The Orbital Angular Momentum of Light (OAM)

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Abstract—**Orbital angular momentum (OAM) of light reveals a new area of research. The quantum nature of OAM in addition to spin angular momentum (SAM) leads a new modulation technique in communication engineering. An ultra high speed data rate and a very high capacity channel can be achieved by using OAM modulation scheme .We are not only increasing the data rate but also improving the BER performance by shifting 2D space to D-dimensional space as Euclidian distance increases between signal constellation points.**

Keywords- OAM, SAM, BER, Quantum nature, datarate, capacity, Constellation point.

I. INTRODUCTION

In last few decades we have introduced several modulation techniques in analog and digital electronics. Now science and technology are advancing day by day and the whole era of technology is now changing due to the revolution of science. Particularly in communication engineering we have seen a drastic change in recent years, but what's next? The next generation modulation technique is modulation of light. We can make a significant revolution in sending the data by modulating the property of light. In this paper we discuss a modulation technique based on orbital angular momentum of light which will create a new capacity of data rates in communication engineering.

The orbital angular momentum of light (OAM) is the component of angular momentum of a light beam that is dependent on the field spatial distribution, and not on the polarization. It can be further split into an internal and an external OAM. The internal OAM is an origin-independent angular momentum of a light beam that can be associated with a helical or twisted wavefront. The external OAM is the origin-dependent angular momentum that can be obtained as cross product of the light beam position (center of the beam) and its total linear momentum.

II. ANGULAR MOMENTUM OF LIGHT

Angular momentum of light is the vector quantity that expresses the amount of dynamical rotation present in the electromagnetic field of the light. In 1619 Johannes Kepler explained that tail of a comet always points away from the Sun because light carries linear momentum. The assertion that light, as electromagnetic radiation, has the property of momentum and thus exerts a pressure upon any surface exposed to it was published by James Clerk Maxwell in 1862, and proven experimentally by Russian physicist Pyotr Lebedev in 1900. According to John Poynting theory of electromagnetic radiation pressure and energy density the polarized light has angular momentum and spin angular momentum which is associated with circular polarization and its value is $\pm h/2\pi$. By means of the Helmholtz theorem on the decomposition of vector fields, the angular momentum of the classical electromagnetic field is decomposed, in a general and manifestly gauge invariant manner, into a spin component and an orbital component.

The method is applied to linearly and circularly polarized plane waves in their classical and quantum forms. All believe that spin angular momentum and orbital angular momentum of light both are associated with circular polarization of light, but it is also true that orbital angular momentum of light is a good deal but less understood. The clarification about orbital angular momentum of light came after a long time. In 1992 Les Allen observed that light beams with an azimuthal phase dependence of exp $(-il\varphi)$ carries an angular momentum independent of polarization state [3]. The angle φ is the azimuthal coordinate in the beam's cross-section and l may take any integer value i.e both positive and negative. This term gives a helical profile to the phase structure of the beam: for $l \neq 0$ a phase singularity, said optical vortex (OV) as shown in Figure-1, appears on the beam axis and the intensity has to vanish there. Hence, the intensity pattern of an optical vortex shows a dark central spot. The integer l is the topological charge of the vortex and it is the winding number of the wave front helix, *i.e*. the number of 2π phase shifts that occur in one revolution of the azimuthal angle φ. According to Poynting theorem the Poyinting vector can be determined by the vector product of electric and magnetic field intensities which represents the directional energy flux density i.e the direction and magnitude of momentum flow.

For a helical phase font the pointing vector must have an azimuthal component which produces an orbital angular momentum parallel to the beam axis which can be observed from the Figure-2. It also tells us that the momentum rotates about the beam axis which contains an optical vortex.

