

# Theoretical Study of Modified Secondary Dynamic Vibration Absorber Concept to Damped Out Forced Vibration of Mass to Minimal Which is Induced by Excitation of Support

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**Abstract** - In this world surrounding must be chaos free for accurate working of everything, otherwise anarchy in control would lead to unbalance and discomforts of system. For body subjected to forced vibration due to excitation of support can be bring back to steady state by using spring-mass-damper system over certain period of time. But as excitation frequency of support varies over time period then fixed combination of spring stiffness  $K_1$  and damping coefficient  $C_1$  won't help for long or for short range only. Main body senses severe or mild shock based on deviation from required value of this parameter. But to achieve minimal vibration of main body irrespective of support excitation, we can use concept of secondary vibration absorber but again it has bandwidth issue. To dealt with this problem I have design secondary vibration absorber in such way that it has flexibility to change its mass  $m_2$  by passing fluid through pipe or to change spring stiffness  $K_2$  such that basic condition of frequency match of support excitation and secondary mass will achieve for entire time period with flexibility. As well to make main body free from external excitation force  $F_0$  irrespective of cycle frequency spring force can be vary by using various combination of springs. In this paper I have mention basic principle and mathematical formulation to achieve complete control over vibration of main body and that to within economical scale as no expensive material is used for this purpose. Finally, we have concluded that this secondary vibration absorber replete with accurate control can be use in automobile, machine platform and other application for vibration isolation purpose.

**Keywords:** *Vibration, Dynamic vibration absorber, forced vibration, Automobile suspension system, Mathematical formulation etc.*

## 1. INTRODUCTION

A body of mass  $m_1$  rests on the support with damper of damping coefficient  $C_1$  and spring of stiffness  $K_1$ . In this case when main body is displaced from equilibrium position by application of external force with excitation frequency of  $w$  then for resonance condition of main body with excitation frequency, given body will start oscillating with very large amplitude. As well for small excitation force cause from support displacement will be large. Hence by using current mass-spring-damper system vibration can be damped out in certain cycles but which means that body has to go through all these vibrations before achieving steady state. Here currently most of suspension systems used in automobiles is either leaf spring type for heavy vehicle and helical spring type for light vehicle.

With advancement of technology all this systems are designed in such way that vibration induced in main body should damped out as early as possible by using under-damped system. But all these systems are giving poor performance as all this system are design for certain bandwidth of input excitation frequency  $w$  as well this system performance cannot be altered as prevailing conditions of input excitation. Now days with the introduction of smart materials and electronic controls, variation in performance can be achieved either by varying damper speed to vary damping force to cope with exact amount of excitation force  $F_0$  or by varying damping coefficient of damper by varying viscosity of smart fluid with application of electric current or by magnetic field. Still option with velocity variation is viable at low cost but again it is local control means intermittent whereas if excitation frequency varies cyclically then it might not give good performance and second option can cover range of input frequency of excitation but it will be costly due to smart material involvement.

To address this problem with care concept of dynamic vibration absorber came into existence, in which secondary mass  $m_2$  attached by spring of stiffness  $K_2$  to main body and it will take entire vibration from support by itself and vibrate freely and provide zero vibration to main body theoretically. This concept has problem as mass  $m_2$  and spring constant  $K_2$  cannot be varied as per input frequency as this dynamic secondary vibration absorber gives good performance only when natural frequency of secondary vibration absorber is equal to frequency of external excitation. The main principle of this concept of secondary vibration absorber is as to increase this bandwidth of input frequency to give ideally zero vibration to main body but practically it would be minimal at least; there should be provision to vary the mass of secondary body  $m_2$  and stiffness of spring  $K_2$  with variation in input frequency  $w$  and this can be achieved with this secondary vibration absorber concept.

