Thermal Stresses In Semi-Infinite Solid Rectangular Plate With Heat Source

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Abstract

This paper is concerned with three dimensional inverse thermoelastic problem of semi-infinite solid rectangular plate due to internal heat source. To determine unknown temperature distribution, displacement and thermal stresses on edge z=h of the rectangular plate due to internal heat sources with known third kind boundary and initial condition by applying Fourier and Laplace transform technique. The results are obtained in terms of infinite series and the numerical calculations are carried out by using MATHCAD -7 software and shown graphically.

Keywords: Three dimensional inverse thermoelastic problem, solid rectangular plate, integral transform.

1. Introduction

Many heat conduction problem encountered in engineering application involve time as independent variable. The goal of analysis is to determine the variation in temperature displacement and stresses with known boundary condition. Tanigawa et al. [1] have studied thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. Khobragade et al. [4] to determine an inverse unsteady-state thermoelastic problem of thick rectangular plate. Lamba et al. [9] have studied thermoelastic problem of thin rectangular plate due to partially distributed heat supply. Himanshu S.Roy and N.W.Khobragade have discussed transient thermoelastic problem of infinite rectangular slab with second kind boundary condition.

In the present paper, an attempt has been made completely the inverse unsteady state thermoelastic problem of semi-infinite solid rectangular plate with internal heat source applied for upper plane surface with third kind boundary conditions. To determine the temperature, displacement and thermal stresses on the edge z= h of semi- infinite rectangular plate due to internal heat by applying Fourier and Laplace transform technique.

2. Statement of the problem

Consider a solid rectangular plate from fig. 1 occupying the space $D: 0 \le x < \infty, 0 \le y < \infty, 0 \le z \le h$.The displacement components ux, uy, uz in the x, y, z direction respectively are in the integral form as in [1]

$$u_x = \int_0^\infty \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right] dx \tag{2.1}$$

$$u_y = \int_0^\infty \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right] dy \tag{2.2}$$

$$u_z = \int_0^h \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2}\right) + \lambda T\right] dz \tag{2.3}$$

where E, ν and λ are the Young modulus, the poisson ratio and the linear coefficient of thermal expansion of the material of the plate respectively, U(x, y, z ,t) is the Airy stress function which satisfies the differential equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\lambda E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T(x,y,z,t) \tag{2.4}$$

Here $T(x,y,z,t)$ denotes the temperature of the thin rectangular plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\psi(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2.5}$$

Where k is thermal conductivity and α is the thermal diffusivity of the material of the plate and $\psi(x,y,z,t)$ is heat generated within the rectangular plate for t>0 subject to initial conditions

$$T(x,y,z,0) = 0 \tag{2.6}$$

The boundary conditions

$$\left[\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=0} = F_1(y,z,t) \tag{2.7}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=0} = F_2(x,z,t) \tag{2.8}$$

$$\left[T(x,y,z,t) + c\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0} = 0 \tag{2.9}$$

$$[T(x,y,z,t)]_{z=h} = g(x,y,t) \text{ Unknown)} \tag{2.10}$$

The interior condition

$$\left[T(x,y,z,t) + c\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=\xi} = f(x,y,t) \quad \text{(Known)} \tag{2.11}$$

The components in term of U(x, y, z, t) are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) \quad (2.12)$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right) \quad (2.13)$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \quad (2.14)$$

the equations (2.1) to (2.14) constitute the mathematical formulation of the problem under consideration.

