

The Optimal Configuration of Turbines Location In A Wind Farm by the Method of Problems Resolution With Satisfaction Constraints

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Abstract—The placement of turbines in a wind park design is an important factor that affects yield, but the automatic placement of turbines is always still a difficult problem. The objective of every wind farm designer is producing as maximum as possible of energy, with minimal cost of installation. The optimization is done by the minimum cost per unit of energy produced.

In this study an algorithm designed on the method of solving constraint satisfaction problems (CSP) based on the wake model of Jensen is developed, it has the capacity to estimate the optimal number of total power produced in wind farm, in comparison with previous studies published in technique literature take much less time and produces better solution.

Keywords—wind farm; cost model; wake effect; the satisfaction constraint of problems (CSP); optimization.

I. INTRODUCTION

Today, the current of the energy situation has become clear that it's very worrying, either economically or environmentally. Economically view the level of energy fossil resources decreased because of human greedy of energy in one hand, and the increase of population on the other hand. Ecologically is the climate change or global warming caused by emission of greenhouse gas like CO₂ because of the combustion of these fossil fuels.

In order to reduce these various risks, it is obviously inevitable that the renewable energy will be used in the production of electrical energy. Among the various sources of clean energy that has some interest, there is wind energy that imposes itself and has resounding success. This is due to the technological progress made by wind industry, and this reduced a lot the cost of kWh that is supplied by wind and made it competitive with other energy sources.

Generally, the use of wind energy as part of the electricity production is in the form of wind farm, where several wind turbines are grouped together on a single site, as shown in Figure 1. This type of configuration allows necessarily reducing cost investment since saving can be made on several points: the electrical infrastructure, especially power lines, which at the same time reduce the loss of energy, expenses related to obtaining concessions on land to establish the wind farm, tasks related to the maintenance and operation of the wind farm will certainly be facilitated and finally ease of connection to the electricity distribution network.

However, in this configuration, wind turbines will be close enough to each other. This therefore would cause losses due to the increase in the wake effect [1]. Indeed, behind each wind turbine takes place a wake where the wind speed decreases and at the same time increases the turbulence, which will inevitably affect the performance of all the wind turbines that are downstream.

Mosetti et al. [2] is the first experience to determine the best distribution of wind turbines can provide optimal production. For this, the authors use an objective function defined as the ratio between the total cost of the wind farm and the total power of the wind farm [2, 3, 4].

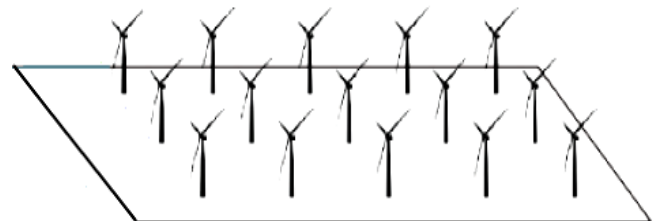


Figure.1. Wind farm

The present study is based on the same patterns used by Mosetti et al [2] and Grady et al [4], but in this study we are looking for more optimal and effective results using a different methodology and similar approach, so that our results are comparable. That is why the last chapter is devoted to a comparison between the results of those three studies.

The algorithm searches the optimization of the park design, through the minimization of the objective function by a program code that has been developed in Matlab.

II. WAKE EFFECTS AND THE COST MODEL

The turbine interact with the wind, capture a portion of its kinetic energy and converts it into useable energy, this extraction of energy creates a gap between the outgoing wind turbine and the oncoming wind turbine. Thus, the wind speeds at the rear of the turbine is lower than the downstream speed of the wind, as a result it decreases the production of

output energy. The wake effect also causes high levels of turbulence in the outgoing wind turbines, giving rise to an additional mechanical stress, which may affect them; this behavior caused by the turbulent is neglected in this study because it does not affect directly output power.

In both works of Mosetti and Grady's [2, 4] the model used is similar to the model developed by Jensen [5] in 1986. Here we assume that the movement is kept inside the wake.

For a single turbine, the downstream wake zone will be considered as a trapezoid such that the average speed of the wind can be expressed by the following equation (1):

$$u = u_0 \left[1 - \frac{2a}{\left(1 + a\left(\frac{x}{r}\right)^2\right)^2} \right] \quad (1)$$

Where α is the entrainment Constant, a is the axial induction factor, x is the distance from the turbine, and r is the radius of the turbine downstream, as shown in Fig. 2.

