Time and Frequency Domain Analysis of Signals: A Review

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Abstract— In this paper, different works of literature have been reviewed that related to the time and frequency analysis of signals. The time domain is the analysis of mathematical functions, physical signals with respect to time. In the time domain, the signal or function's value is known for all real numbers, for the case of continuous-time, or at various separate instants in the case of discrete-time. An oscilloscope is a tool commonly used to visualize real-world signals in the time domain. A time-domain graph shows how a signal changes with time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. The frequency-domain refers to the analysis of mathematical functions or signals with respect to frequency, rather than time Put simply, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid to be able to recombine the frequency components to recover the original time signal. And finally, the time-frequency signal analysis introduced, it's a new method in which the problem that had on the frequency signal analysis will be solved.

Index Terms— Auto spectral density, digital signal, frequency domain, time domain.

I. INTRODUCTION

Any attempt to detect different types of machine faults reliably at an early stage requires the development of improved signal processing methods. Signals are the foundation of information processing, transmission, and storage. Signal representations are unique; a signal is either analog or digital, time domain or frequency domain.

Table 1: Major Signal Properties [1].

Signal Properties		
Linear	Nonlinear	
Deterministic	Stochastic (random)	
Stationary (time invariant)	Nonstationary	

The two most basic signal measurements, the mean and root mean squared (rms) value, quantify these two differences. The equations we use to calculate these measurements depend on whether the domain used to represent the signal is discrete or continuous. Although the same measurement requires a different equation for discrete versus continuous signals, the two equations are related. For a discrete signal that is represented by a series of numbers, determination of mean value is intuitive; it is the average of all the numbers in the series. To determine the average of a series of numbers, simply add the numbers together and divide by the length of the series

(i.e., the number of numbers in the series). For a series of N numbers:

$$x_{avg} = \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n \tag{1}$$

Where n is an integer indicating a specific member in the series and ranging from 1 to N

For signals represented in the continuous domain, summation becomes integration. The discrete set of numbers, xn, becomes a continuous time series, x (t), and the length of the series, N, becomes a fixed time period, T. This leads to the equation:

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt \tag{2}$$

The other basic difference between the two signals is their range of variability. The signal on the left clearly shows greater and more prevalent fluctuations. This signal property is quantified by the rms value. The equation for the rms of a signal follows this measurement's name: first square the signal, then take its mean, then take the square root of this mean:

$$x_{ms} = \left[\frac{1}{N} \sum_{n=1}^{N} x_n^2\right]^{1/2} \tag{3}$$

Having the above the basic relations between the two methods are summarized in the table below.

Table 2. Relationship between Discrete and Continuous
Operations [1].

Operation	Discrete	Continuous
Summation/integration	$\sum_{n=1}^{N} x_n$	$\int_{t=0}^{T} x(t)$
Variable name	x_n	$t = n/f_s$
Signal length	N	T
	$N = T f_s$	$T = N/f_s$

Vibration measurements provide a good basis for condition monitoring. Early vibration measurements by means of mechanical or optical instruments used displacement x = x(t). The next step was the adoption of velocity i.e. x(1) signals, which were obtained either by differentiating displacement or using sensors whose output was directly x(1). The drawback with the signals x and x(1) is that they do not usually allow the detection impact-like faults sufficiently early stage. Acceleration measurements have been performed more frequently upon the introduction of accelerometers. The signals x and x (1) can be obtained from the x(2) signal through analogue or numerical integration [2]. Solution of dynamic systems is sometimes simplified when modeled in alternative coordinate frames. Floquet theory

provides a robust transformation, via the system's transition matrix, to convert a linear time-periodic (LTP) system into a system with linear time-invariant (LTI) state matrix [3].

The time-domain representation s (t) reveals information about the actual presence of the signal, its start and end times, its strength and temporal evolution, and it indicates how the signal energy is distributed along the t axis.

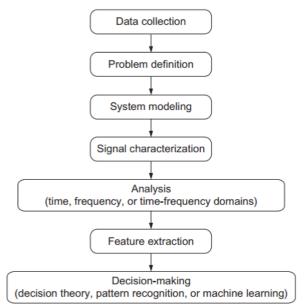


Figure 1: Typical flowchart for a basic DSP problem-solving and decision-making procedure (other procedures can include preprocessing stages, filtering, and post processing stages) [4].

It is required to develop mathematical tools that will allow us to quantitatively analyze measurement systems.