3. Solution of the problem

The Fourier integral transform of f(x), $0 \le x < \infty$ is defined to be

$$\bar{F}(\beta) = \int_{x'=0}^{\infty} K(\beta, x') F(x') dx' \quad (3.1)$$

then inverse formula

$$F(x) = \int_{\beta=0}^{\infty} K(\beta, x) \bar{F}(\beta)\, d\beta \quad (3.2)$$

Where $K(\beta, x) = \sqrt{\frac{2}{\pi}} \cos \beta x$ (3.3)

By applying the Fourier transform two times and Laplace transform to equation (2.5) and their inverses, we obtain

$$\frac{d^2 \bar{\bar{T}}^*}{dz^2} - p^2 \bar{\bar{T}}^* = \propto \left(\phi + \frac{\bar{\bar{\psi}}^*}{k}\right) \quad (3.4)$$

Where $p^2 = \beta^2 + \gamma^2 + \frac{s}{\propto}$, $\phi = \overline{\phi_1}^* + \phi_2{}^*$

$\phi_1 = [K(\beta, x)]_{x=0} F_1$, $\phi_2 = [K(\gamma, y)]_{y=0} F_2$, upper limit vanishes

$$\left[\bar{\bar{T}}^*(\beta,\gamma,z,t) + c\frac{d\bar{\bar{T}}^*(\beta,\gamma,z,t)}{dz}\right]_{z=0} = 0 \quad (3.5)$$

$$\left[\bar{\bar{T}}^*(\beta,\gamma,z,t)\right]_{z=h} = \bar{\bar{g}}^*(\beta,\gamma,z,t) \quad (3.6)$$

$$\left[\bar{\bar{T}}^*(\beta,\gamma,z,t) + c\frac{d\bar{\bar{T}}^*(\beta,\gamma,z,t)}{dz}\right]_{z=\xi} = \bar{\bar{f}}^*(\beta,\gamma,t) \quad (3.7)$$

The equation (3.4) is a second order differential equation whose solution is in form

$$\bar{\bar{T}}^* = Ae^{pz} + Be^{-pz} + PI \quad (3.8)$$

Where $PI = \frac{\propto (Q^* - \frac{\bar{\bar{\psi}}^*}{k})}{D^2 - p^2}$, $D \equiv \frac{d}{dz}$ and A, B are constant. Using equation (3.5), (3.7) in (3.8) we obtain the values of A and B substituting these values (3.8) and then apply inverse of Laplace transform and Fourier integral transform. We obtain

$$T(x,y,z,t) = \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{(1-c^2p^2)}\, l[\sin(pz) - cp\cos(pz)]\right\} \int_0^t \left[\bar{\bar{f}}(\beta,\gamma,t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma$$
$$- \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{(1-c^2p^2)}\, l[\sin p(z-\xi) - cp\cos p(z-\xi)]\right\} \times \int_0^t \left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$$
$$d\beta d\gamma + \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) [\ L^{-1}\{PI\}] \int_0^t e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma \quad (3.9)$$

where $p = \frac{\pi l}{\xi}$

$$g(x,y,t) = \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{(1-c^2p^2)}\, l[\sin(ph) - cp\cos(ph)]\right\} \int_0^t \left[\bar{\bar{f}}(\beta,\gamma,t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma$$
$$- \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{(1-c^2p^2)}\, l[\sin p(h-\xi) - cp\cos p(h-\xi)]\right\} \times \int_0^t \left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$$
$$d\beta d\gamma + \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) [\ L^{-1}\{PI\}] \int_0^t e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma \quad (3.10)$$

where

$\phi_1(z) = [\sin(pz) - cp\cos(pz)]$

$\psi_1(t) = \int_0^t \left[\bar{\bar{f}}(\beta,\gamma,t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$

$\phi_2(z) = [\sin p(z-\xi) - cp\cos p(z-\xi)]$

$\psi_2(t) = \int_0^t \left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$

$\psi_3(t) = \int_0^t e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$

4. Determination of the airy stress function

Substituting the value of T(x, y, z, t) from equation (3.9) in the equation (2.4), one obtains

$$U(x,y,z,t) = -\lambda E \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)}\, l[\sin(pz) - cp\cos(pz)]\right\} \int_0^t \left[\bar{\bar{f}}(\beta,\gamma,t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma$$
$$+\lambda E \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2} \left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)}\, l[\sin p(z-\xi) - cp\cos p(z-\xi)]\right\} \times \int_0^t \left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right] e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'$$
$$d\beta d\gamma - \lambda E \int_{\beta=0}^{\infty} K(\beta,x)\ K(\gamma,y)\ [\ L^{-1}\{PI\}] \int_0^t e^{-\propto(\beta^2+\gamma^2+p^2)(t-t')} dt'\ d\beta d\gamma \quad (4.1)$$

