5	6	8	8
$M_1\ v$	3	3	4	4	6	8	9	9
$M_2\ v$	-	4	5	5	8	9	-	10
$M_3\ v$	-	5	-	6	-	-	-	-

First set $f(j)=0, \forall j \in V$. In Step 2, set $i=1, D_1=\phi$ and $L=\phi$, that is initially D_1 and L are empty. Step 2 repeats for n times. Here $n=8$, the number of vertices in the induced sub graph $G_1 = < V-D >$.

Now we will illustrate the iterations in the following way:

Iteration (1) :

For the first iteration $i=1$

$$N[1]=\{1,3\}$$

$$w_1(f)=f(N[1])$$

$$w_1(f)=f(1)+f(3)=0$$

The first condition of if-end if is satisfied. Since $w_1(f)=0$, we find $M_1(1)=3, M_0(1)=1$

Then set $f(1)=1, f(3)=1$

Also set

$$D_1=\phi \cup \{1,3\} \Rightarrow D_1=\{1,3\}, L=\{1\}, i=i+1=2$$

Iteration (2):

For the second iteration $i=2$

$$N[3]=\{1,3,4,5\}$$

$$w_3(f)=f(N[3])$$

$$w_3(f)=f(1)+f(3)+f(4)+f(5)=1+1+0+0=2$$

That is the vertex 2 is dominated by two vertices 1 and 3 of D_1. So, in this iteration D_1 could not be calculated. Hence D_1 and L remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

$$N[4]=\{3,4,5\}$$

$$w_4(f)=f(N[4])$$

$$w_4(f)=f(3)+f(4)+f(5)=1+0+0=1$$

Here the domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now, we check either $f(M_2(4))=1$ or not. We see that $f(M_2(4))=f(5)=0$ and hence set $f(5)=1$. Update D_1 by $D_1 \cup \{5\}=\{1,3,5\}$ and L by $L \cup \{3\}=\{1,3\}$. The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i=4$

$$N[5]=\{3,4,5,6\}$$

$$w_5(f)=f(N[5])$$

$$w_5(f)=f(3)+f(4)+f(5)+f(6)=1+0+1+0=2$$

In this iteration D_1 could not be calculated. Hence D_1 and L remains same and i is being increased to 3. The iteration number i is being increased to 5.

Iteration (5):

For the fifth iteration $i = 5$

$N \; 6 \; = \; 5,6,8$

$W_6 \; f \; = f(N[6])$

$W_6 \; f \; = f \; 5 \; + f \; 6 \; + f \; 8 \; = 1 + 0 + 0 = 1$

Here also the domination criteria is not satisfied. Now, we check either $f(M_2(6)) = 1$ or not.

We see that

$f(M_2(6)) = f(8) = 0$ and hence set

$f(8) = 1,$

$D_1 = D_1 \cup \{8\} = \{1,3,5,8\}, L = L \cup \{5\} = \{1,3,5\}.$

The iteration number i is being increased to 6.

Iteration (6):

For the sixth iteration $i = 6$

$N \; 8 \; = \; 6,8,9$

$W_8(f) = f(N[8])$

$W_8(f) = f(6) + f \; 8 \; + f \; 9 \; = 0 + 1 + 0 = 1$

Here also the domination criteria is not satisfied. Now, we check either $f(M_2(8)) = 1$ or not.

We see that

$f(M_2(8)) = f(9) = 0$ and hence set

$f(9) = 1,$

$D_1 = D_1 \cup \{9\} = \{1,3,5,8,9\}, L = L \cup \{6\} = \{1,3,5,6\}.$

The iteration number i is being increased to 7.

Iteration (7):

For the seventh iteration $i = 7$

$N \; 9 \; = \; 8,9$

$W_9 \; f \; = f(N[9])$

$W_9 \; f \; = f \; 8 \; + f \; 9 \; = 1 + 1 = 2$

In this iteration D_1 and L remain unchanged. The iteration number i is being increased to 8.

Iteration (8):

For the eighth iteration $i = 8$

$N \; 10 \; = \; 8,9,10$

$W_{10} \; f \; = f(N[10])$

$W_{10} \; f \; = f \; 8 \; + f \; 9 \; + f(10) = 1 + 1 + 0 = 2$

In this iteration D_1 and L remain unchanged.

$\therefore D_1 = \{1,3,5,8,9\}$, $L = \{1,3,4,6\}$

$|D_1|$ = Cardinality of D_1

$= 5$.