The relationships between r , r_r the radius of turbine and C_T the thrust coefficient are represented in the equations (2) and (3):

$$r = r_r \sqrt{\frac{1-a}{1-2a}} \quad (2)$$

$$C_T = 4a(1 - a) \quad (3)$$

The entrainment Constant is empirically given by (4):

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_0}\right)} \quad (4)$$

Where z is the hub height of the wind turbine, and z_0 is the surface roughness of the site which depends on:

- h the height of each obstacle.
- S the cross section facing the wind.
- Ah the average of the projection surface on the floor.

As z_0 can be expressed by the following expression (5):

$$z_0 = 0.5 \left[\frac{s}{A_H} \right] h \quad (5)$$

Evaluations of z_0 in deferent grounds lead to the values listed in Table 1

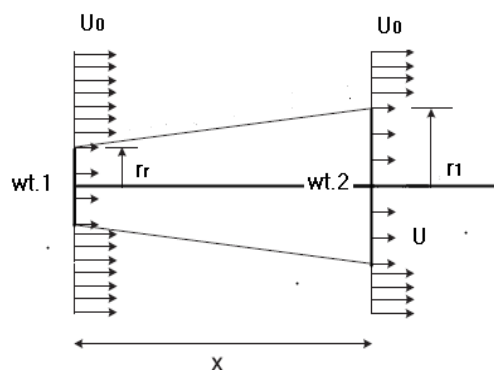


Figure.2 The wake effect

Table 1: Z_0 values in deferent grounds

z_0	Grounds	Class Roughness
10^{-4}	Water bodies	
3×10^{-4}	Surface smooth sand	0
10^{-3}	Smooth surface snow	
5×10^{-4}	Smooth and bare soil	
0.01	Airport Runways and taxiways	
0.03	Agricultural land with very few buildings, trees, ...	1
0.05	Agricultural land open look	
0.10	Gated farmland look	2
0.20	Lot of trees and bushes	
0.30	Shafting windbreaks	
0.50	Suburbs	3
1.00	Foret, city	4

For multiple wakes we supposed that the loss of kinetic energy is equal to the sum of the energy losses. So, for N turbines, the downstream speed can be expressed by the following expression (6):

$$u_i = u_0 \left[1 - \sqrt{\sum_{i=1}^N \left(1 - \frac{u}{u_0}\right)^2} \right] \quad (6)$$

The electricity generated by an aero generator, is a function of the local wind speed. Furthermore, the hub height, the thrust coefficient and the rotor diameter also affects the extracted power.

The total power P extracted from the wind is a function of the local section and wind speed, as shown in the following expression (7):

$$P = \sum_{i=0}^N 0.3u_i^3 \quad (7)$$

To calculate the total cost, we modeled the investment cost such a way that only the number of wind turbines must be taken into consideration.

The total cost per year for the entire wind farm can be expressed as follows (8):

$$\text{cost} = N \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174N^2} \right) \quad (8)$$

Where N is the total number of wind turbines.

The objective function that will lead to optimization (minimum cost per unit of energy produced) is expressed as follows (9):

$$\text{objective function} = \frac{\text{cost}}{P_{\text{total}}} \quad (9)$$

Where P_{total} is the total production, while the cost is calculated as mentioned in equation (8).

Minimize the objective function leads to a solution with the lowest cost of producing wind energy.

III. FORMALIZATION PROBLEM OF TURBINES LOCATION

A. Introduction to the method of constraint satisfaction problems

The family of constraint satisfaction problems (CSP) covers a wide range of problems. These problems share a common structure description, based on a very simple formalism, which allows a clear and intuitive modeling [1].

A constraint satisfaction problem (CSP) is generally presented as a set of variables, which are associated with fields and a set of constraints. Each constraint is defined on a subset of the set of variables and limit values of the combinations of these variables can take these variables. Solving a CSP consists in finding an assignment of each variable values so that all constraints are satisfied. For some problems, the objective is to find all these assignments.

A constraint satisfaction problem (CSP) is defined by a triple (X, D, C) where:

- $X = \{x_1, \dots, x_n\}$ is a finite set of n variables of the problem,
- $D = \{D_{x_1}, \dots, D_{x_n}\}$ is the set of n finite domains for variables. D_{x_i} is the set of possible values for the variable x_i ,
- $C = \{c_1, \dots, c_m\}$ is the set of m constraints.

Constraints can be expressed in different forms: table of compatible values, mathematical formulas, etc. It is a relationship between variables that define the structure of the problem.

