Theoretical Analysis of the Design of Different Shape Diaphragm for Piezoresistive Pressure Sensor

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Abstract- Pressure sensors are widely used in various domains like automobile, medical fields, pneumatics, military etc. out of various transduction mechanism, piezoresistive type pressure sensor is widely used because it has simple read out circuitry, low cost sensor fabrication. A piezoresistive pressure sensor consists of a sensing element i.e. on diaphragm and suitable circuitry to convert deflection of diaphragm into electrical signal. The design of a diaphragm play important role in the sensitivity of a sensor. Hence in this paper, a theoretical analysis of a design of a piezoresistive pressure sensor with different diaphragm designs is carried out. It is studied that square diaphragm is suitable for design of high sensitive pressure sensor.

1. INTRODUCTION
Pressure sensor is referred as an electronic device, used to find the changes in element state by applying pressure. Pressure sensor converts mechanical stress to the electrical signal due to the applied pressure. The diaphragm undergoes stress hence elements placed on diaphragm produces electrical signal by using electric circuit. The electrical signal may be current, capacitance, resistance or voltage. Pressure sensors are widely used in automobiles, biomedical, aerospace and military application due to the sensing mechanism and accuracy.

Pressure sensors are categorized depending on measurement 1. Absolute pressure sensor, it measures pressure referring to the vacuum. This kind of pressure sensor is uses in vehicle launching, satellite and maintaining cabin pressure. 2. Gauge pressure sensor, measures pressure referring with atmospheric pressure. This kind of pressure sensor is uses in intra-cranial pressure sensor, gas cylinder pressure and blood pressure measurements. 3. Differential pressure sensor, measures the difference between two pressures. One side of pressure is constant. This kind of pressure sensor is uses in high pressure oxidation systems. Pressure sensor also categorized based on technology. 4. Piezoelectric pressure sensor, works on piezoelectric effect. Induced electrical charge when material undergoes a stress or physical movement. Piezoelectric sensing element such as Lead Zirconate Titanate(PZT) or Zinc Oxide(ZnO) are used to induce electrical charge by placing on silicon diaphragm.

5. Piezoresistive pressure sensor, works on the principle of piezoresistive effects. Change the resistivity due to mechanical stress. Piezoresistive pressure sensors (PZR) are widely used compare to other pressure sensor because silicon properties which are used in PZR, high sensitivity, simple fabrication, less expensive good linearity high gain and high gauge factor with 0.27% per °C of temperature coefficient Piezoresistivity (TCP).

The design of a piezoresistive pressure sensor must be consider diaphragm shape, placement of resistor, side length of the diaphragm and material which is uses to diaphragm and to sensing elements. Sensitivity, burst pressure, total displacement and output voltage are the performance parameter of the piezoresistive pressure sensor. Sensitivity is ratio of change in output voltage for given input voltage to the applied pressure. This is shown mathematically in eq(1)

\[
S = \frac{\Delta V}{V_{in}} \times \frac{1}{\Delta P} \tag{1}
\]
The maximum pressure which can be sustained by the diaphragm above which the diaphragm ruptures this is known as burst pressure. Burst pressure can be calculated using eq(2).

\[ P_B = \frac{3.4 \sigma_{\text{fracture}}^2}{1 - \varphi^2} A \]  

\((2)\)

Where \(\varphi\) is Poisson’s ratio, \(\sigma_{\text{fracture}}\) is Fracture stress, \(t\) is Thickness of diaphragm, \(A\) is area of diaphragm. The output voltage across the Wheatstone bridge depends on the input voltage and applied pressure. Output voltage across the bridge is given in eq(3)

\[ V_{\text{out}} = \left( \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) V_{\text{in}} \] 

\((3)\)

Where ‘S’ is the sensitivity of the pressure sensor, \(p\) is the applied pressure, \(V_{\text{in}}\) is the input voltage, \(R_1, R_2, R_3\) and \(R_4\) are piezoresistors and \(V_{\text{out}}\) is the output voltage.

The material, thickness and placement of piezoresistors are the design concept to achieve high sensitivity with low burst pressure. Low cost fabrication and different pressure sensors are obtained depends on the different applications, analysis one of such pressure sensor is piezoresistive pressure sensor. Diaphragm shapes are considered to analyze stress region and deflection region. Preferably using the shape of diaphragm is circular, rectangular or square.

II. LITERATURE SUVEY:

Various articles on Piezoresistive pressure sensor has been studied with respect to their sensing mechanism, sensing pressure, material, pressure range, merits and demerits.

K.N.Bhat et.al explains the gauge factor of different material in that silicon has high gauge factor and next is carbon nano tube (CNT). Discussed static and dynamic analysis of silicon (si) using both square and rectangular flat diaphragm and concluded that in both diaphragm maximum stress is occurs at center of the edges. Using si thin diaphragm discusses the position of piezoresistive for better sensitivity.

Suja K J et.al estimated high sensitivity of pressure sensor with silicon material with high pressure measurement. Square shape diaphragm has got high stress. Silicon and SOI pressure sensor has modeled and at 800μm length of SOI pressure sensors gives better sensitivity of 0.36 V/MPa/V and pressure range from 2-4MPa.