2.5 Theorem

Let $I = \{i_1, i_2, ..., i_n\}$ be an interval family and G be an interval graph of I. If i, j, k are three consecutive intervals such that $i < j < k$ and $j \in D$, i intersects j, j intersect k and i intersect k then the induced sub graph $< V - D >$ is connected. Also the cardinality of 2-tuple dominating set of $< V - D >$ is $|D_1| = n - 3$.

Proof:-

Let $I = \{i_1, i_2, ..., i_n\}$ be an interval family and G be an interval graph of I. Let i, j, k be three consecutive intervals satisfy the hypothesis. Now i and k intersect implies that i and k are adjacent in $< V - D >$. So that there will not be any disconnection in $< V - D >$. Further we wil find the 2-tuple dominating set of G_1. Since G_1 is an induced subgraph. Now we will prove that the cardinality of 2-tuple dominating set $|D_1| = n - 3$ by using the Algorithm 2DIG as discussed in Theorem 1.

Now we will find the 2-tuple dominating set as follows, consider the following interval family

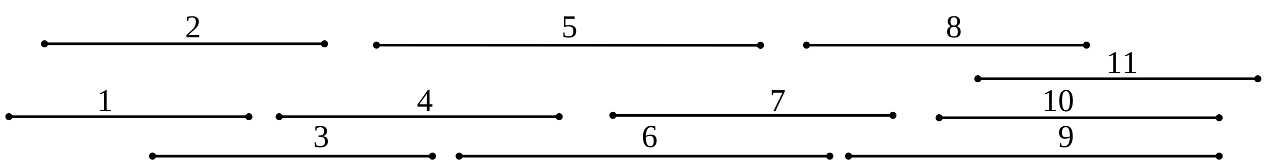

Fig.1: Interval Family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4}, nbd [3] = {1,2,3,4,5}, nbd [4] = {2,3,4,5,6},
nbd [5] = {3,4,5,6,7}, nbd [6] = {4,5,6,7,8}, nbd [7] = {5,6,7,8,9}, nbd [8] = {6,7,8,9,10,11},
nbd [9] = {7,8,9,10,11}, nbd [10] = {8,9,10,11}, nbd [11] = {8,9,10,11}

Dominating set of the interval graph is $D = \{3,8\}$

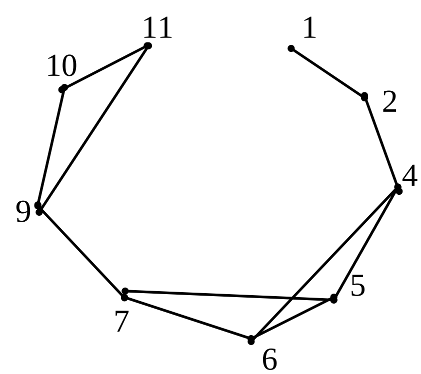

Fig.2: Vertex induced graph $< V - D >$- Connected graph

nbd [1] ={1,2}, nbd [2] ={1,2,4}, nbd [4] ={2,4,5,6}, nbd [5] ={4,5,6,7}, nbd [6] ={4,5,6,7},
nbd [7] ={5,6,7,9}, nbd [9] ={7,9,10,11}, nbd [10] ={9,10,11}, nbd [11] = {9,10,11}

To find the minimum 2-tuple dominating set, we have to compute all p - th numbered adjacent vertices follows.

$M_i(v)\backslash v$	1	2	4	5	6	7	9	10	11
$M_0(v)$	1	1	2	4	4	5	7	9	9
$M_1(v)$	2	2	4	5	5	6	9	10	10
$M_2(v)$	-	4	5	6	6	7	10	11	11
$M_3(v)$	-	-	6	7	7	9	11	-	-

First set $f(j)=0, \forall j \in V$. In Step 2, set $i=1, D_1=\phi$ and $L=\phi$, that is initially D_1 and L are empty. Step 2 repeats for n times. Here $n=9$, the number of vertices in the induced sub graph $G_1=<V-D>$.

