Thermal Radiation Effect on MHD Flow of A Dusty Fluid Over An Exponentially Stretching Sheet

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Abstract

The present investigation deals with the effect of thermal radiation on MHD boundary layer flow and heat transfer of an incompressible viscous dusty fluid over an exponentially stretching surface. The similarity transformations are used to reduce the given problem into set of non-linear ordinary differential equations. The transformed equations are then solved numerically using Runge-Kutta-Fehlberg fourth-fifth order method. The effects of thermal radiation are carried out for two cases of heat transfer analysis known as Prescribed exponential order surface temperature (PEST) and Prescribed exponential order heat flux (PEHF). The obtained numerical solutions are compared with the exist results and it shows the influence of different physical parameters like fluid-particle interaction parameter, Prandtl Number, Eckert Number, magnetic parameter and radiation parameter on velocity and temperature fields are discussed in detail and are shown graphically.

KeyWords: Thermal radiation, boundary layer flow, exponentially stretching sheet, dusty fluid, numerical solution.

AMS Subject Classification (2010): 76T15, 80A20;

1. INTRODUCTION

The problem of boundary layer flow and heat transfer over a stretching surface has an importance due to its many applications in polymer industry, fibre industry, chemical drying, paper production, glass blowing, etc. In all these cases, the quality of the final product depends on the rate of heat transfer at the stretching surface. Also the heat-transfer analysis of boundary-layer flows with radiation is also important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas.

Initially Sakiadis [16, 17] investigated the boundary layer behavior and encouraged many researchers. Crane [6] has obtained the exact solutions for boundary layer flow caused by stretching surface. The effect of temperature field in flow over a stretching sheet with uniform heat flux was analyzed by Dutta et al [8]. Some of the authors made attempts with non-standard stretching, known as exponential stretching. Magyari et al [11] was the first who considered the boundary layer over an exponentially stretching surface aiming to derive the new type of similarity solution of the governing equations. They examined both analytical and numerical solution for heat and mass transfer in the boundary layers on an exponentially stretching surface. The laminar boundary layer equations describing heat and flow in a quiescent fluid driven by an exponentially stretching in presence of suction parameter was studied by Elbashbeshy [9] and found that there is a significant effect of Prandtl number and suction parameter on fluid flow. Al-odat et al [1] carried out the study with aiming to introduce a local similarity solution of an exponentially stretching surface with an exponential dependence of the temperature distribution in the presence of the magnetic field effect.

The effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet was studied by Sajid et al [15]. Here the homotopy analysis method (HAM) is employed to determine the velocity and temperature profiles. Recently Partha et al [13] and Dulal Pal [7] investigated the effect of viscous dissipation and magnetic field on the mixed convection heat transfer in the boundary layers on an exponentially stretching surface respectively. On the other hand, Anuar Ishak [2] carried out a flow due to an exponentially stretching sheet problem under the effect of magnetic and radiation parameter. Later, Bikash Sahoo et al [4] discussed the flow and heat transfer analysis of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition. A numerical solution is obtained by Reddy et al [3] for the effect of thermal radiation on hydro-magnetic flow due to an exponentially stretching sheet. Singh et al [18] studied the heat transfer analysis in a second grade fluid over an exponentially stretching sheet through porous medium with thermal radiation and elastic deformation under the effect of magnetic field for PST and PHF cases.

Above all investigations deals with the flow and heat transfer analysis only for pure fluids induced by stretching sheet. The study of the flow of dusty fluid has an important application in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, and purification of crude oil, electrostatic precipitation, polymer technology and fluid droplets sprays. Initially Saffman [14] describes the fluid-particle system and obtained the motion of fluid equations carrying the dust particles. Vajravelu et al [20] obtained the solution for the hydromagnetic flow of a dusty fluid over a stretching sheet. The heat transfer effects on dusty gas flow past a semi-infinite inclined plate was analyzed by Palani et al [12]. Recently, Gireesha et al [10] have discussed the flow and heat transfer of heat transfer analysis of a dusty fluid over a stretching sheet in presence of MHD and viscous dissipation. Further, they concluded that for effective cooling of the stretching sheet the strength of external magnetic field should be as mild as possible and the effect of Chandrasekhar number is to increase temperature distributions in the flow region for both the phases.