$U(x,y,z,t) =$

$-\lambda E \int_{\beta,\gamma=0}^{\infty} K(\beta,x)\ K(\gamma,y) \frac{2k\pi}{\xi^2}$

$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}d\beta d\gamma$$

$$-\lambda E\int_{\beta=0}^{\infty}K(\beta,x)\ K(\gamma,y)[\ L^{-1}\{PI\}]\psi_3(t)d\beta d\gamma \quad (4.2)$$

5. Determination of the displacement components

Substituting the value of U(x, y, z, t) from equation (4.1) in the equation (2.1), (2.2), (2.3) one obtains

$$u_x=-\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$-\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx$$
$$\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma[\ L^{-1}\{PI\}]\psi_3(t)$$
$$-\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi''_1(z)\psi_1(t)-\phi''_2(z)\psi_2(t)]\right\}$$
$$-\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx\frac{\partial^2[\ L^{-1}\{PI\}]}{\partial z^2}$$
$$K(\mu_n,y)\psi_3(t)$$
$$+\lambda\upsilon\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K''(\beta,x)d\beta\right)dx$$
$$\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$+\lambda\upsilon\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K''(\beta,x)d\beta\right)dx$$
$$\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma[\ L^{-1}\{PI\}]\psi_3(t)$$
$$+\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$+\lambda\int_{x=0}^{\infty}\left(\int_{\beta=0}^{\infty}K(\beta,x)d\beta\right)dx\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma$$
$$[\ L^{-1}\{PI\}]\ \psi_3(t) \quad (5.1)$$

$$u_y=-\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K(\beta,x)d\beta$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi''_1(z)\psi_1(t)-\phi''_2(z)\psi_2(t)]\right\}$$
$$-\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K(\beta,x)d\beta$$
$$\frac{\partial^2[\ L^{-1}\{PI\}]}{\partial z^2}\ \psi_3(t)$$
$$-\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K''(\beta,x)d\beta$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$-\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K''(\beta,x)d\beta$$
$$[\ L^{-1}\{PI\}]\psi_3(t)$$
$$+\lambda\upsilon\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K(\beta,x)d\beta$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$+\lambda\upsilon\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma\right)dy$$
$$\int_{\beta=0}^{\infty}K(\beta,x)d\beta[\ L^{-1}\{PI\}]\psi_3(t)$$
$$+\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy\int_{\beta=0}^{\infty}K(\beta,x)d\beta$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$+\lambda\int_{y=0}^{\infty}\left(\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\right)dy$$
$$\int_{\beta=0}^{\infty}K(\beta,x)d\beta\ [\ L^{-1}\{PI\}]\ \psi_3(t) \quad (5.2)$$

$$u_z=-\lambda\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\int_{\beta=0}^{\infty}K''(\beta,x)d\beta\frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l\left[\int_0^h\phi_1(z)dz\,\psi_1(t)-\int_0^h\phi_2(z)dz\,\psi_2(t)\right]\right\}$$
$$-\lambda\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\int_{\beta=0}^{\infty}K''(\beta,x)d\beta$$
$$\int_0^h[\ L^{-1}\{PI\}]dz\ \psi_3(t)$$
$$-\lambda\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma\frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l\left[\int_0^h\phi_1(z)dz\,\psi_1(t)-\int_0^h\phi_2(z)dz\,\psi_2(t)\right]\right\}$$
$$-\lambda\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma\int_0^h[\ L^{-1}\{PI\}]dz\ \psi_3(t)$$
$$+\nu\lambda\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l\left[\int_0^h\phi''_1(z)dz\,\psi_1(t)-\int_0^h\phi''_2(z)dz\,\psi_2(t)\right]\right\}$$
$$+\lambda\nu\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma$$
$$\int_0^h[\ L^{-1}\{PI\}]dz\ \psi_3(t)$$
$$+\lambda\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K(\gamma,y)d\gamma\frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l\left[\int_0^h\phi_1(z)dz\,\psi_1(t)-\int_0^h\phi_2(z)dz\,\psi_2(t)\right]\right\}$$
$$+\sum_{m,n=1}^{\infty}K(\lambda_m,x)\,K(\mu_n,y)\int_0^h[\ L^{-1}\{PI\}]dz\ \psi_3(t) \quad (5.3)$$