Now we will illustrate the iterations in the following way:

Iteration (1) :

For the first iteration $i=1$

$$N[1]=\{1,2\}$$

$$w_1(f)=f(N[1])$$

$$w_1(f)=f(1)+f(2)=0$$

The first condition of if-end if is satisfied. Since $w_1(f)=0$, we find $M_1(1)=2, M_0(1)=1$. Then set $f(1)=1, f(2)=1$

Also set

$$D_1=\phi\cup\{1,2\} \Rightarrow D_1=\{1,2\}, L=\{1\}, i=i+1=2$$

Iteration (2):

For the second iteration $i=2$

$$N[2]=\{1,2,4\}$$

$$w_2(f)=f(N[2])$$

$$w_2(f)=f(1)+f(2)+f(4)=1+1+0=2$$

That is the vertex 2 is dominated by two vertices 1 and 2 of D_1. So, in this iteration D_1 could not be calculated. Hence D_1 and L remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

$$N[4]=\{2,4,5,6\}$$

$$w_4(f)=f(N[4])$$

$w_4(f)=f(2)+f(4)+f(5)+f(6)=1+0+0+0=1$ Here the domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now, we check either $f(M_3(4))=1$ or not. We see that $f(M_3(4))=f(6)=0$ and hence set $f(6)=1$. Update D_1 by $D_1 \cup \{6\}=\{1,2,6\}$ and L by $L \cup \{3\}=\{1,3\}$. The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i=4$

$N\ 5\ =\ 4,5,6,7$

$w_5\ \ f\ \ =f(N[5])$

$w_5\ \ f\ \ =f\ \ 4\ +f\ \ 5\ +f(6)+f(7)=0+0+1+0=1$

Here also the domination criteria is not satisfied. Now, we check either $f(M_3(5))=1$ or not. We see that

$f(M_3(5))=f(7)=0$ and hence set $f(7)=1, D_1=D_1 \cup \{7\}=\{1,2,6,7\}, L=L\cup\{4\}=\{1,3,4\}$.

The iteration number i is being increased to 5.

Iteration (5):

For the fifth iteration $i=5,\ N\ 6\ =\ 4,5,6,7$

$W_6\ \ f\ \ =f(N[6]), W_6\ \ f\ \ =f(4)+f\ \ 5\ +f\ \ 6\ +f\ \ 7\ \ =0+0+1+1=2$

In this iteration D_1 could not be calculated. Hence D_1 and L remains same. The iteration number i is being increased to 6.

Iteration (6):

For the sixth iteration $i=6$

$N\ 7\ =\ 5,6,7,9$

$W_7(f)=f(N[7])$

$W_7(f)=f(5)+f(6)+f\ \ 7\ +f\ \ 9\ \ =0+1+1+0=2$

In this iteration D_1 could not be calculated. Hence D_1 and L remains same. The iteration number i is being increased to 7.

Iteration (7):

For the seventh iteration $i=7$

$N\ 9\ =\ 7,9,10,11$

$W_9\ \ f\ \ =f(N[9])$

$W_9\ \ f\ \ =f(7)+f\ \ 9\ +f\ \ 10\ +f(11)=1+0+0+0=1$

Here also the domination criteria is not satisfied. Now, we check either $f(M_3(9))=1$ or not. We see that

$f(M_3(9))=f(11)=0$ and hence set

$f(11)=1,$

$D_1=D_1\cup\{11\}=\{1,2,6,7,11\}, L=L\cup\{7\}=\{1,3,4,7\}$.

The iteration number i is being increased to 8.

Iteration (8):

For the eighth iteration $i=8$

$N[10] = \{9,10,11\}$

$W_{10}(f) = f(N[10])$

$W_{10}(f) = f(9) + f(10) + f(11) = 0 + 0 + 1 = 1$

Here also the domination criteria is not satisfied. Now, we check either $f(M_2(10)) = 1$ or not. We see that

$f(M_2(10)) = f(11) = 1$ and hence set

$f(M_1(10)) = f(10) = 1,$

$D_1 = D_1 \cup \{10\} = \{1,2,6,7,10,11\}, L = L \cup \{8\} = \{1,3,4,7,8\}.$

The iteration number i is being increased to 9.

Iteration 9:

For the ninth iteration $i = 9$

$N[11] = \{9,10,11\}$

$W_{11}(f) = f(N[11])$

$W_{11}(f) = f(9) + f(10) + f(11) = 0 + 1 + 1 = 2$

In this iteration D_1 could not be calculated. Hence D_1 and L remains same.

$\therefore D_1 = \{1,2,6,7,10,11\}, L = \{1,3,4,7,8\}$

$|D_1| = \text{Cardinality of } D_1 = 6$

3 References

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