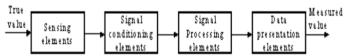


Figure 2: General structure of an engineering measurement system

The purpose of control is to make a plant (i.e., the "dynamic" system to be controlled) behave in a desired manner, according to some performance specifications. The overall system that includes at least the plant and the controller is called the control system. The control problem can become challenging due to such reasons as:

- Complex system (many inputs and many outputs, dynamic coupling, nonlinear, etc.)
- Rigorous performance specifications
- Unknown excitations (unknown inputs/disturbances/noise)
- Unknown dynamics (incompletely known plant)
 Signals can be measured using analog and digital method. The analog method is a continuous time measuring method. In a digital control system, a digital device is used as the controller. The digital controller may be a hardware device that uses permanent logic circuitry to generate control signals. Such a device is termed a hardware controller. The following are some of the important advantages of digital control.
- 1. Digital control is less susceptible to noise or parameter variation in instrumentation because data can be

- represented, generated, transmitted, and processed as binary words, with bits possessing two identifiable states.
- 2. Very high accuracy and speed are possible through digital processing. Hardware implementation is usually faster than software implementation.
- 3. Digital control can handle repetitive tasks extremely well, through programming.
- 4. Complex control laws and signal-conditioning methods that might be impractical to implement using analog devices can be programmed.
- High reliability in operation can be achieved by minimizing analog hardware components and through decentralization using dedicated microprocessors for various control tasks.
- 6. Large amounts of data can be stored using compact, high-density data storage methods.
- Data can be stored or maintained for very long periods of time without drift and without being affected by adverse environmental conditions.
- 8. Fast data transmission is possible over long distances without introducing excessive dynamic delays, as in analog systems.
- 9. Digital control has easy and fast data retrieval capabilities.
- 10. Digital processing uses low operational voltages (e.g., 0–12. V dc).
- 11. Digital control is cost effective.

II. OBJECTIVE

The objective of this paper is to review the literatures that deals with time domain and frequency domain signals analysis of different systems and surmised the basic concept.

III. SIGNIFICANCES OF THE VIBRATION SIGNAL IS ANALYSIS

The value of using the concepts of signal analysis and processing in situations outside, as well as within, the field of communications may best be illustrated by considering the types of problem to which they are conventionally applied. To represent an apparently complex signal waveform by a limited set of parameters which, although not necessarily describing that waveform completely, are sufficient for the task in hand (such as deciding whether or not the signal may be faithfully transmitted through a particular communication channel). Most important of all, careful analysis of a signal may often be used to learn something about the source which produced it; in other words, certain detailed characteristics of a signal which are not immediately apparent can often give important clues to the nature of the signal source, or to the type of processing which has occurred between that source and the point at which the Signal is recorded or detected.

In addition to the above points, when the characteristics of a signal have been adequately defined, it is possible to determine the exact type of processing required to achieve a particular object. For example, it might be required to pass the signal undistorted through a communications system, to detect the occurrence of a particular signal waveform in the presence of random disturbances, or to extract by suitable processing some significant aspect of a signal or of the relationship between two signals. Once again, the techniques used to process signals in such ways are of interest in other fields; for example, it is often important to be able to clarify particular features or trends in

Vol. 9 Issue 12, December-2020

data of all types, or to examine the relationships between two recorded variables [5].

IV. METHODS OF SIGNAL ANALYSIS

Two types of mathematical tools:

- 1) Time Domain Analysis
- Time domain analysis examines the amplitude vs. time characteristics of a measuring signal.
- 2) Frequency Domain Analysis
- Frequency domain analysis replaces the measured signal with a group of sinusoids which, when added together, produce a waveform equivalent to the original.
- The relative amplitudes, frequencies, and phases of the sinusoids are examined.

V. TIME DOMAIN ANALYSIS

The time-domain representation s(t) reveals information about the actual presence of the signal, its start and end times, its strength and temporal evolution, and it indicates how the signal energy is distributed along the t axis [4].

- \checkmark In time-domain analysis the response of a dynamic system to an input is expressed as a function of time c (t).
- ✓ It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.
- \checkmark The time response of a system can be obtained by solving the differential eq. governing the system.
- \checkmark Alternatively, the response c(t) can be obtained from the transfer function of the system and the input to the system.