Eswaram P et.al explains the capacitor pressure sensor principle of various types such that absolute, gauge, Electrostatic Tuned, comb drive and so on. Silicon and silicon component material and their features and applications, deflection of diaphragm of square and circular are stated with equation. Different capacitor pressure sensor application and working also discussed.

K Y Madhavi et.al proposed the piezoresistive pressure sensor and got the sensitivity of 41.6 mV/V/Bar for the square diaphragm of 1000μm×1000μm side length and 17.2μm at 100KPa pressure by considering safety and linearity.

Bhanu Pratap Chaudhary et.al simulated circular and square diaphragm in CoventorWare and in MATLAB. Silicon material is used for both diaphragm and piezoresistors. Parameter used are deflection and stress for both square and circular for applied pressure and observed that more stress is occurs in square diaphragm than circular for same pressure but deflection is less in square diaphragm.

Ankitha E Bangera et.al discussed the modeling of a piezoresistive pressure sensor using a single walled CNTs in three different orientation on a square diaphragm in COMSOL Multiphysics. And sensitivity of these three position is analyzed and observed better sensitivity that placing two are opposite edge and two are centre of the diaphragm that is 0.23mV/kPa.

Nural Amiziaiah Md Yunus et.al discuss about the pressure sensing mechanism, stress is measured when changes the resistivity of the piezoresistors. The pressure ranges from 0 to 25 kPa. The material is used are silicon, silicon dioxide and aluminum. The parameter observed that deflection and voltage with respective to the pressure. The advantages of this paper gives the better performance and low cost. The NEMS technologies are not suitable for large pressure applications.

Vladimir Kutis et.al proposed a ring and circular diaphragm of piezoelectric pressure sensor. By using micro-Raman spectroscopy effect measured the residual stress for the pressure 10kPa. Material used for the diaphragm is ALGAN/GAN for both models. Measured maximum deflection and induced stress as a function of pressure. Micro-Raman effect gives the optimal location and size of electrode for prescribed loading and boundary condition but the fabrication of ALGAN/GAN C-HEMT structure piezoelectric pressure sensor.

P.K.Kinnell et.al proposed a model of resonant pressure sensor. Measured the resonant sensor offset drift and span drift in ppm f.s as a function of time for the pressure range from 1bar to700bar. Long term stability can be achieved by placing resonant sensor in fully isolated hermetic package but fabrication is complex.

M.Rajavelu et.al proposed a square silicon diaphragm of PZR pressure sensor. They measured the deflection and output voltage as a function of pressure and area of performance as a function of sensitivity. Sensitivity can be achieved using thick perforated and thin non-perforated diaphragm but the fabrication is complex compared to simple diaphragm.
Nabiollah Abol Fathi et al. discussed the different types of pressure sensor and has conducted experimental study of the diaphragm with rectangular holes. Pressure sensing mechanism where the pressure applied is converted into electrical fluctuation and measured the deflection, stress and sensitivity by applying pressure of the range 10MPa using ABAQUS tool. It was observed that rectangular hole on circular diaphragm gives high sensitivity but gives less sensitivity without holes.

M. A. Fraga et al. have conducted experimental study of fabrication and characteristics of SiO/SiO2/SiC material diaphragm and pressure sensing mechanism is strain is sensed by changing the resistance of the element on diaphragm. Observer output voltage by given pressure range from 0 to 12MPa. It was observed that good linearity and sensitivity can be achieved by consider 6 SiC layer on the SiO2/Si diaphragm but the fabrication is complex.

U. Sampath Kumar et al. discuss about piezoresistive pressure sensing with two Wheatstone bridge and pressure sensing mechanism where the stress is induced by the applied pressure is converted into change of electric resistance. Measured the deflection by applying pressure range is 0-20kPa and it was observed that sensitivity is increased by using double Wheatstone bridge but the designing cost is more.

S. Maflin Shaby et al. discussed about piezoresistive pressure sensor using changing resistance due to stress mechanism for pressure sensor. Analyses stress and deflection by applying 2MPa pressure. It was observed that the deflection is more in circular and stress is more in square diaphragm compare to rectangular diaphragm but rectangular diaphragm gives less performance compare to the other two diaphragms.

III. DIAPHRAGM DESIGN

To achieve linear operation the deflection must be less than thickness of the diaphragm. From the plate theory the deflection is less than the five times of the diaphragm thickness. Consider the different diaphragm such that they have the same area and determining the stress deflection by applying uniform pressure on the surface of diaphragm.

1. Square Diaphragm:

From Lagrange equation the deflection w(x,y) is given for square diaphragm for uniform pressure(P) on surface of the diaphragm of side length $\sqrt{\pi a}$ shown in Fig 1 is given in eq(6)

$$w(x,y) = \frac{1}{4} \frac{P a^4}{E t} \left(1-x^2\right)^2 \left(1-y^2\right)^2$$

(6)

Where deflection $w(x,y)$ at the center is more when applied pressure uniformly on surface of the square diaphragm is given as eq(7)

$$W(\text{max}) = \frac{P a^4}{E t g_1}$$

(7)

Where P is pressure applied on the diaphragm, ‘a’ is the length, t is thickness of diaphragm, ‘E’ is the young’s modulus of the material and $g_1$ is the constant which is depends on Poisson’s ratio $\nu$ [1].