So based on these, the objective of the present investigation is to study the effect of thermal radiation on MHD flow and heat transfer of a dusty fluid over an exponentially stretching sheet. Further we have considered two cases of heat transfer analysis namely Prescribed exponential order surface temperature and Prescribed exponential order heat flux. The governing equations have been simplified using suitable similarity transformations and then are solved numerically using Runge-Kutta-Fehlberg fourth-fifth order method with the help of Maple.

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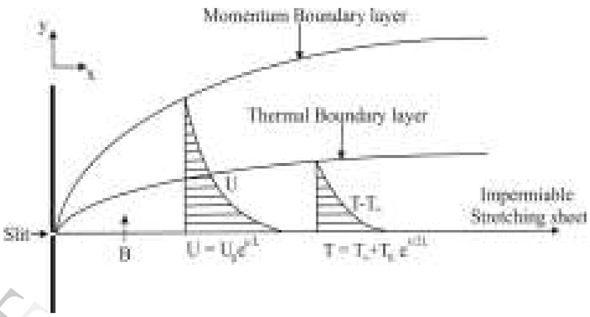
2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

A steady two-dimensional laminar boundary layer flow and heat transfer of an incompressible viscous dusty fluid near an impermeable plane wall stretching with velocity $U_w = U_0 e^{(x/L)}$ is considered. The x-axis is chosen along the sheet and y-axis normal to it. Two equal and opposite forces are applied along the sheet so that the wall is stretched exponentially. A uniform magnetic field is applied in y-direction.



Schematic representation of boundary layer flow.

Under these assumptions, the two dimensional boundary layer equations can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

(2.1)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(u_p - u) - \frac{\sigma B^2 u}{\rho},$$

(2.2) $$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0,$$

(2.3) $$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{K}{m}(u - u_p),$$

(2.4)

where x and y represents coordinate axes along the continuous surface in the direction of motion and perpendicular to it, respectively. (u, v) and (u_p, v_p) denotes the velocity components of the fluid and particle phase along the x and y directions respectively, ν is the coefficient of viscosity of fluid, ρ is the density of the fluid phase, K is the Stoke's resistance, N is the number density of dust particles, m is the mass concentration of dust particles, $\tau_v = m/K$ is the relaxation time of particle phase.

In order to solve the governing boundary layer equations consider the following appropriate boundary conditions on velocity;

$$u = U_w(x), \quad v = 0 \quad \text{at } y = 0,$$

$$u \to 0,\ u_p \to 0,\ v_p \to 0, \text{ as } y \to \infty, \tag{2.5}$$

where $U_w(x) = U_0 e^{(x/L)}$ is the sheet velocity, U_0 is reference velocity and L is the reference length.

Equations (2.1) to (2.4) are subjected to boundary condition (2.5), admits self-similar solutions in terms of the similarity function f and the similarity variable η as

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \quad v = -\sqrt{\frac{u_0 \nu}{2L}} e^{\frac{x}{2L}} [f(\eta) + \eta f'(\eta)],$$

$$u_p = U_0 e^{\frac{x}{L}} F'(\eta),$$

$$v_p = -\sqrt{\frac{u_0 \nu}{2L}} e^{\frac{x}{2L}} [F(\eta) + \eta F'(\eta)], \quad \eta = \sqrt{\frac{u_0}{2\nu L}} e^{\frac{x}{2L}} y,$$

$$B = B_0 e^{\frac{x}{2L}}, \tag{2.6}$$

Where B_0 is the magnetic field flux density

These equations identically satisfy the governing equation (2.1) and (2.3). Substitute (2.6) into (2.2)-(2.4) and on equating the co-efficient of $(x \setminus L)^0$ on both sides, then one can get

$$f'''(\eta) + f(\eta) f''(\eta) - 2f'(\eta)^2 + 2l\beta[F'(\eta) - f'(\eta)] - Mf'(\eta) = 0, \tag{2.7}$$

$$F(\eta) F''(\eta) - 2F'(\eta)^2 + 2\beta[f'(\eta) - F'(\eta)] = 0, \tag{2.8}$$

where prime denotes the differentiation with respect to η and $l = mN/\rho$ is the mass concentration, $\beta = L/\tau_v U_0$ is the fluid-particle interaction parameter for velocity.