## 2. SECONDARY DYNAMIC VIBRATION ABSORBER CONCEPT

### 2.1 Layout

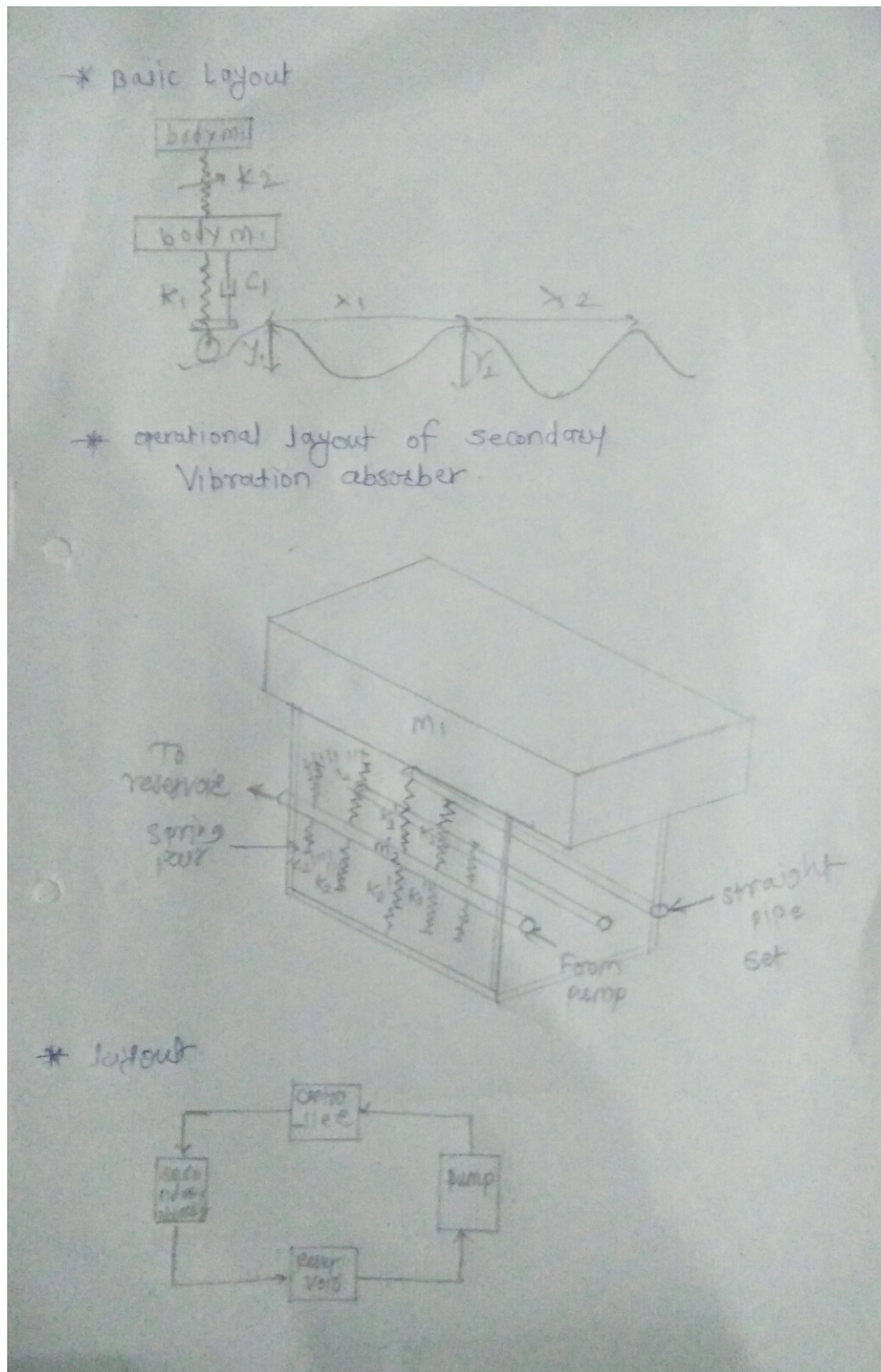


Figure 1. Modified Secondary vibration absorber concept and design layout

### 2.2 Construction

1. This secondary vibration absorber should be connected to main body of system.
2. It should have either hollow helical coiled body or straight pipes connected through pairs of springs of stiffness  $K_2^1, K_2^{11}, \dots$  etc. in parallel between main body and secondary mass  $m_2$  in such way that each pair

of spring will come into operation after certain displacement to increase stiffness of secondary system due to parallel arrangement and spring should be rigidly connected main body except lateral pair of spring, these spring will come into operation only when certain displacement has achieved as per following relation  

$$K_2 = 2K_2^1 + 2K_2^{11} + \dots$$

- This hollow helical tube or hollow straight pipe should be connected through pump and reservoir through flow control valve and variation in flow rate controlled by controller will bring change in mass  $m_2$  of secondary vibration absorber.  
Here  $m_2 = m_2^1 \cdot \text{time}$   
As  $m_2^1 =$  mass flow rate through system  
Here working fluid could smart material or any fluid available at low cost like water.
- In this way this small unit coupled with controller taking input from speedometer to sense speed variation and sensor to sense change in excitation amplitude if excitation source is road. In this case velocity variation and wavelength variation will give current excitation frequency.