![Square diaphragm](image)

Fig.1: Square diaphragm of side length $\sqrt{\pi a}$.[2].

Stress is also analyses the sensitivity of a pressure sensor. The stress of a square diaphragm is given in eq(8a&b). Two types of stress are acting on the square diaphragm namely longitudinal and transverse stress which are given as in eq(5a) and eq(5b)

$$\sigma_r = \frac{P a^2}{t}$$

(8a)

$$\sigma_\theta = \frac{r P a^2}{t}$$

(8b)

The maximum stress is at the middle of the edges of the diaphragm is given as shown in eq(9)[1]

$$\sigma_{\text{max}} = 1.2 \frac{P a^2}{t}$$

(9)

2. Circular Diaphragm:

Fig.2 shows the isotropic circular membrane of radius ‘r’. The maximum deflection w(x,y) when pressure is applied uniformly on the diaphragm is given by eq(10)[2]

$$W(\text{max}) = \frac{3 P r^4 (1-\nu^2)}{16 E t^2}$$

(10)

Where r is the radius of diaphragm, t is the thickness of the diaphragm, P is pressure and $\nu$ is the Poisson’s ratio.

![Circular diaphragm](image)

Fig.2. Circular diaphragm.

Radial and tangential are two acting on the circular diaphragm when pressure is applied uniform. Radial force active along the radius of circular diaphragm and tangential force is acting edges of diaphragm. Which are given as $\sigma_r$ and $\sigma_\theta$ respectively,

$$\sigma_r \text{max} = \frac{3 d}{4 \pi t^2}$$

(11a)

$$\sigma_\theta \text{max} = \frac{3 d}{4 \pi t^2}$$

(11b)

Maximum stress is at the edges of the diaphragm is given by eq(12).

$$\sigma \text{max} = \frac{3 d}{4 \pi t^2}$$

(12)
Where \( r \) is the radius, \( t \) is the thickness of the circular diaphragm and \( d \) is the equivalent load [12].

### 3. Rectangular Diaphragm:

Fig.3. shows the rectangular membrane of width \( 0.5a \) and length \( 2a \) gives a deflection \( w(x,y) \) when apply uniform pressure on to the diaphragm gives in eq.(13)

\[
w(x,y) \cong \frac{P(1-\nu)}{2EI} \left( \frac{\pi^4 a^4}{16 b^4} + \frac{\pi^4 a^4}{256 b^4} \right) \left( a^2 - y^2 \right)
\]

Longitudinal and transverse stress are acting on the rectangular diaphragm which gives in eq (14a&b)

\[
\sigma_l = \frac{2Pb^2}{t^2a^4} \frac{\pi^4 a^4}{256 b^4}
\]

\[
\sigma_t = \frac{\pi^4 a^4}{256 a^4}
\]

Where \( \sigma_l \) is longitudinal stress, \( \sigma_t \) is transverse stress, \( P \) is applied pressure, \( t \) is thickness of diaphragm, ‘a’ is side length of diaphragm, \( \nu \) is poisson’s ratio.

### IV. COMPARATIVE ANALYSES:

To achieve good sensitivity and linearity, the deflection, stress and placement of sensing element plays a major role in the diaphragm design.

In circular diaphragm, the deflection is more but the stress occur at the edges is less compared to other diaphragm.

The deflection is more at the centre of circular membrane and the stress is more at the edges of the circular membrane. Due to low stress this type of pressure sensor are used in harsh environments. The deflection of the diaphragm is such that one quarter of the diaphragm thickness according to plate theory.

In rectangular membrane when applied uniform pressure, the deflection is more at the centre and stress is at the middle of edges. Non uniform stress is occurred at the edges in the rectangular membrane and placing of sensing element is difficulty in these cases. The magnitude of maximum stress is higher in rectangular diaphragm.

In square diaphragm, the maximum stress is distributed uniformly at the edges of the diaphragm and the deflection is more in centre when applied uniform pressure on to the diaphragm. According to plate theory the deflection of the diaphragm is less than or equal to the one fifth of the thickness of the diaphragm. The stress is high in the square diaphragm compared to other diaphragm. Also square diaphragm with whetstone bridge connection of sensing element gives high sensitivity and linear output.

### V. CONCLUSION

In this paper, three different shapes [square, circular and rectangular] of diaphragm is studied theoretically. Based on the study, it is observed the deflection is maximum at the centre in all three designs and equation for calculation is given above. It is also learnt that the circular diaphragm gives maximum deflection for an applied pressure compared to other diaphragms, but the stress induced is less. The stress is high in square diaphragm and deflection is less compared to circular diaphragm. As the length of the rectangular diaphragm increases, the stress increases along the length edges. The magnitude of stress is higher in rectangular diaphragm. In square diaphragm, sensitivity is more and non-linearity is less because of the uniform stress distribution at all the four edges of a diaphragm.

### REFERENCES


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