Similarity boundary conditions (2.5) will become,

$$f'(\eta) = 1, \quad f(\eta) = 0 \quad \text{at } \eta = 0,$$

$$f'(\eta) = 0, \quad F'(\eta) = 0,$$

$$F(\eta) = f(\eta) + \eta f'(\eta) - \eta F'(\eta) \tag{2.9}$$

3. HEAT TRANSFER ANALYSIS

The governing steady, dusty boundary layer heat transport equations are given by,

$$\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \frac{N c_p}{\tau_T} \left(T_p - T \right) + \frac{N}{\tau_v} \left(u_p - u \right)^2 - \frac{\partial q_r}{\partial y}, \tag{3.1}$$

$$N c_m \left[u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right] = -\frac{N c_p}{\tau_T} \left(T_p - T \right), \tag{3.2}$$

where T and T_p are the temperatures of the fluid and dust particle inside the boundary layer, c_p and c_m are the specific heat of fluid and dust particles, τ_T is the thermal equilibrium time i.e., it is time required by a dust cloud to adjust its temperature to the fluid, k is the thermal conductivity, τ_v is the relaxation time of the of dust particle and q_r is the radiative heat flux.

Using the Rosseland approximation [5] for radiation, radiative heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \tag{3.3}$$

Where σ^* and k^* are the Stefan-Boltzman constant and the mean absorption co-efficient respectively. T^4 is expressed as a linear function of the temperature, and hence we get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \tag{3.4}$$

Using (3.3) and (3.4), (3.1) can be written as,

$$\rho c_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\left(k+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{Nc_p}{\tau_T}\left(T_p-T\right)+\frac{N}{\tau_v}\left(u_p-u\right)^2 \tag{3.5}$$

We considered the heat transfer phenomenon for two types of heating process, namely [20]

(1) Prescribed exponential order surface temperature (PEST) , and

(2) Prescribed exponential order heat flux (PEHF)

Case 1: Prescribed exponential order surface temperature:

For this heating process, we employ the following boundary conditions;

$$T=T_w(x) \quad \text{at } y=0,$$
$$T\to T_\infty, \quad T_p\to T_\infty \quad \text{as} \quad y\to\infty, \tag{3.6}$$

where $T_w=T_\infty+T_0 e^{c_1 x/2L}$ is the temperature distribution in the stretching surface, T_0 is a reference temperature and c_1 is constant.

Introduce the dimensionless variables for the temperatures $\theta(\eta)$ and $\theta_p(\eta)$ as follows:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}, \tag{3.7}$$

where $T-T_\infty=T_0 e^{c_1 x/2L}\theta(\eta)$.

Using the similarity variable η and (3.7) into (3.2) to (3.5) and on equating the co-efficient of $(x\setminus L)^0$ on both sides, one can arrive the following system of equations;

$$\left(1+\frac{4R}{3}\right)\theta''(\eta)+\Pr[f(\eta)\theta'(\eta)-c_1 f'(\eta)\theta(\eta)]+\frac{2N}{\rho}\beta_T\Pr[\theta_p(\eta)-\theta(\eta)]+\frac{2N}{\rho}\beta\Pr Ec[F'(\eta)-f'(\eta)]^2=0 \tag{3.8}$$

$$c_1F'(\eta)\theta_p(\eta)-F(\eta)\theta_p'(\eta)+2\beta_T\gamma[\theta_p(\eta)-\theta(\eta)]=0 \tag{3.9}$$

where $R=4\sigma^* T_\infty^3/kk^*$ is the radiation parameter, $\Pr=\mu c_p/k$ is the Prandtl number, $Ec=U_0^2/c_pT$ is the Eckert number, $\beta=L/\tau_v U_0$ and $\beta_\tau=L/\tau_T U_0$ are the fluid-particle interaction parameter for velocity and temperature and $\gamma=c_p/c_m$ is the ratio of specific heat.

Corresponding thermal boundary conditions reduceses to,

$$\theta(\eta)=1 \quad \text{at } \eta=0,$$
$$\theta(\eta)\to 0, \quad \theta_p(\eta)\to 0, \quad \text{as} \quad \eta\to\infty. \tag{3.10}$$

Case 2: Prescribed exponential order heat flux:

For this heating process, we employ the following boundary conditions:

$$\frac{\partial T}{\partial y}=-\frac{q_w(x)}{k} \quad \text{at } y=0,$$
$$T\to T_\infty, \quad T_p\to T_\infty \quad \text{as } y\to\infty, \tag{3.11}$$

Where $q_w(x)=T_1 e^{(c_2+1)x/2L}$, T_1 is reference temperature and c_2 is constant.

