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The $L(2,1)$ - Labeling of Dragon and Armed Crown Graphs

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Abstract

An L(2,1)-labeling of a graph G is a function f from the vertex set V(G) to the set of all nonnegative integers such that $|f(x)-f(y)|\geq 2$ *if* $d(x,y)=1$ *and* $|f(x)-f(y)|\geq 1$ *if* $d(x,y)=2$. *The L(2,1)-labeling number of G, denoted by* $\lambda(G)$ *or* λ*, is the smallest number k such that* $\max\{f(v): v\in V(G)\}=k$. *In this paper I will completely determine* $\lambda(G)$ *-number for dragon and armed crown graphs.*

key - words:
L(2,1)-labeling, λ*-number , dragon, armed crown.*

1. Introduction

The channel assignment problem is to assign a channel (non negative integer) to each TV or radio transmitters located at various places such that communication do not interfere. In 1980, Hale [5] introduced the notion of T – coloring of a graph. He formulated the channel assignment problem as a graph coloring problem.

In 1988, Roberts proposed a variation of the channel assignment problem in which close transmitters must receive channels that are at least two apart. In a graph model of this problem, the transmitters are represented by the vertices of a graph. Two vertices are very close if they are adjacent in the graph and close if they are at a distance two apart in the graph. In [4], Griggs and Yen motivated by the above problem proposed by Roberts introduced $L(2,1)$ – labeling.

Definition 1.1: An $L(2,1)$ – labeling (or distance two labeling) of a graph $G=(V(G),E(G))$ is a function f from vertex set $G=(V(G),E(G))$ to the set of all nonnegative integers such that the following conditions are satisfied:

(1) $|f(x)-f(y)|\geq 2$ *if* $d(x,y)=1$

(2) $|f(x)-f(y)|\geq 1$ *if* $d(x,y)=2$

A $L(2,1)$ - labeling in which no label is greater than k is called a k - $L(2,1)$ - labeling. The smallest number k such that G has a k - $L(2,1)$ – labeling is called the $L(2,1)$ – labeling number of G and is denoted by $\lambda(G)$ or λ. The $L(2,1)$ – labeling has been studied extensively in recent past by many researchers like Yeh [8], Sakai [6], Georges, Mauro and Whittlesey [3], Georges and Mauro [2], and Vaidya, Vihol, Dani and Bantva[7].

Throughout this work, I will consider the finite, connected and undirected simple graph $G=(V(G),E(G))$. Following is the brief summary of definitions and information which are prerequisites for the present work.

Definition 1.2: A Dragon, $D_n(m)$ is a graph in which path P_m is attached to any vertex of cycle C_n.

Definition 1.3: An armed crown, $C_n \odot P_m$ is a graph in which path P_m is attached at each vertex of the cycle C_n.

Proposition 1.4: [4] The λ-number of a star $K_{1,\Delta}$ is $\Delta+1$, where Δ is the maximum degree.

Proposition 1.5: [4] The λ-number of a complete graph K_n is $2n-2$.

Proposition 1.6: [1] $\lambda(H)\leq\lambda(G)$, for any subgraph H of a graph G.

2. Main Result

Theorem 2.1: The λ -number of a Dragon $D_n(m)$ for $n\geq 3$ and $m\geq 1$ is 4.

Proof: Let $v_0,v_1,\ldots,v_{n-1}$ be the vertices of the cycle of $D_n(m)$ such that v_i is adjacent to v_{i+1}, $0\leq i\leq n-2$ and v_0 is adjacent to v_{n-1}. Also, let $u_0,u_1,\ldots,u_{m-1}$ be the vertices of the path or tail of

$D_n\ m$ such that u_j is adjacent to u_{j+1}, $0 \le j \le m-2$ and u_0 is adjacent to v_0. The graph $K_{1,3}$ is a subgraph of $D_n\ m$ and hence by Proposition 1.4 and 1.6 $\lambda\ D_n\ m \ \ge 4$. Now define $f : V\ D_n\ m \ \rightarrow\ 0,1,2,3,4$ as follows. We start vertex labeling from v_0.