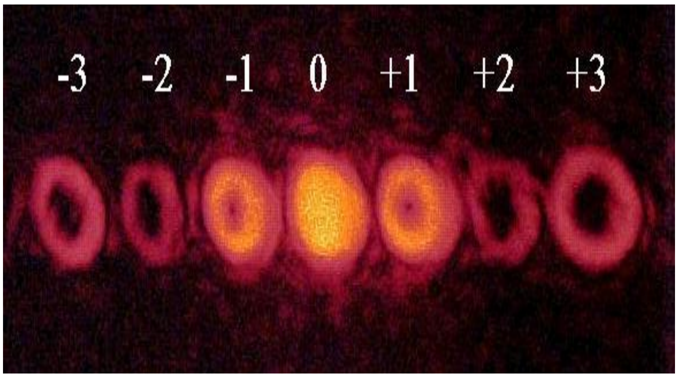

Figure- 1: Observation of Optical Vortex (OV) for different values of l=0, ±1, ±2 and ±3.

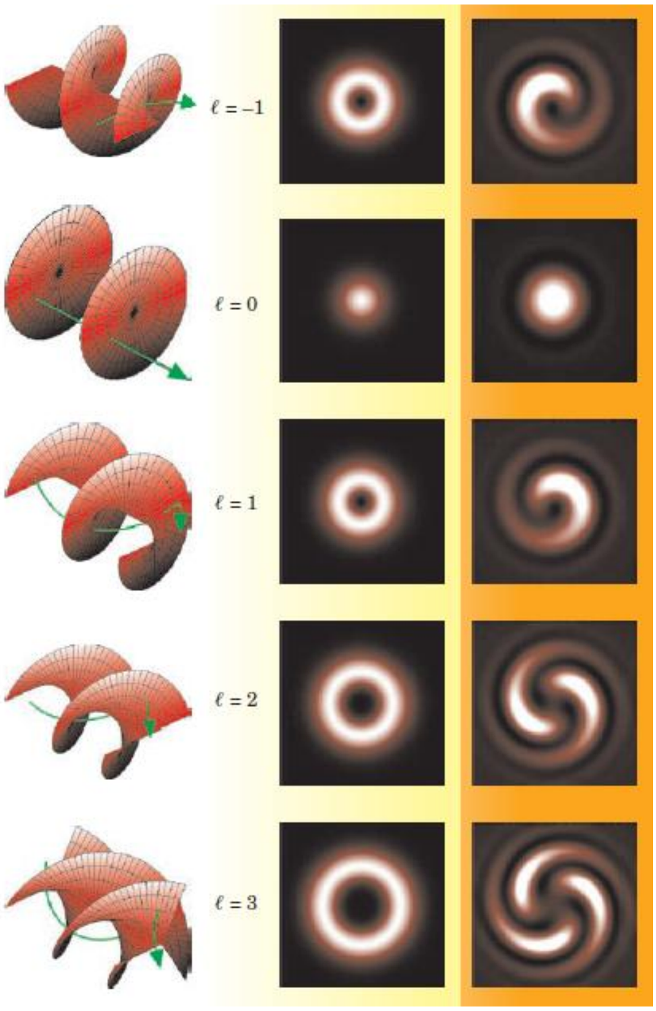

Figure- 2: OAM of light beam arises from helical phase font as shown in left column. Poyinting vector is given by the green arrow which follows the helical trajectory around the axis for all values of OAM except $l=0$. The middle column shows the intensity profile of the helical wavefront . The right column shows the spiral intensity pattern when the beam is made to interfere with a plane wave. [1]

III. Generation of OAM Beams

There are several methods of generation of OAM beams by using different techniques and systems. Let us discuss some efficient methods used in the process of generation of light that carry an orbital angular momentum.

A. Using a Pair of cylindrical lenses

As presented in Figure 3 by using a pair of cylindrical lenses we can generate light that carry the OAM. A pair of cylindrical lenses transforms the Hermite Gaussian (HG) into Laguerre Gaussian (LG) mode that carries orbital angular momentum. Therefore basically the pair of cylindrical lenses works as a converter. The separation distance between two lenses determines the handedness of the LG modes like a half wave plate reverses circular polarization. Generally reversing LG modes can be achieved easily by the help of an Dove prism which acts as an image inverter [1].