### 2.3 Working

- When excitation frequency change  $\Delta \omega$  is noted from sensor then controller will adjust flow rate by actuating flow control valve to maintain the natural frequency  $\omega_{n2}$  of secondary vibration absorber as equal to  $\Delta \omega$ .
- In this way by varying mass flow rate bandwidth of input frequency will enhance.
- Frequency change of excitation source can also be control by either varying mass flow rate through secondary straight pipes or by varying damping coefficient of damper  $C_1$  between main body and support as theoretically calculated in this paper.
- Either combination of variation in  $dC_1$  and variation in  $dm_2$  or by using any of them alone vibration of main body can be damped out completely.
- Displacement  $X_2$  of secondary vibration has relation to excitation force as excitation force will completely neutralize by spring force as  
 $F_o = \text{Excitation force} = K_2 \cdot X_2$
- As  $F_o$  changes with wavelength variation and amplitude variation of road and to compensate for this spring force should be changed by either varying  $K_2$  or varying  $X_2$ . Displacement  $X_2$  has some limit for displacement so to change  $K_2$  with requirement will be favorable option and which will be done by engaging different spring for different displacement.
- In this way input excitation frequency maintain equals to natural frequency of secondary mass  $m_2$  by varying flow rate  $m_2^1$  and by varying spring stiffness  $K_2$  and excitation force  $F_o$  variation is achieved by varying  $K_2$ . So no force will be acting on main body  $m_1$  as well cycle gives best performance for resonance of  $\omega$  and  $\omega_{n2}$  and main body will have vibration if all system combinations are executed properly.
- Here  $F_o$  value is mostly taken by current available damper, which are using two fluid to counteract this force and variation above certain level can be balanced by varying  $K_2$  and  $m_2$ . In this way complete balance can be obtained and vibration would reduce to minimal.

### 2.4 Mathematical formulation for secondary vibration absorber and Results

- Secondary vibration absorber system is two mass system and governing equation for it as follows:

Here for mass  $m_1$ ,  $m_1 \cdot X_1'' + K_1 \cdot (X_1 - X_2) + C_1 \cdot X_1' = F_o \cdot \sin(\omega \cdot t)$

For mass  $m_2$ ,  $m_2 \cdot X_2'' + K_2 \cdot (X_2 - X_1) = 0$

- For better performance of this secondary absorber and to give  $X_1 = 0$

Excitation frequency of support = natural frequency of mass  $m_2$

Here  $\omega = \omega_{n2} = (K_2/m_2)^{0.5}$  ..... equation 1

That is if excitation source is road with amplitude  $Y$  and wavelength  $\lambda$  then angular frequency of excitation is given as  $\omega = 2\pi \cdot V / \lambda$

On differentiating we get that

As  $\Delta \omega = \{ \lambda \cdot (2\pi \cdot dV) - 2\pi \cdot V \cdot d\lambda \} / \lambda^2$  ..... equation 2

- Here for better performance and to meet condition posed by equation 1, we must change mass  $m_2$  and stiffness  $K_2$  as follows

On differentiating equation 1, we get that

As  $\Delta \omega = \{ [-K_2 \cdot dm_2 / 2 \cdot \omega \cdot m_2^2] + [dK_2 / 2 \cdot m_2 \cdot \omega] \}$  ..... equation 3

- As well displacement of main body  $X_1$  with excitation  $Y$  from support then  $X_1$  is given as

$X_1 = Y \cdot (1 + 2 \cdot \zeta \cdot (\omega / \omega_{n1}))^{0.5}$

For  $X_1 = 0$  we get that

As  $\omega = \pm (1/2 \cdot \zeta) \cdot \omega_{n1}$

As  $\omega = \pm (C_c / 2 \cdot C_1) \cdot \omega_{n1}$

$C_c$  = critical damping coefficient

On differentiating we get that

As  $\Delta \omega = (\omega_{n1} \cdot C_c / 2) \cdot (-1/C_1^2) \cdot dC_1$  ..... equation 4

As practically  $C_1$  can only be vary by varying viscosity of fluid as in smart material or by changing damper dimension.