Using the similarity variable η and (3.11) into (3.2) to (3.5) and on equating the co-efficient of $(x\setminus L)^0$ on both sides, one can arrive the following system of equations:

$$\left(1+\frac{4R}{3}\right)\theta''(\eta)+\Pr[f(\eta)\theta'(\eta)-c_1 f'(\eta)\theta(\eta)]+\frac{2N}{\rho}\beta_T\Pr[\theta_p(\eta)-\theta(\eta)]+\frac{2N}{\rho}\beta\Pr Ec[F'(\eta)-f'(\eta)]^2=0 \tag{3.12}$$

$$c_1F'(\eta)\theta_p(\eta)-F(\eta)\theta_p'(\eta)+2\beta_T\gamma[\theta_p(\eta)-\theta(\eta)]=0, \tag{3.13}$$

where $Ec = kU_0^2 / c_p T_1 \sqrt{U_0 / 2\nu L}$ is the Eckert number.

Corresponding thermal boundary conditions becomes,

$$\theta'(\eta) = -1 \quad \text{at } \eta = 0,$$

$$\theta(\eta) \to 0, \quad \theta_p(\eta) \to 0, \quad \text{as } \eta \to \infty$$
(3.14)

4. NUMERICAL SOLUTION

Two-dimensional MHD boundary layer flow and heat transfer of a dusty fluid over an exponential stretching sheet is considered. The system of of highly non-linear ordinary differential equations (2.7)-(2.8), (3.8)-(3.9) for PEST case or (3.12)-(3.13) for PEHF case are solved numerically using Runge-Kutta-Fehlberg fourth-fifth order method with the help of Maple software. In this method, we choose suitable finite values of $\eta \to \infty$ say $\eta = 5$.

Here we have given the comparison of our results of $-\theta'(0)$ in presence of thermal radiation with [5] as in Table 1 and in absence of thermal radiation with [11] as in Table 2 for various values of $\Pr$. From these tables, one can notice that there is a close agreement with this approach and thus verifies the accuracy of the method used. Further, we have considered the effect of magnetic field and thermal radiation on velocity and temperature profiles and are depicted graphically for different values of fluid-particle interaction parameter (β), Magnetic parameter (M), Radiation parameter (R), Prandtl number ($\Pr$) and Eckert number (Ec).

5. RESULTS AND DISCUSSION

Numerical calculations are performed for velocity and temperature profiles for various values of physical parameters such as fluid-particle interaction parameter (β), Magnetic parameter (M), radiation parameter (R), Prandtl number ($\Pr$) and Eckert number (Ec) and are depicted graphically (from figure 2 to figure 8). Comparison values of wall-temperature gradient are tabulated in Table 1 and 2 and are found in good agreement.

Figures 2 and 3 represents the velocity profiles for different values of β and M. From figure 2, the observation shows that increase of fluid particle interaction parameter β decreases the fluid velocity $f'(\eta)$ and increases the particle velocity $F'(\eta)$ with $\Pr = 1$, $M = 0.5$, $R = 1$ and $Ec = 0.5$. From this one can noticed that at a particular values of β, the velocity of fluid and dust particle will be same. The velocity profiles $f'(\eta)$ and $F'(\eta)$ versus η for different values of the magnetic parameter M presented in the figure 3. The variation of M leads to the variation of the Lorentz force and it produces more resistance to the transport phenomena. So that the rate of transport is considerably reduced with the increase of M.

Figure 4 depicts the temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ versus η for different values of fluid particle interaction parameter β. We infer from this figure that temperature decreases with increases in fluid particle interaction parameter β and also it indicates that the temperature profile is higher in PEHF case than in PEST case. We have used $\Pr = 1$, $M = 0.5$, $R = 1$ and $Ec = 0.5$. The figure 5 shows the temperature distributions $\theta(\eta)$ and $\theta_p(\eta)$ versus η for different values of magnetic parameter M with $\beta = 0.5$, $R = 1$, $Ec = 0.5$ and $\Pr = 1$. From this figure it is observed that temperature increases with increases in M for both the cases.