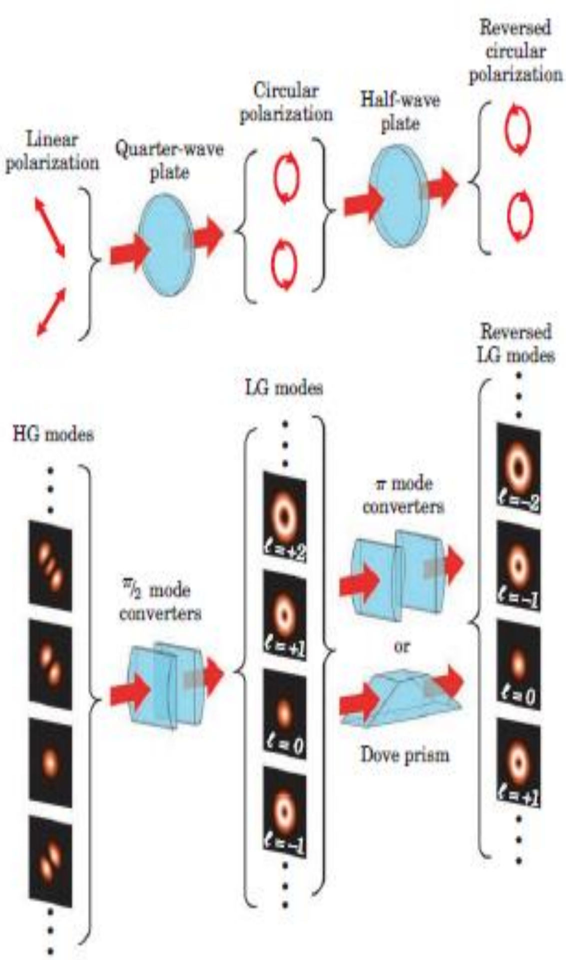

Figure- 3: Generation of OAM beams by using a pair of cylindrical lenses.[1].

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B. *Using computer generated holograms*

A computer generated hologram can also be used to generate orbital angular momentum of light as shown in figure 4. The generated OAM depends on the holographic pattern which is done by means of grating in the hologram. We can add or subtract OAM of two light beams using a hologram and can generate another beam according to our requirement. The grating pitch variation can be produced by local magnification variation in the hologram fabrication process. The local magnification variation can be due to a deflection error of the electron beam lithography system used for the hologram fabrication. Other fabrication systems based on raster scan may have a similar deflection error. Similar deformation can appear for holograms fabricated by such a system.

A possible source of the beam deformation is that the grating pitch of the hologram varies with the horizontal location. For example it may be broader near the right and left edges of the hologram compared to the center. For this reason, light diffracted from regions near the right and left edges shifts towards the direction of lower-order diffraction.

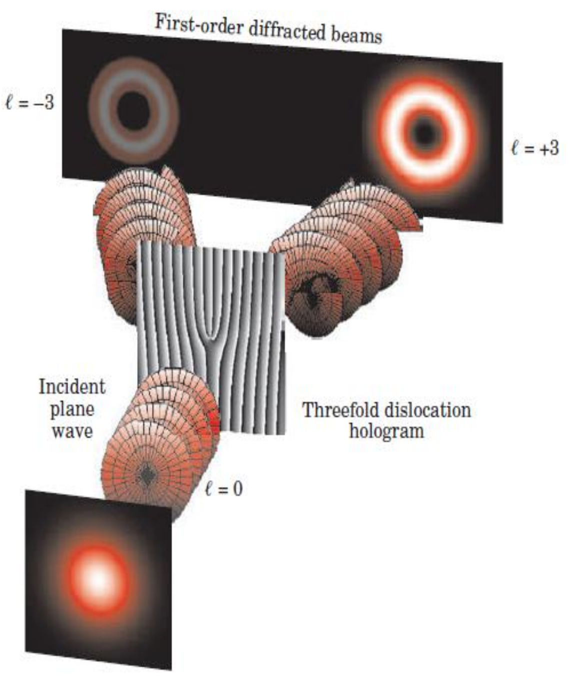

Figure- 4 : It shows how a hologram can be used to generate a helical phase font carrying an OAM $l\ \hbar$ per photon. [1]

IV. ORBITAL ANGULAR MOMENTUM MODULATION FOR FIBER OPTIC COMMUNICATION

We have discussed about orbital angular momentum of light briefly now let us focus on the modulation of orbital angular momentum of light. There is a approach of OAM modulation of light beams in the fiber optic communication for a high speed data rate and a high capacity [9].