Whether a CSP (X, D, C) , a constraint $c \in C$ on variables x_{i_1}, \dots, x_{i_k} for $i_1, \dots, i_k \in \{1, \dots, n\}$ is a relationship in $D_{x_{i_1}} \times \dots \times D_{x_{i_k}}$, variable fields of x_{i_1}, \dots, x_{i_k} . Therefore (10):

$$c = \prod_{j=1}^{i_k} D_{x_j} \quad (10)$$

Given a CSP, its resolution consists in assign values to variables such that all constraints are satisfied. For this we introduce the following definitions and notations:

a) *Definition 1 [Assignment]* for a CSP, assignment is instantiate some variables by values (taken in their respective areas).

An assignment is a function (11):

$$s : X \rightarrow \prod_{i=1}^n D_{x_i} \quad (11)$$

Such that $s(x_i) \in D_{x_i}$ for $i \in [1, \dots, n]$.

To simplify, we note $s = (d_1, d_2, \dots, d_n)$ a variable assignment with a value d_1 for the variable x_1 , d_2 for the variable x_2, \dots, d_n for the variable x_n .

b) *Definition 2 [Research space]* The search space of a CSP is the set of possible assignments which will be denoted S .

$$S = D_{x_1} \times D_{x_2} \times \dots \times D_{x_n} .$$

The search space is equal to the Cartesian product of all variable domains and an assignment s will be treated as an element of this set S .

Assignment $s \in S$ satisfies a constraint $c_k \in C$ if all the variables $\text{var}(c_k)$ are instantiated by s and if the

relationship defined by c_k holds for values of variables $\text{var}(c_k)$ given by s . To simplify, we note $s \in c_k$.

c) *Definition 3 [Solution]* A solution of a CSP is an assignment $s \in S$ that satisfies all constraints. There $\text{Sol}(P)$ the set of solutions $\text{Sol}(P) = \{s \in S \mid \forall c \in C, s \in c\}$. The set of solutions is any element of the search space (assignments) as belonging to the allowed values for each constraint.

The definition of a CSP can represent a large number of problems, but there is no universal method for efficient resolution. Various techniques have been developed, including using the notions of consistency, that we will develop later, and combinations of these methods to form the algorithm of resolution.

Given a CSP, the aim of a solver is to provide solutions satisfying the constraints to the extent possible. So the properties of solvers can be stated as the following:

- A solver is complete if it is still able to answer with true or false to the existence of a solution.
- A solver is correct if only calculates solutions.
- A solver is reliable if it calculates all the solutions to a given problem.

In an ideal case, the execution of a solver must end in a finite time; provide all the solutions, satisfying of course all the constraints.

To avoid testing all variable - value combinations inherent in the formulation of a CSP, in most time we will use filtering consistency that is to remove values of the domain which can not satisfy certain constraints and thereby to reduce the search space. As our aim is to find an assignment for each variable satisfying all the constraints, the idea is to reduce the search space (the space of possibilities). To do this, the solver will try to remove some values in the fields, so-called inconsistent values. A value is deemed inconsistent in so far as it is not joined to one or more constraints.

A notion of consistency is associated to the notion of constraint. Indeed, constraint forces the variables to take only certain values, consistency occurs where the values of a field may in no case satisfy this constraint. The consistency property for a constraint is reached when no value can be deleted.

We will consider here the local consistency properties, i.e. a consistency considering each constraint independently.

The filter of inconsistent values is useful to instantiate the variables and allows to tap out certain values of the domains while maintaining the solutions. The objective is to reduce the field to make the CSP relatively consistent with the properties of each constraint.

B. Optimization problems with constraints

An optimization problem with constraints is to find an optimal solution among a group of sets of feasible solutions (that do not violate constraints). The problem is twofold, on one hand we have a research for feasible solutions and on the other hand a research for an optimal solution. A function, called objective function is defined as

the problem of assessing the quality of an assignment. This function associates a value to each instantiation, the objective is to find the assignment that minimizes or maximizes this function [6].

Given a CSP: $P = (X, D, C)$, S denote the search space defined by the areas of D . Let $(K, <)$ with K a totally ordered by the strict relation $<$, and f a cost function, also called objective function from S to K :

$$f: S \rightarrow K$$

$$s \rightarrow k$$

We define an optimization problem with constraints as minimizing (respectively maximization) of f ; the objective is to find a feasible solution $(s \in \text{Sol}(P))$ with the smallest value of k for the function f [7]. With more formal manner:

$$s \in \text{Sol}(P) / \forall s' \in \text{Sol}(P), f(s') \geq f(s)$$

C. Algorithms for positioning of the turbine

In the problem of positioning the turbine we use a specific type of wind turbine with the characteristics shown in Table 2. The available land can be subdivided into cells that will be the possible location of wind turbines.