The observations from figure 6 shows the effect of radiation parameter R on temperature profiles with $\beta = 0.5$, $M = 1$, $Ec = 0.5$ and $\Pr = 1$. As thermal radiation increases, temperature profiles for both fluid and dust phases increases in both PEST and PEHF case. It is observed that an increase in the thermal radiation parameter produces a significant increase in the thickness of thermal boundary. The effect of R is to enhance the heat transfer and hence the thermal radiation effect is used for better cooling process. Further one can see that the combined effect of increasing values of magnetic parameter and the thermal radiation is to increase the temperature significantly in the boundary layer.

Figure 7 illustrates the effect of Prandtl number $\Pr$ on temperature profiles with η for $\beta = 0.5$, $R = 1$, $Ec = 0.5$ and $M = 1$. The ratio of viscous diffusion rate to thermal diffusion rate is the Prandtl number in which for small values of $\Pr$ heat diffuses very quickly compared to the velocity. Hence thickness of the thermal boundary layer is much bigger than the velocity boundary layer. From this figure, it reveals that the temperature decreases with increase in the value of $\Pr$.

Figure 8 explains the effect of Eckert number Ec on temperature profiles with η for $\beta = 0.5$, $R = 1$, and $M = 1$. From this one can see that the temperature increases with increasing values of Ec. This is due to the fact that the heat energy is stored in the liquid due to the frictional heating.

We have used the following values $N = 1$, $\rho = \beta_T = 0.5$ and $l = 0.1$ in throughout our thermal analysis.

6. CONCLUSIONS

The present work deals with the boundary layer flow and heat transfer of a steady dusty fluid over an exponential stretching sheet with magnetic field and thermal radiation for both in PEST and PEHF cases. The set of non-linear ordinary differential equations (2.7)-(2.8), (3.8)-(3.9) for PEST Case or (3.12)-(3.13) for PEHF Case are solved numerically by applying RKF-45 order method using the software Maple. The results of the thermal characteristics at the wall are examined for the values of temperature gradient function $\theta'(0)$ in PEST case and the temperature function $\theta(0)$ in PEHF case, which are tabulated in Table 3. The velocity and temperature profiles are obtained for various values of physical parameters like fluid-particle interaction parameter (β), Magnetic parameter (M), radiation parameter (R), Prandtl number ($\Pr$) and Eckert number (Ec). The numerical results obtained are agrees with previously reported cases available in the literature [5] and [11].

The major findings from the present study can be summarized as follows:

- Fluid phase temperature is higher than the dust phase temperature both in PEST and in PEHF cases.
- The study reveals that the momentum boundary layer thickness reduced by magnetic field and thermal boundary layer thickness increased by radiation parameter.
- The effect of increasing the values of M, Ec and R is to increase the wall temperature gradient function $\theta'(0)$ and wall temperature function $\theta(0)$ for both in PEST and PEHF cases.
- The rate of heat transfer $\theta'(0)$ and $\theta(0)$ decreases with increasing in fluid-particle interaction parameter as well as in number density of dust particle.
- If the thermal radiation affects the boundary layer, its thickness increases.
- The Prandtl number decreases the temperature profile for both in PEST and PEHF cases.
- In presence of thermal radiation, our results of $\theta'(0)$ will coincides with Bidin et al [19].
- If $\beta \to 0, M \to 0, R \to 0, N \to 0$ then our results coincides with the results of Magyari et al [5] for different values of Prandtl number.

10. References

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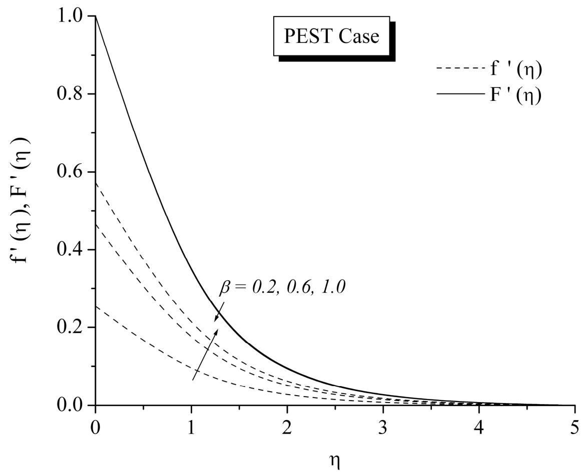