- In this way to satisfy the condition of equal frequency of support and secondary body, we can achieve  $\Delta \omega = \omega_{n2}$  by following practice of using either equation 3 and 4 together or any of them alone.

From equation 3, equation 4 and equation 5, we can write as

As  $\Delta \omega = \omega_{n2} = \{ \lambda \cdot (2\pi \cdot dV) - 2\pi \cdot V \cdot d\lambda \} / \lambda^2 = \{ [-K_2 \cdot dm_2 / 2 \cdot \omega \cdot m_2^2] + [dK_2 / 2 \cdot m_2 \cdot \omega] \} = (\omega_{n1} \cdot C_c / 2) \cdot (-1/C_1^2) \cdot dC_1$

Hence combine by controlling mass flow rate  $m_2$  or by controlling variation in  $C_2$  we can match frequency and frequency of main body can turned to zero.

In this way by controlling this entire variable shown with yellow background as sensing change in variable with red background and controlling few will bring steady state to main body as  $X_1 = 0$

- Whereas to avoid unbalance of forces on main body arise from excitation support  $F_o$  should be damped out by spring force of secondary mass system, at steady state relation as

$F_o = -K_2 \cdot X_2$

On differentiating we get

As  $dF_o = -X_2 \cdot dK_2 - K_2 \cdot dX_2$  ..... equation 6

As well excitation force  $F_o$  due to amplitude  $Y$  of support would be as follows

$F_o = Y \cdot (K_1^2 + C_1^2 \cdot \omega^2)^{0.5}$

On differentiating we get that



As

$$dF_o = \{ [2*Y*dY*(K1^2+C1^2*w^2)] + [(2*C1^2*Y^2*w*dw) + [2*C1*w^2*Y^2*dC1]] \} / 2*F_o$$

On putting value of dw from support in terms of velocity and wavelength we get it as

$$dF_o = \{ [2*Y*dY*(K1^2+C1^2*w^2)] + [(2*C1^2*Y^2*w*(\lambda*(2*\pi*dV)-2*\pi*V*d\lambda)/\lambda^2)] + [2*C1*w^2*Y^2*dC1] \} / 2*F_o$$

.....equation 7

7. As well current technology has excitation force damped out by damping force as follows  $F_o = C1*X1^1$

On differentiating we get that  $dF_o = C1*dX1^1 + X1^1*dC1$  .....equation 8

As far now we can just control acceleration of damper and by doing that we are opposing excitation force  $F_o$

8. In this way equation 7 tells that wavelength, velocity, damping coefficient and amplitude can control and act as input to produce excitation force  $F_o$  and it can be opposed by secondary absorber by varying spring stiffness  $K2$  and displacement  $X2$  as predicted from equation 6. As well by controlling damper speed can also opposed excitation force but secondary vibration absorber will do it in cyclic mode. So we can write it As

$$dF_o = \{ [2*Y*dY*(K1^2+C1^2*w^2)] + [(2*C1^2*Y^2*w*dw) + [2*C1*w^2*Y^2*dC1]] \} / 2*F_o = C1*dX1^1 + X1^1*dC1 = -X2*dK2 - K2*dX2$$

### 3. CONCLUSIONS

1. As in secondary vibration absorber by varying mass flow rate by sensing speed and wavelength, we can maintain change in external excitation frequency  $w$  as equal as natural frequency  $w_n$ . As this condition is satisfied then main body will have no vibration and performance will be high depends upon how fast mass flow rate variation achieved and to get this done many sets of straight hollow pipe should be installed in system with booster.
2. As external excitation force is completely damped out by spring force and that to in cyclic manner by varying stiffness  $K2$ , then main body will have no unbalanced force acting on it and complete balanced steady system can be achieved.
3. This system can be develop as small package along with required sensor for speed and road profile reading along with controller to damped out vibration of system induced by support excitation in Automobile, machine mounting without vibration isolation, precision machine and many more application where vibration are linear in motion.

### 4. ACKNOWLEDGMENT

I want to extend my sincere thanks to Dr. Amar pandhare and Mr. Paresh Kothawade for their constant guidance throughout my work. As well I want to extend my sincere thanks to my parent, Rohit, Rohan, Eknath, Akshay Waghchoure, Varsharani Patil and my friends for cheering they gave me during my work.

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