Let us discus the modulation of OAM taking the Laguerre-Gaussian (LG) vortex beam as this can easily be implemented. As we are discussing about fiber optic communication so let us take the cylindrical co ordinate system that is convenient for mathematical expression. The field distribution of LG beams travelling in the Z- direction can be given as

$$E_{m,p}(r,\phi,z) = \sqrt{\frac{2p!}{\pi(p+|m|)}}\,\frac{1}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|m|} L_p^m\left[\frac{2r^2}{w^2(z)}\right]\times$$
$$exp\left[\left(\frac{r^2}{w^2(z)}\right)-\frac{jkr^2 z}{2(z^2+z_R{}^2)}+j\left(2p+|m|+1\right)\tan^{-1}\left(\frac{z}{z_R}\right)-jm\phi\right] \quad (1)$$

Where, $w(z)$ is the radius at which the Gaussian term falls to $(1/e)$ of its on-axis value and $w(z)=w_0\sqrt{[1+z/z_R]}$ (w_0 is the zero order Gaussian radius at the waist or we can say it is the minimum value of the beam radius that occurs at z= 0 . L_p^m is an associated Laguerre polynomial, m and p are called azimuthal and radial mode numbers respectively, $z_R=[\pi w_0{}^2/\lambda]$ Rayleigh range which is called the "confocal parameter", λ is the wavelength, $k=2\pi/\lambda$. The phase term $exp\ (-j\phi m)$ in (1) creates helical phase font. For m=0, $E(r,\phi,z)$ becomes a zero-order Gaussian beam, that is the TEM00 mode. It can be shown that for a fixed p, the following principle of orthogonality is satisfied [9]

$$\int E^*_{m,p}(r,\phi,z).E_{m,p}(r,\phi,z)rdrd\phi = \begin{cases} \int |E_{m,p}|^2 rdrd\phi, n=m \\ 0, n\neq m \end{cases} \quad (2)$$

Thus different OAM states corresponding to a constant radial mode (p) are mutually orthogonal to each other. Therefore those can be used as a principal functions for OAM modulation. To create different modes in the fiber we can use the methods which are already discussed in the previous section.

Now let us discuss about the OAM modulation scheme which can provide ultra high speed data communication in the optical fiber. The scheme provides in-phase/quadrature channels, taking two SAM states and N OAM states resulting in D=4N-dimensional signal-space [9]. Data rate can be increased by increasing number of OAM states keeping uninterrupted transmission at this ultra high speed using capacity approaching linear dimensional parity check (LDPC) codes. We are not only increasing the data rate but also improving the BER performance by shifting 2D space to D-dimensional space as Euclidian distance increases between signal constellation points. The overall system configuration is described below.

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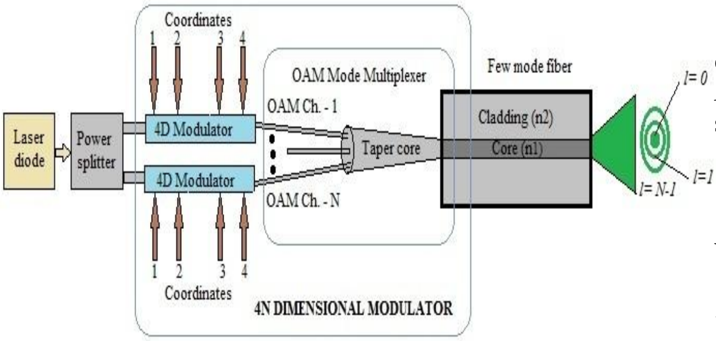

Figure- 5: A 4N- dimensional transmitter for ultra high speed LDPC coded modulation system