Figure 2: Effect of β on velocity profiles

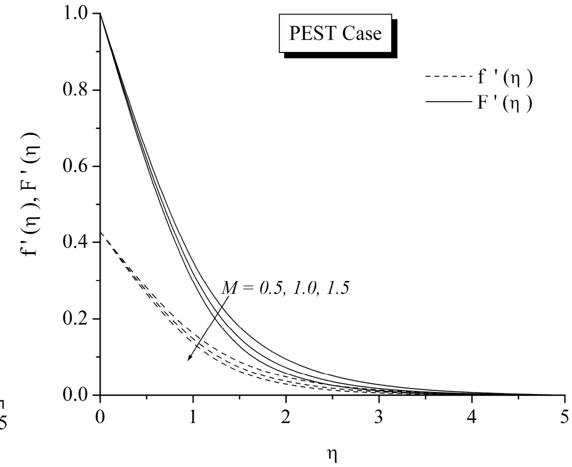

Figure 3: Effect of M on velocity profiles

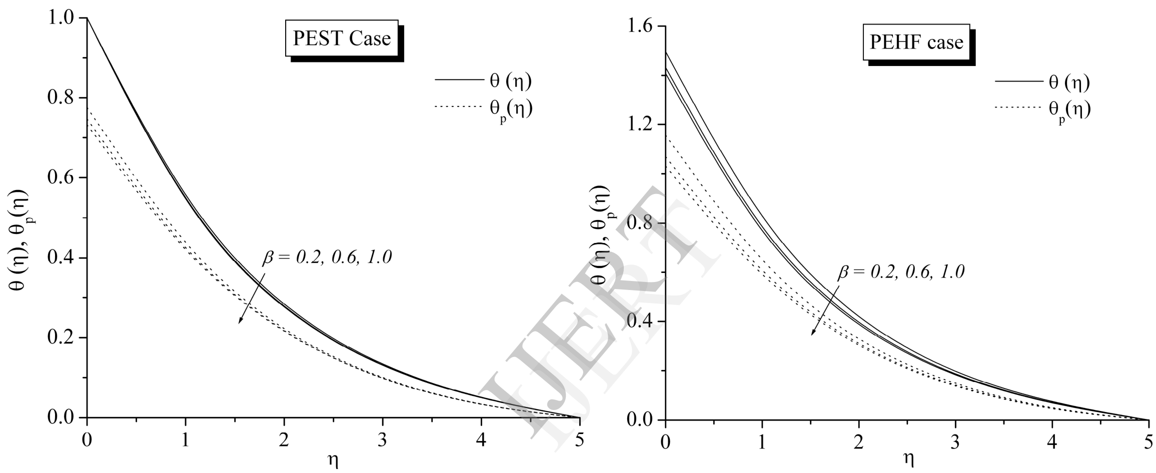

Figure 4: Effect of β on temperature profiles for both PEST and PEHF case.

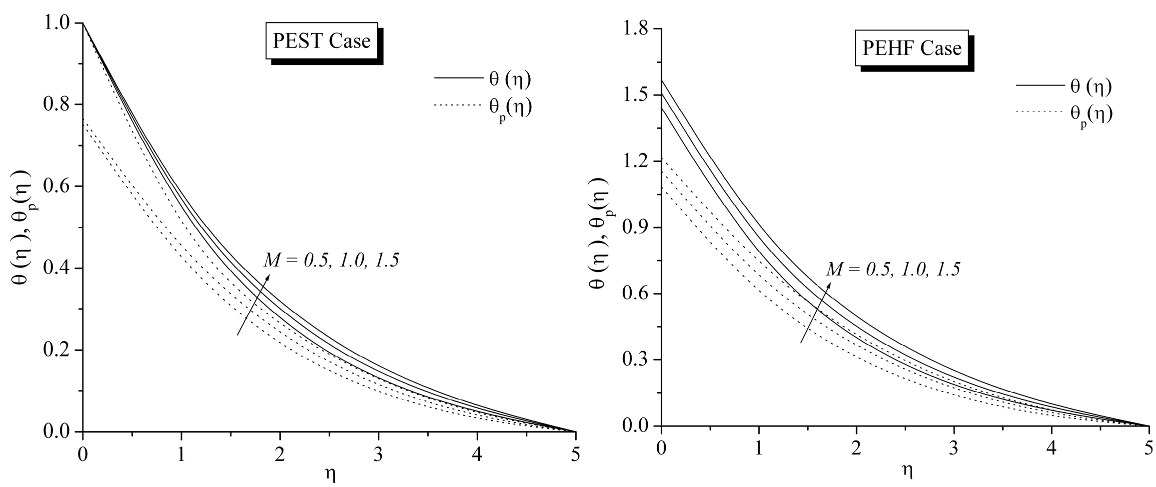

Figure 5: Effect of M on temperature profiles for both PEST and PEHF case.