For a closed loop transfer function,

C(s)/R(s) = G(s)/[1+G(s) H(s)]Response in s-domain, C(s) = R(s)*M(s)

Response in t-domain, c(t) = InvLap[C(s)]

A. 5.1. Time Domain Specifications

- ✓ For specifying the desired performance characteristics of a measurement control system.
- ✓ These characteristics of a system of any order may be specified in terms of transient response to a unit step input signal.
- ✓ The response of a second order system for an input is,

$$\frac{C(s)}{R(s)} = \frac{W_n^2}{s^2 + 2zW_n s + W_n^2}$$
 (4)

Time Domain Specifications

- 1. Delay time
- 4. Peak overshoot
- 2. Rise time
- 5. Settling time
- 3. Peak time
- 6. Steady-state error

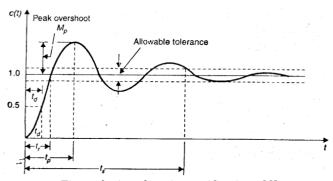


Figure 3: time domain specifications [6].

Time Domain Specifications

- 1. *Delay time*: It is the time required for the response to reach 50% of the final value in first attempt.
- 2. *Rise time*: It is the time required to rise from 0 to 100% of the final value for the under damped system.
- 3. *Peak time*: It is the time required for the response to reach the peak of time response or the peak overshoot.
- 4. **Settling time**: It is the time required for the response to reach and stay within a specified tolerance band (2% or 5%) of its final value.
- 5. *Peak overshoot*: It is the normalized difference between the time response peak and the steady output and is defined as,

$$M_p = \frac{\hat{C}(t_p) - c(\infty)}{c(\infty)} \times 100\%$$
 (5)

6. *Steady-state error*: It indicates the error between the actual output and desired output as't' tends to infinity.

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)] \tag{6}$$

VI. FREQUENCY DOMAIN ANALYSIS

approaches for frequency-domain characterization of time-periodic networks have been outlined, mainly focusing on switched networks. Time-domain characterization can also be performed in direct or indirect form, nonetheless, by these three approaches. Floquet method is primarily based on coordinate transformation of state space representation of a system while providing zero/pole characterization for stability and control design studies. Harmonic domain dynamic transfer function provides a dynamic transfer function that relates frequency content of input/output variables. Generalized transfer function permits to replace a sampled signal by a synthetic continuous signal; poles in s- and z-domain are readily available. Though the main direction of analysis depends on the establishment of input-output relationships among different components, still none of the approaches lends itself easily to application. Finally, it is worth noting that analysis of periodic systems is still an active area of research that requires further attention [3].

The Fourier theorem indicates that, under some conditions, any bounded signal can be written as a weighted sum of sines and cosines (or complex exponentials) at different frequencies f. The weights are given by the complex coefficient S(f). If s(t) is periodic, we obtain a line spectrum.

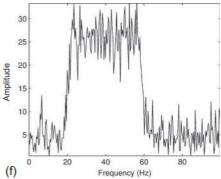


Figure 4: Frequency-domain representations [4].

Advantages

- Stability of closed loop system can be estimated
- Transfer function of complicated systems can be determined experimentally by frequency tests
- Effects of noise disturbance and parameter variations are relatively easy to visualize.
- Analysis can be extended to certain nonlinear control systems.

A. Frequency Domain Specifications

Resonant Peak
 Resonant Frequency
 Gain Margin
 Bandwidth
 Phase Margin

Frequency Domain Specifications

- 1 **Resonant Peak** Maximum value of the closed loop transfer function.
- 2 **Resonant Frequency** Frequency at which resonant peak occurs.
- 3 **Bandwidth** range of frequencies for which the system normalized gain is more than -3db.
- 4 *Cut-off rate-* It is the slop of the log-magnitude curve near the cut off frequency.
- 5 *Gain Margin* The value of gain to be added to system in order to bring the system to the verge of instability.
- 6 *Phase Margin* Additional phase lag to be added at the gain cross over freq. in order to bring the system to the verge of instability.

B. Frequency Response Plots

Frequency domain analysis of a system can be carried either analytically or graphically.

The various graphical techniques are:

Bode Plot
 Nichols Plot
 Polar Plot
 M and N circles

Frequency response plots are used to determine the frequency domain specifications, to study the stability of the system.

VII. TIME-FREQUENCY SIGNAL ANALYSIS

The time domain and the frequency domain are two modes used to analyze data. Both time domain analysis and frequency domain analysis are widely used in electronics, acoustics, telecommunications, and many other fields.

 Frequency domain analysis is used in conditions where processes such as filtering, amplifying, and mixing are required.

- Time domain analysis gives the behavior of the signal over time. This allows predictions and regression models for the signal.
- Frequency domain analysis is very useful in creating desired wave patterns, such as binary bit patterns of a computer.
- Time domain analysis is used to understand data sent in such bit patterns over time.