(1) $\boldsymbol{n \equiv 0 \bmod\ 3}$

$$f\ v_i\ = \begin{cases} 0 & ,i \equiv 0 \bmod\ 3 \\ 2 & ,i \equiv 1 \bmod\ 3 \\ 4 & ,i \equiv 2 \bmod\ 3 \end{cases}$$

and

$$f\ u_j\ = \begin{cases} 3 & ,j \equiv 0 \bmod\ 4 \\ 1 & ,j \equiv 1 \bmod\ 4 \\ 4 & ,j \equiv 2 \bmod\ 4 \\ 0 & ,j \equiv 3 \bmod\ 4 \end{cases}$$

(2) $\boldsymbol{n \equiv 1 \bmod\ 3}$ then redefine the above f at $v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}$ as

$$f\ v_i\ = \begin{cases} 0 & ,i = n-4 \\ 3 & ,i = n-3 \\ 1 & ,i = n-2 \\ 4 & ,i = n-1 \end{cases}$$

and

$$f\ u_j\ = \begin{cases} 3 & ,j \equiv 0 \bmod\ 4 \\ 1 & ,j \equiv 1 \bmod\ 4 \\ 4 & ,j \equiv 2 \bmod\ 4 \\ 0 & ,j \equiv 3 \bmod\ 4 \end{cases}$$

(3) $\boldsymbol{n \equiv 2 \bmod\ 3}$ then redefine f in **(1)** at v_{n-2} and v_{n-1} as

$$f\ v_i\ = \begin{cases} 1 & ,i = n-2 \\ 3 & ,i = n-1 \end{cases}$$

and

$$f\ u_j\ = \begin{cases} 4 & ,j \equiv 0 \bmod\ 4 \\ 1 & ,j \equiv 1 \bmod\ 4 \\ 3 & ,j \equiv 2 \bmod\ 4 \\ 0 & ,j \equiv 3 \bmod\ 4 \end{cases}$$

Hence, $\lambda\ D_n\ m\ = 4$.

Illustration 2.2: In the following Figure-1 the $L(2,1)$-labeling of graph $D_6\ 4$ is shown.

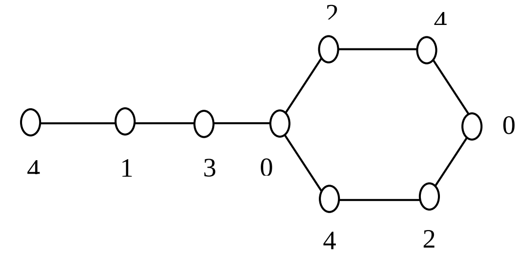

Figure - 1

Theorem 2.3: The λ-number of an armed crown $C_n \odot P_m$ for $n \ge 3$ and $m \ge 1$ is 5.

Proof: Let $v_0, v_1, \ldots, v_{n-1}$ be the vertices of the cycle of $C_n \odot P_m$ such that v_i is adjacent to v_{i+1}, $0 \le i \le n-2$ and v_0 is adjacent to v_{n-1}. Also, let $u_0, u_1, \ldots, u_{m-1}$ be the vertices of the path which is attached to every vertex of C_n such that u_j is adjacent to u_{j+1}, $0 \le j \le m-2$. We denote the vertices of the path adjacent to the vertex v_i , , as $u_{i0}, u_{i1}, \ldots, u_{i\ m-1}$ where u_{i0} adjacent to v_i $0 \le i \le n-1$. The graph $K_{1,3}$ is a subgraph of $D_n\ m$ and hence by Proposition 1.4 and 1.6 $\lambda\ C_n \odot P_m\ \ge 4$. Now define $f : V\ C_n \odot P_m\ \rightarrow\ 0,1,2,3,4,5$ as follows.

(1) $\boldsymbol{n \equiv 0 \bmod\ 3}$

$$f\ v_i\ = 0,\ i \equiv 0 \bmod\ 3$$

$$f\ u_{ij}\ = \begin{cases} 5 & ,j \equiv 0 \bmod\ 3 \\ 3 & ,j \equiv 1 \bmod\ 3 \\ 0 & ,j \equiv 2 \bmod\ 3 \end{cases}$$

$$f\ v_i\ = 2,\ i \equiv 1 \bmod\ 3$$

$$f\ u_{ij}\ = \begin{cases} 5 & ,j \equiv 0 \bmod\ 3 \\ 0 & ,j \equiv 1 \bmod\ 3 \\ 2 & ,j \equiv 2 \bmod\ 3 \end{cases}$$

and

$$f\ v_i\ = 4,\ i \equiv 2 \bmod\ 3$$

$$f\ u_{ij}\ = \begin{cases} 1 & ,j \equiv 0 \bmod\ 3 \\ 3 & ,j \equiv 1 \bmod\ 3 \\ 5 & ,j \equiv 2 \bmod\ 3 \end{cases}$$