To keep the required distance between two adjacent turbines, the size of a cell is suitably chosen and all turbines are solely installed on the center of a cell so that the wake of a column of turbines would not be affect the turbines in the other columns.

Table 2: Caractéristiques of turbines

Description	Parameter	value
Hub height	z	60 m
Radius of the rotor	r_r	40 m
Thrust coefficient	C_t	0.88

IV. NUMERICAL PROCEDURE AND RESULTS

A. Generate-and-test and backtracking

A simple method for the resolution of CSP is to generate all possible configurations, in other word all possible combinations of values of variables and test if they

satisfy the constraints. This approach is known as the Generate-and-Test.

The number of possibilities tested is the cardinal of the Cartesian product of the domains of the variables, which for problems of large sizes is impossible to consider.

Backtracking is undoubtedly the most widespread method for systematic research. For this method, variables are assigned one after the one until a complete assignment is achieved. Only, unlike a generate-and-test, this method tests the feasibility of each resolution step [8].

So, when in a given stage, the current partial assignment violates a constraint, backtracking removes the sub-search space below the point of choice. In an ideal case, the execution of a solver must end in a finite time; provide all the solutions, satisfying all constraints. To avoid testing all combinations variable value inherent in the formulation of a CSP, we use filtering consistency. This filter is to remove the domain values can't satisfy certain constraints and thereby reduce the search space.

B. Results and analysis

In order to estimate the optimal number of wind turbines and for comparative purposes we will take the following basic conditions: uniform wind direction and speed steady wind of 12 m / s. This case has been discussed in detail in [1,2,3] where different approaches were used. Choosing this case study was performed for comparison.

In this case, turbines affect each other. This makes it difficult to place the turbines through the experience. Figure 3 shows the proposed solutions by Mosetti et al., Grady et al. and by this study.

In this scenario of simple wind, turbines of a column can not influence those of other columns. Grady noted that optimizing for such a column can be extended to the whole field. The results obtained using the CSP model and the results produced by other methods for the same case study are presented numerically in Table 3.

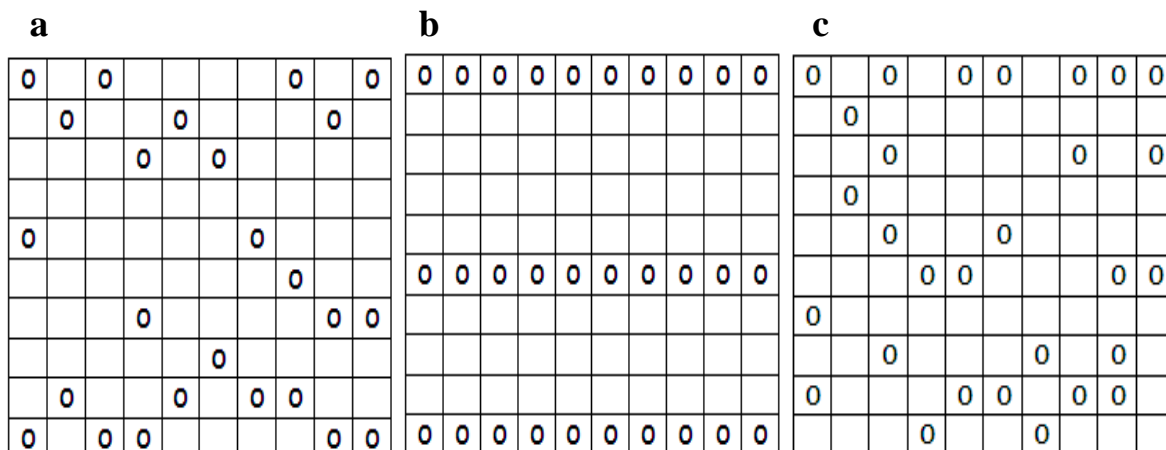


Figure.3 The results of these three studies: (a) Mosetti et al. (b) Grady et al. (c) this study.