Time frequency analysis identifies the time at which various signals frequencies are present, usually by calculating a spectrum at regular intervals of time. The consideration of non-stationary signals requires an assortment of analysis tools. Many scientific and technical activates are interested on such, for medical purpose, for earth quick study, for machine maintenance, for astronomy, etc. Time-varying signals may be transformed to the frequency domain by using various transformations. Fourier transformation is suitable if the signal is stationary and frequency components do not change with time. But real-world signals like brain and speech signals change with time. These signals can be analyzed using sliding-window-based Fourier transformation methods by choosing an appropriate window size and overlapping successive sliding windows. The uncertainty principle limits resolutions related to time and frequency in a constant-size window. Wavelet transformation is suitable to analyze such signals.

The short-time Fourier transformation (STFT) function is simply Fourier transformation operating on a small section of the data. After the transformation is complete on one section of the data, the next selection is transformed, and the output "stacked" next to the previous transformation output. This method is very similar to Gabor transformation, as mentioned above; the only difference is the types of window used. Popular types of window functions are rectangular, Hamming, Hanning, and Blackman-Tukey [13].

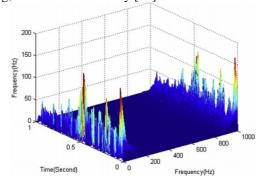


Figure 5: The three-dimensional representation of the wavelet for the signal [14].

VIII. LITERATURES OF TIME DOMAIN AND FREQUENCY DOMAIN SIGNAL ANALYSIS FOR DIFFERENT SYSTEMS

Bivariate signals are a special type of multivariate time series corresponding to vector motions on the 2D plane or equivalently in R². They are specific because their time samples encode the time evolution of vector valued quantities (motion or wave field direction, velocity, etc). In most of these scientific fields, the physical phenomena (electromagnetic waves, currents, elastic waves, etc.) are described by various

types of quantities over different ranges of frequencies. As frequency components evolve with time, time–frequency representations are necessary to accurately describe the evolution of the recorded signal [4].

In recent years, compressive sensing (CS) came up as a promising method of sub-Nyquist sampling. It is a powerful tool for signal analysis, and it allows to acquire signals with fewer measurements than previously thought possible. This is accomplished by exploiting the fact that many real-world signals have far fewer degrees of freedom than the signal size might indicate. For instance, spectrally sparse (narrowband) signal depends upon only a few degrees of freedom, although its total bandwidth is exceptionally wide. The main goal of CS is transforming a signal to a suitable descriptive domain and condensing it into a few samples, which is referred to as analog-to-information conversion (AIC). This compressed information is then sent to the receiver, where the original signal is reconstructed. No information loss and exact reconstruction is theoretically possible with CS, with high potential compression ratios [6].

Analog-to-information conversion

Conventional ADCs acquire signal samples at equidistant time instants, according to the Nyquist sampling theorem. With signals highly sparse at a certain domain, such sampling means great redundancy in acquired data, since the amount of information contained within a sparse signal is limited. AIC exploits this fact, and only takes a limited number of samples, sufficient to represent the information content. High bandwidth sparse signals can be precisely measured by time equivalent sampling as well, and reconstructed by a very simple algorithm. This approach however relies on the signal being stationary for a prolonged time period. CS employs different sampling and reconstruction strategies and in general does not require the measured signal to be stationary.

Reconstruction

At the receiving end of CS framework, the information signally, and both bases Ψ and Φ are known. The only unknown required for reconstruction o is if A was a square matrix (which would mean applying conventional Nyquist sampling), the problem could be solved simply by inverting A. But since A is a rectangular matrix, it cannot be simply inverted, and an undetermined system of M equations and L unknowns is to be solved. Here the importance of sparsity turns out, because based on this requirement a unique solution can be found. Out of all the possible solutions, the right solution is the one that is the most spars.

Stochastic sampling

In order to subsample the Nyquist grid, SS exploits the incoherence of time and frequency. Subsampling is random—time instants at which the signal is sampled are selected randomly. Thus there is no coherent aliasing effect, meaning no frequency information loss, and the original signal can be recovered via nonlinear processing.

Power networks are, in nature, nonlinear time-variant systems. Starting with synchronous machines, which function as generators, passing by transmission and distribution networks whose topologies are altered due to different switching actions in the network, and ending with uncontrolled loads, which depend to a large extent on customer behavior and their dynamics [3].