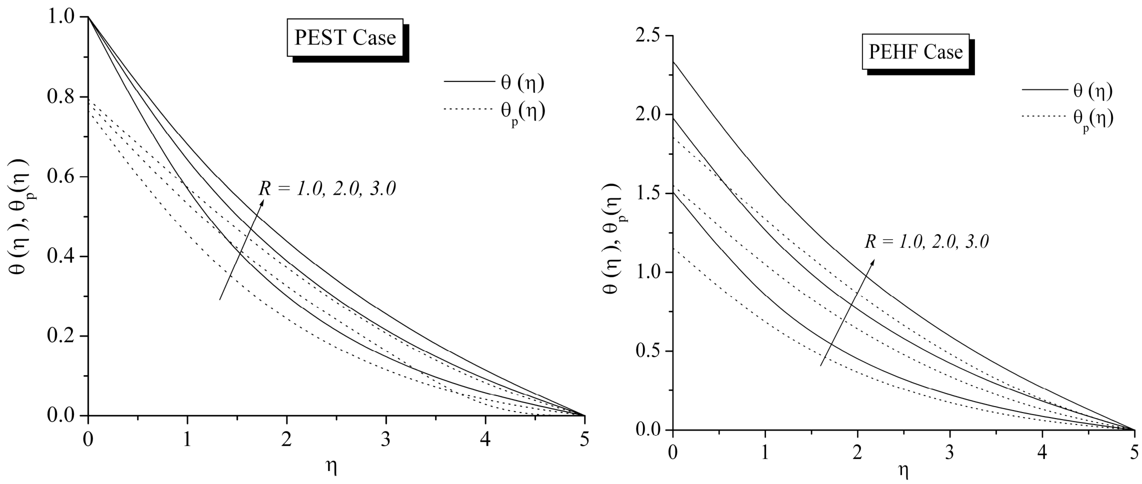

Figure 6: Effect of R on temperature profiles for both PEST and PEHF case.

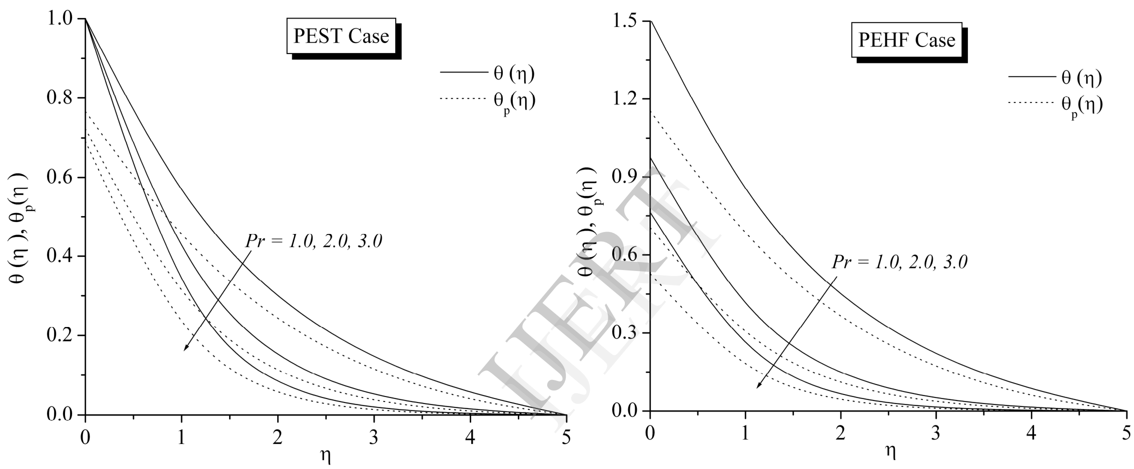

Figure 7: Effect of $\Pr$ on temperature profiles for both PEST and PEHF case.