6. Determination of stress functions

Using (4.1) in (3.14), (3.15) and (3.16) the stress functions are obtained as

$$\sigma_{xx}=-\lambda E\int_{\beta=0}^{\infty}K(\beta,x)d\beta\int_{\gamma=0}^{\infty}K''(\gamma,y)d\gamma$$
$$\frac{2k\pi}{\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{p^2(1-c^2p^2)}l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$

$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K''(\gamma,y)d\gamma$$
$$[\ L^{-1}\{PI\}]\psi_3(t)$$
$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma \frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)} l[\phi''_1(z)\psi_1(t)-\phi''_2(z)\psi_2(t)]\right\}$$
$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma$$
$$\frac{\partial^2[\ L^{-1}\{PI\}]}{\partial z^2}\psi_3(t) \tag{6.1}$$

$$\sigma_{yy} = -\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma \frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)} l[\phi''_1(z)\psi_1(t)-\phi''_2(z)\psi_2(t)]\right\}$$
$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma$$
$$\frac{\partial^2[\ L^{-1}\{PI\}]}{\partial z^2}\psi_3(t)$$
$$-\lambda E \int_{\beta=0}^{\infty} K''(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma \frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)} l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$-\lambda E \int_{\beta=0}^{\infty} K''(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma$$
$$[\ L^{-1}\{PI\}]\psi_3(t) \tag{6.2}$$

$$\sigma_{zz} = -\lambda E \int_{\beta=0}^{\infty} K''(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma \frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)} l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$-\lambda E \int_{\beta=0}^{\infty} K''(\beta,x)d\beta \int_{\gamma=0}^{\infty} K(\gamma,y)d\gamma$$
$$[\ L^{-1}\{PI\}]\psi_3(t)$$
$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K''(\gamma,y)d\gamma \frac{2k\pi}{\xi^2}$$
$$\left\{\sum_{l=1}^{\infty} \frac{(-1)^{l+1}}{p^2(1-c^2p^2)} l[\phi_1(z)\psi_1(t)-\phi_2(z)\psi_2(t)]\right\}$$
$$-\lambda E \int_{\beta=0}^{\infty} K(\beta,x)d\beta \int_{\gamma=0}^{\infty} K''(\gamma,y)d\gamma$$
$$[\ L^{-1}\{PI\}]\psi_3(t) \tag{6.3}$$

7. Special case

Setting

$$f(x,y,t) = \delta(x)\,\delta(y)(1-e^{-\xi})^2 e^{-t} \tag{7.1}$$
$$\psi(x,y,z,t) = \delta(x)\,\delta(y)e^{(z-z_0)}e^{-\omega t}$$
$$\omega > 0 \tag{7.2}$$
$$\bar{\bar{f}}^* = \sqrt{\frac{2}{\pi}}\cos\beta x\sqrt{\frac{2}{\pi}}\cos\eta y\,(1-e^{-\xi})^2 e^{-t} \tag{7.3}$$
$$\bar{\bar{\psi}}^* = \sqrt{\frac{2}{\pi}}\cos\beta x\sqrt{\frac{2}{\pi}}\cos\eta y\, e^{(z-z_0)}\frac{1}{s+\omega} \tag{7.4}$$
$$\square\square = \frac{1}{1-p^2}\sqrt{\frac{2}{\pi}}\cos\beta x\sqrt{\frac{2}{\pi}}\cos\eta y\, e^{(z-z_0)}e^{-\omega t} \tag{7.5}$$