(2) $\boldsymbol{n \equiv 1 \bmod\ 3}$ then redefine the above f at $v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}$ and the corresponding u_{ij}'s as

$$f\ v_i\ = 0,\ i = n-4\ ,$$

$$f\ u_{ij} = \begin{cases} 5 & , j \equiv 0 \bmod\ 3 \\ 3 & , j \equiv 1 \bmod\ 3 \\ 0 & , j \equiv 2 \bmod\ 3 \end{cases}$$

$f\ v_i = 3,\ i = n-3$,

$$f\ u_{ij} = \begin{cases} 5 & , j \equiv 0 \bmod\ 3 \\ 2 & , j \equiv 1 \bmod\ 3 \\ 0 & , j \equiv 2 \bmod\ 3 \end{cases}$$

$f\ v_i = 1, i = n-2$,

$$f\ u_j = \begin{cases} 5 & , j \equiv 0 \bmod\ 3 \\ 2 & , j \equiv 1 \bmod\ 3 \\ 0 & , j \equiv 2 \bmod\ 3 \end{cases}$$

and

$f\ v_i = 4,\ i = n-1$,

$$f\ u_{ij} = \begin{cases} 2 & , j \equiv 0 \bmod\ 3 \\ 5 & , j \equiv 1 \bmod\ 3 \\ 0 & , j \equiv 2 \bmod\ 3 \end{cases}$$

(3) $\boldsymbol{n \equiv 2 \bmod\ 3}$ then redefine f in (1) at v_{n-2} and v_{n-1} and the corresponding u_{ij}'s as

$f\ v_i = 4,\ i = n-3$,

$$f\ u_{ij} = \begin{cases} 0 & , j \equiv 0 \bmod\ 3 \\ 3 & , j \equiv 1 \bmod\ 3 \\ 5 & , j \equiv 2 \bmod\ 3 \end{cases}$$

$f\ v_i = 1,\ i = n-2$,

$$f\ u_{ij} = \begin{cases} 5 & , j \equiv 0 \bmod\ 3 \\ 0 & , j \equiv 1 \bmod\ 3 \\ 2 & , j \equiv 2 \bmod\ 3 \end{cases}$$

and

$f\ v_i = 3,\ i = n-1$,

$$f\ u_{ij} = \begin{cases} 5 & , j \equiv 0 \bmod\ 3 \\ 0 & , j \equiv 1 \bmod\ 3 \\ 2 & , j \equiv 2 \bmod\ 3 \end{cases}$$

Hence, $\lambda\ D_n\ m = 5$.

Illustration 2.4: In the following Figure-2 the $L(2,1)$-labeling of graph $C_5 \square P_3$ is shown.

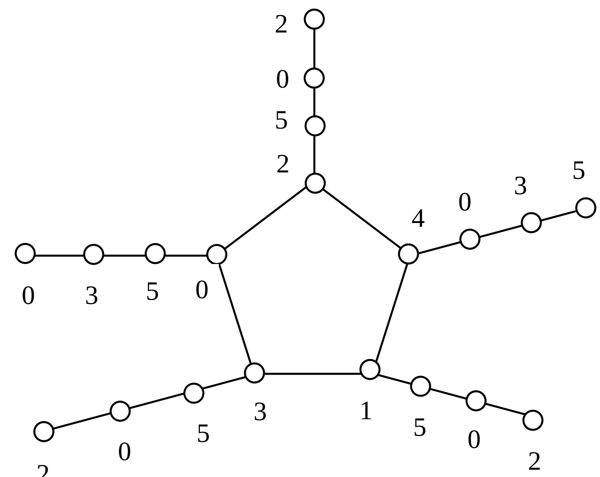

Figure – 2

3. Conclusion

Here I have proved that the λ number of a dragon is 4 and the λ number of an armed crown is 5.

4. References

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