Frequency-domain (FD) provides a convenient domain to solve a set of differential equations that describes the system as algebraic equations for the aforementioned studies, though limited to linear systems. In performing power system studies, periodic nature of rotating machines is accounted for by Park's transformation, which is able to mask the time-periodic nature of the machine in a manner that does not compromise the simplicity of the solution. Yet, the introduction of converters sidestepped FD for network analyses to the favor of time-domain (TD) analysis tools, albeit their relative complexity and large computational times. However, since the inception of the need to analyze time-periodic (TP) systems in general, numerous techniques evolved with varying degrees of success to provide a practical tool for network studies. Harmonic Domain Dynamic Transfer Functions, and Equivalent Signal theory as tools for analysis of TP power systems, putting special attention to switched networks. Also, the paper compares those tools from the points of view of:

- (i) Computational effort for model preparation,
- (ii) Possible applications to modern power networks, and
- (iii) Suitability for integration with present network analysis tools.

Flexible structures subject to vibrations experience material fatigue, often referred to as vibration fatigue. In most cases the excitation vibrations are of a random nature, harmonic or impulse. Harmonic and impulse loads are deterministic and can be described analytically for linear systems in the time 5 and frequency domains. Random loads are stochastic, and a frequency-domain analysis is possible using the assumptions of linearity, stationarity and Gaussianity. Three types of loads are typical in vibration fatigue: random, harmonic and impact. In an application, any combination of these loads is possible. In vibration fatigue the random loads can be investigated by using the frequency response function of the structure [10]. The Short-Time Fourier Transform (STFT) is widely used to convert signals from the time domain into a time-frequency representation. This representation has well-known limitations regarding time-frequency resolution. In this paper we use the basic concept of the Short-Time Fourier Transform, but fix the window size in the frequency domain instead of in the time domain. This approach is simpler than similar existing methods, such as adaptive STFT and multi-resolution STFT, and in particular it requires neither the band-pass filter banks of multi-resolution techniques, nor the evaluation of local signal characteristics of adaptive techniques [11].

Integrating a sampled time signal is a common task in signal processing, for example in vibration engineering applications. It is common in vibration engineering to convert a measured acceleration signal into velocity or displacement. This may be important when time data from sensors producing different output units are analyzed, for example combining accelerometers and laser Doppler vibro-meters, or accelerometers and geophone sensors, or when combining accelerometers with strain gauges whose output is proportional to displacement. Despite the frequent need of time domain integration, not much has been published on best practices or best methods to be used [12].

Both time-domain and frequency-domain analyses were used to calculate the Auto Spectral Density (ASD) of the response, which is the basis for evaluating the criterion for assessing the

vibration serviceability of floors [13]. By utilizing Discrete Fourier Transform (DFT), the ASD of a time history is given as $ASD(f) = |DFT(f)|^2 \times 2df$

Where f is the frequency, and df is the frequency resolution of the DFT.

The dynamic response of a system to an applied excitation force f (t) can be obtained by solving the equation of motion given as:

$$m\ddot{x}(t) + c\dot{x}(t) + Kx(t) = f(t)$$

Where $\ddot{x}(t)$, \dot{x} , and x(t) are acceleration, velocity, and displacement of the system, respectively, m is the mass, c is the damping coefficient, and k is the system stiffness. To solve this equation, a numerical time integration method is usually utilized, such as the Newmark Integration. For such a technique, initial conditions must be introduced and those are given as initial velocity and displacement $\dot{x}(0)$ and x(0)

Then, the solution at each step can be calculated based on the integration time step. Cytosolic calcium signals play important roles in processes such as cell growth and motility, synaptic communication and formation of neural circuitry. These signals have complex time courses and their quantitative analysis is not easily accomplished; in particular, it may be difficult to evidence subtle differences in their temporal patterns. In this paper, we use wavelet analysis to extract information on the structure of Ca^{2+} oscillations [14].

CONCLUSION

Every measurement system require analysis of its features or performance to work as a system. Time domain analysis gives the behavior of the signal over time. This allows predictions and regression models for the signal. Frequency Analysis is much easier. Some equations can't be solved in time domain while they can be solved easily in frequency domain. Signals are the foundation of information processing, transmission, and storage. Signal representations are unique; a signal is either analog or digital, time domain or frequency domain.

One of the drawbacks of frequency analysis was that, with no time domain data associated with the signals, it was only useful for static signals. Time frequency analysis sometimes called joint time-frequency analysis allows a work around to this problem. Time frequency analysis is the process of taking multiple FFT's (Fast Fourier Series Transform) are taken of small enough portions of data the frequencies will not have had time to change, these FFT's can then be combined to see how the power spectrum of the signal change over time.

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