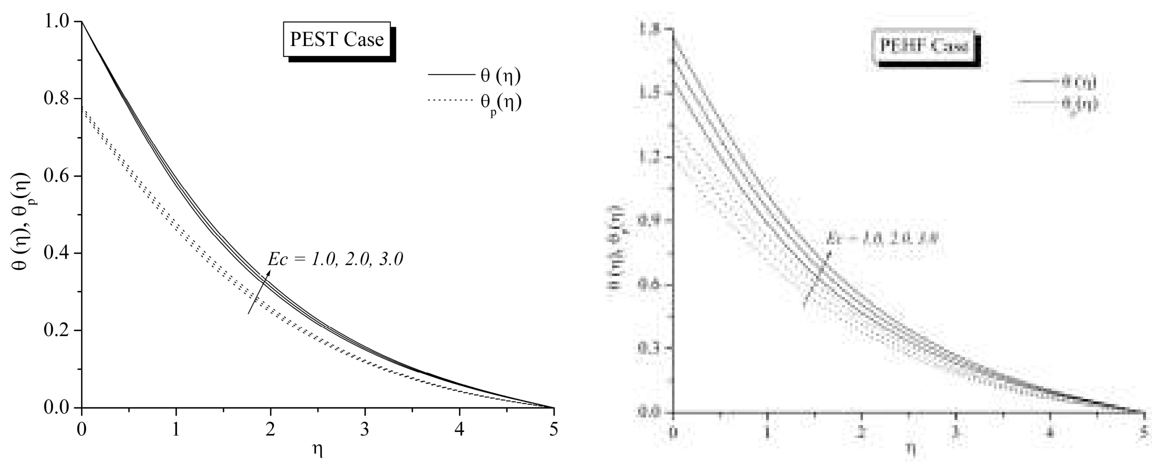

Figure 8: Effect of Ec on temperature profiles for both PEST and PEHF case.

Table 1: Comparison of the results for the dimensionless temperature gradient $-\theta'(0)$ by varying the radiation parameter R with $\beta = N = M = 0$.

Pr	Bidin et al [19] with R = 0	Present Study	Bidin et al [19] with R=0.5	Present Study	Bidin et al [19] with R=1	Present Study
1	-0.9548	-0.9548	-0.6765	-0.6766	-0.5315	-0.5315
2	-1.4700	-1.4700	-1.0735	-1.0735	-0.8627	-0.8628
3	-1.8691	-1.8690	-1.3807	-1.3807	-1.1214	-1.1214

Table 2: Comparison of the results for the dimensionless temperature gradient $-\theta'(0)$ for various values of $\Pr$ and c_1 with $\beta = M = R = N = 0$.

Pr	$c_1 = 0$		$c_1 = 1$		$c_1 = -0.5$	
	Magyari [5]	Present Study	Magyari [5]	Present Study	Magyari [5]	Present Study
0.5	-0.33049	-0.33048	-0.59433	-0.59433	-0.17581	-0.17582
1.0	-0.54964	-0.54964	-0.95478	-0.95478	-0.29987	-0.30000
3.0	-1.12218	-1.122018	-1.86907	-1.86907	-0.63411	-0.63411
5.0	-1.52124	-1.52123	-2.50013	-2.50013	-0.87043	-0.87043
8.0	-1.99184	-1.99183	-3.24212	-3.24212	-1.15032	-1.15032
10	-2.25742	-2.25742	-3.66037	-3.66036	-1.30861	-1.30860

Table 3: Values of wall temperature gradient $-\theta'(0)$ (for PEST Case) and wall temperature $\theta(0)$ (for PEHF Case).

β	M	Ec	$\Pr$	N	R	$-\theta'(0)$ (PEST)	$\theta(0)$ (PEHF)
0.2	1.0	2.0	1.0	1.0	1.0	-0.345442	1.774325
0.6						-0.620168	1.443245
1.0						-0.718472	1.326283
0.6	1.0	2.0	1.0	1.0	1.0	-0.620168	1.443245
	2.0					-0.587550	1.519160
	3.0					-0.560914	1.587520
0.6	1.0	1.0	1.0	1.0	1.0	-0.738550	1.305099
		2.0				-0.620168	1.443245
		3.0				-0.501786	1.581392
0.6	1.0	2.0	1.0	1.0	1.0	-0.620168	1.443245
			2.0			-0.930208	1.051043
			3.0			-1.16282	0.908536
0.6	1.0	2.0	1.0	1.0	1.0	-0.620168	1.443245
				2.0		-0.708780	1.250744

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				3.0		-0.775779	1.156850
0.6	1.0	2.0	1.0	1.0	1.0	-0.620168	1.443245
					2.0	-0.478867	1.820668
					3.0	-0.407353	2.131224