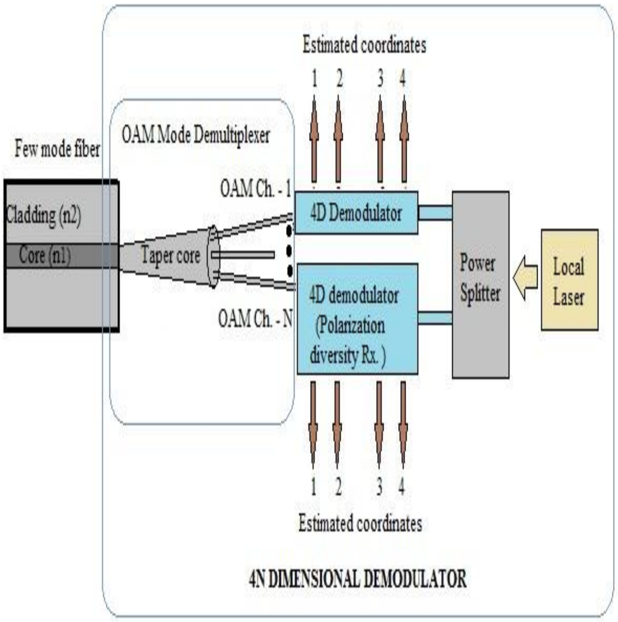

Figure- 6: A 4N- dimensional receiver for ultra high speed LDPC coded modulation system.

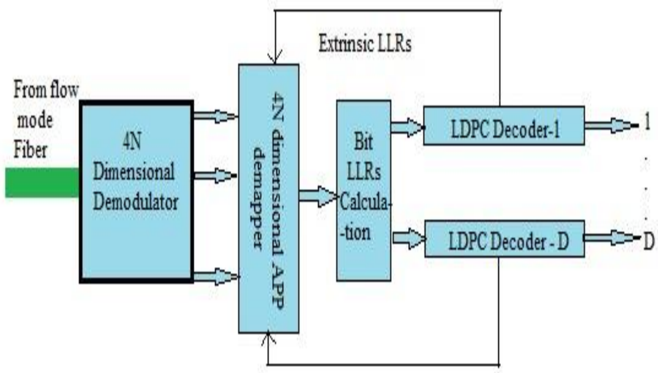

Figure- 7: Block diagram of 4N- dimensional APP demappers and LDPC decoders.

Figure- 5, 6 and 7 describe overall system configuration for LDPC coded OAM modulation scheme. The *D*- dimensional transmitter as shown in figure- 5 generates the signal constellation points as [9]

$$S_i = C_D \sum_{d=1}^{D} \Psi_{i,d} \Phi_d \qquad (3)$$

Where, $\Psi_{i,d}$ denotes the *d*th coordinate (*d= 1,2,3 .....,D*) of the *i*th signal-constellation point, and the set $\{\Phi_1, \Phi_2 ...., \Phi_d\}$ denotes a set of d-orthogonal basis functions. $\Psi_{i,d}$ denotes signal constellation point coordinates. A laser diode signal is split into *N* branches by using a power splitter to feed 4D electro-optical modulators, each corresponding to one out of *N* OAM modes. The 4D electro-optical modulator is composed of polarization-beam splitter (PBS), two I/Q modulators and polarization-beam combiner (PBC). The OAM mode multiplexer is composed of *N* waveguides, taper-core fiber and FMF, properly designed to excite orthogonal OAM modes in FMF. The 4*N*-dimensional demodulator architecture is shown in Figure-5. We first perform OAM mode demultiplexing in the OAM-demux block (see Figure-6), whose outputs are 4D projections along *N* OAM states. Each OAM mode undergoes polarization-diversity coherent detection and corresponding outputs are forwarded to 4*N*-dim. *a posteriori* probability (APP) demapper as shown in Figure-7. In APP demaper we calculate symbol LLRs, which are used to calculate bit LLRs needed for LDPC decoding, as shown in Figure-6. After LDPC decoding, extrinsic bit LLRs are used to calculate the prior symbol LLRs for APP demapper. we show the transmitter architecture.

V. Conclusion

We have gone through the concept of orbital angular momentum of light along with generation of beams that carry OAM. Then we described the LDPC coded ultra high speed modulation scheme which is a revolution in the communication engineering in today's world. OAM modulation not only enhances the data rates but also significantly increases channel capacity. It can be used in quantum cryptography to secure the data and much more.

Further research into OAM multiplexing in the radio and mm wavelength frequencies has been shown in preliminary tests to be able to transmit 32 gigabits of data per second over the air. There is an ongoing discussion if this will add any capacity on top of other schemes, such as MIMO.

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