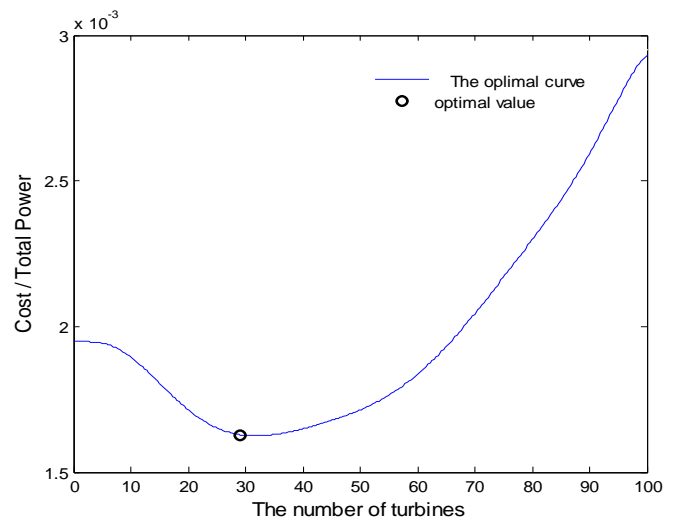
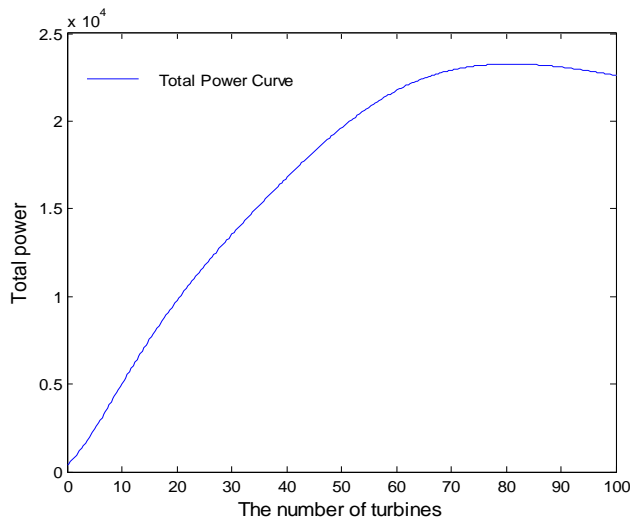


Figure.4. The total power and the objective curve

Table 3. The comparison of numerical results

	a	b	c
Number of the turbines	26	30	29
The total power (KW/ year)	12352	14310	14664
objective function	0.0016197	0.0015436	0.0014710

Seen the results, the study (a) reaches a value of 0.0016197, the study (b) reaches a value of 0.0015436 and this study achieved an even lower price 0.0014710, the higher the percentage, when using the same number of turbines, between the average total power output based on the previous two methods and this study increases the total power has a value near 1.5%, which clearly implies that the this proposed model reached more optimal results, in comparison with the previous two studies.

V. CONCLUSION

In this work, the design problem is defined as an optimization problem. The proposed algorithm can specify the placement of a wind farm with a minimum investment and more efficient use of the wind resource. To reach this objective a method based on the resolution of constraint satisfaction problems (CSP) algorithm is processed to estimate the optimal number of wind turbines and the total

power generated in a wind farm. The results of the developed model is very accurate and comparable with those produced by previous studies, thing which clearly implies that the proposed CSP method is well worked, more this method can be as a support tool for the study of park designers wind, with the intention of producing the maximum possible power from a wind farm, minimizing installation costs.

REFERENCES

- [1] Tony LAMBERT. Hybridization of complete and incomplete methods for solving CSPs, Nantes, 2006
- [2] G. Mosetti, C. Poloni, B. Diviacco, Optimization of wind turbine positioning in large wind farms by means of a genetic algorithm m, Journal of Wind Engineering and Industrial Aerodynamics 51 (1) (1994) 105–116.
- [3] G. Marmidis, S. Lazarou, E. Pyrgioti, Optimal placement of wind turbines in a wind park using Monte Carlo simulation, Renewable Energy 33 (2008) 1455–1460.
- [4] Grady SA, Hussaini MY, Abdullah MM. Placement of wind turbines using genetic algorithms. Renewable Energy Feb 2005;30(2):259–70.
- [5] Jensen NO. A note on wind generator interaction; 1983. Technical Report RISØ-M-2411, Denmark.
- [6] R. Dechter and I. Meiri. Experimental evaluation of preprocessing techniques in constraint satisfaction problems. In Proc.of the 11th IJCAI, pages 271-277, Detroit, MI, 1989.
- [7] E. Aarts and J.K. Lenstra, editors. Local Search in Combinatorial Optimization. John Wiley and Sons, 1997.
- [8] Chompoo-inwai C, Leelajindakraierk M, Banjongjit S, Fuangfoo P, Lee W-J. Design optimization of wind power planning for a country with low-medium wind-speed profile. IEEE Transactions on Industry Applications 2008;44(5):1341–7.