Substitute the values in the equation (3.9),(3.10),(4.1),(5.1),(5.2)(5.3),(6.1),(6.2)and(6.3)one obtains

$$T(x,y,z,t) =$$
$$\left(1-e^{-\xi}\right)^2 e^{\xi-z_0}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)$$
$$\times\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{(1-c^2p^2)} l[\sin(pz)-cp\cos(pz)]\right\}$$
$$e^{-\propto(p^2+A)t}$$
$$-\left(1-e^{-\xi}\right)^2 e^{\xi-z_0}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)$$
$$\times$$
$$\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{(1-c^2p^2)} l[\sin p(z-\xi)-cp\cos p(z-\xi)]\right\}$$
$$e^{-\propto(p^2+B)t}$$
$$+\frac{1}{1-p^2}e^{(z-z_0)}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)e^{-\propto(p^2+1)t} \tag{7.6}$$

$$g(x,y,t) = \left(1-e^{-\xi}\right)^2 e^{\xi-z_0}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)$$
$$\times\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{(1-c^2p^2)} l[\sin(ph)-cp\cos(ph)]\right\}$$
$$e^{-\propto(p^2+A)t}$$
$$-\left(1-e^{-\xi}\right)^2 e^{-z_0}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)$$
$$\times\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{(1-c^2p^2)} l[\sin p(h-\xi)-cp\cos p(h-\xi)]\right\}$$
$$e^{-\propto(p^2+B)t}$$
$$+\frac{1}{1-p^2}e^{(h-z_0)}\operatorname{erf}\left(\frac{x}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{y}{\sqrt{4\pi\alpha}}\right)e^{-\propto(p^2+1)t} \tag{7.7}$$

8. Numerical results

For Aluminum metal

Modulus elasticity $E = 6.9\times10^{11}$ Poisson ratio $\upsilon = 0.35$, $\lambda = 12.84\times10^{-6}$

Thermal Expansion coefficient $\alpha_t = 25.5\times10^{-6}$

Thermal Diffusivity k=117, c=1

Thermal Conductivity $\propto = 3.33$, h=1,$\xi = 0.5$

A=3, B=2, p=$\frac{\pi l}{\xi}$, x=2, y=3, $z_0 = 0.2$

$$T(x,y,z,t) =$$
$$(1-e^{-0.5})^2 e^{0.5-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$
$$\times\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\left(\frac{\pi l}{\xi}z\right)-\frac{\pi l}{\xi}\cos\left(\frac{\pi l}{\xi}z\right)\right]\right\}$$
$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+3\right)t}$$
$$-(1-e^{-0.5})^2 e^{-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$
$$\times\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\frac{\pi l}{\xi}(z-\xi)-\frac{\pi l}{\xi}\cos p(z-\xi)\right]\right\}$$
$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+2\right)t}$$
$$+\frac{1}{1-\left(\frac{\pi l}{\xi}\right)^2}e^{(z-0.2)}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+1\right)t} \tag{7.8}$$

$$g(x,y,t) =$$
$$(1-e^{-0.5})^2 e^{0.5-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$\times \frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\left(\frac{\pi l}{\xi}\right)-\frac{\pi l}{\xi}\cos\left(\frac{\pi l}{\xi}\right)\right]\right\}$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+3\right)t}$$

$$-\left(1-e^{-0.5}\right)^2 e^{-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$\times$$

$$\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\frac{\pi l}{\xi}(h-\xi)-\frac{\pi l}{\xi}\,cp\cos\frac{\pi l}{\xi}(h-\xi)\right]\right\}$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+2\right)t}$$

$$+\frac{1}{1-\left(\frac{\pi l}{\xi}\right)^2}e^{(h-0.2)}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+1\right)t} \tag{7.9}$$

$$U(x,y,t)$$

$$=\lambda E\left(1-e^{-0.5}\right)^2 e^{0.5-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$\times \frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(\frac{\pi l}{\xi}\right)^2\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\left(\frac{\pi l}{\xi}\right)-\frac{\pi l}{\xi}\cos\left(\frac{\pi l}{\xi}\right)\right]\right\}$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+3\right)t}$$

$$-\lambda E\left(1-e^{-0.5}\right)^2 e^{-0.2}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$\times$$

$$\frac{8\propto}{\pi\xi^2}\left\{\sum_{l=1}^{\infty}\frac{(-1)^{l+1}}{\left(\frac{\pi l}{\xi}\right)^2\left(1-\left(\frac{\pi l}{\xi}\right)^2\right)} l\left[\sin\frac{\pi l}{\xi}(h-\xi)-\frac{\pi l}{\xi}\,cp\cos\frac{\pi l}{\xi}(h-\xi)\right]\right\}$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+2\right)t}$$

$$+\lambda E\frac{1}{1-\left(\frac{\pi l}{\xi}\right)^2}e^{(h-0.2)}\operatorname{erf}\left(\frac{2}{\sqrt{4\pi\alpha}}\right)\operatorname{erf}\left(\frac{3}{\sqrt{4\pi\alpha}}\right)$$

$$e^{-\propto\left(\left(\frac{\pi l}{\xi}\right)^2+1\right)t} \tag{7.10}$$

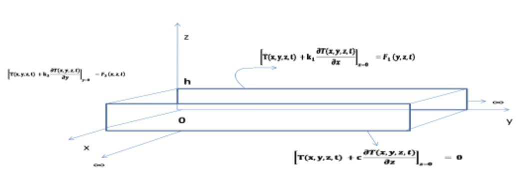

Fig.1 : Three dimensional semi-infinite solid rectangular plate

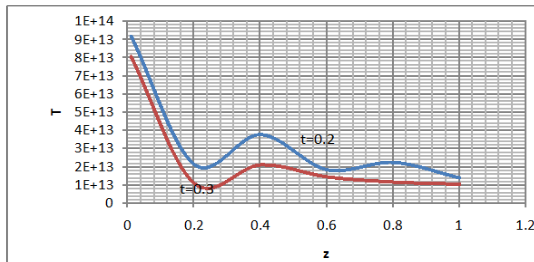

Fig.1 Temperature verses z with different time

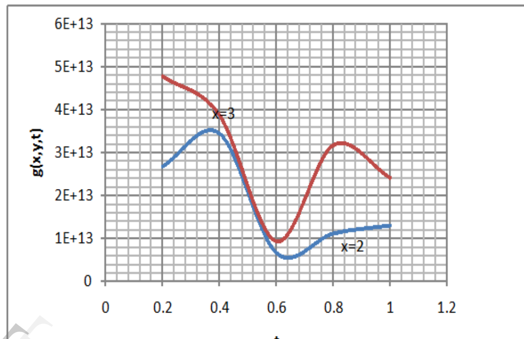

Fig.2 Unknown temperature verses t

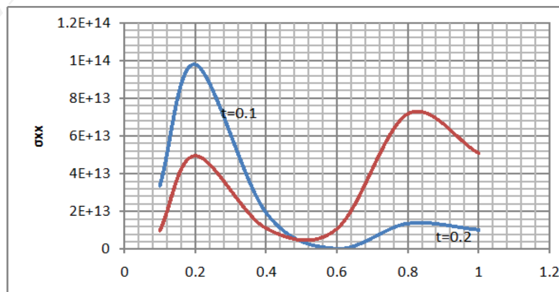

Fig.3 Stresses verses z with different time

7. Conclusion

In this paper, we discussed completely the inverse unsteady state thermoelastic problem of semi-infinite solid rectangular plate with internal heat source applied for upper plane surface with known third kind boundary and initial condition. The Fourier and Laplace integral transforms are used to obtain the numerical results. The temperature, Displacement and thermal stresses that are obtained can be applied to the design of useful structure or machines in engineering application. Any particular case of special interest can be derived by assigning suitable value of the parameters and function in the expression.

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