The Influence of Wall Thickness and Additional Mass Flux on Peristaltic Pumping of A Jeffrey Fluid

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***Abstract* - This paper discusses the influence of wall thickness and additional mass flux on peristaltic pumping of a Jeffrey fluid in a cylindrical tube. The flow is investigated in a wave frame of reference moving with velocity of the wave. Using the perturbation technique, for modified Reynolds number, solutions are obtained for the non-linear differential equations with suitable boundary conditions. The axial velocity, pressure rise, friction force are obtain. The effect of various parameters of interest on flow rate and frictional force are discussed with help of graphs.**

Keywords: wall thickness, additional mass flux, Jeffrey fluid, Peristaltic pumping.

1. INTRODUCTION

Peristaltic motion is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube or channel containing the fluid. Peristalsis is a well known major mechanisms for fluid transport in many biological systems like swallowing food through the esophagus, urine transport from the kidney to the bladder through the ureter, movement of chyme in the gastro intestinal tract, movement of ovum in the female fallopian tubes, the transport of spermatozoa in the ducts efferent us of the male reproductive tract and in the cervical canal, the transport of lymph in the lymphatic vessels, and in the vasomotion in small blood vessels, such as arterioles and capillaries. Peristaltic pumping is one of the most important characteristics of fluid transport mechanism in many biological systems. It pumps the fluid against pressure rise since the first investigation of Latham reference [1], a number of analytical, numerical and experimental studies of peristaltic pumping of different fluids have been reported under different conditions with reference to physiological and mechanical situations. Srinivas et al. [2], discussed the influence of heat and mass transfer, wall properties and slip conditions for Newtonian fluid. Most of bio-fluids such as blood exhibit the behavior of non-Newtonian fluids may help to get better understanding of the working of biological systems.

Ravi Kumar et al.[3], studied unsteady peristaltic pumping in a finite length tube with permeable wall. Vajravelu et al.[4], considered the peristaltic transport of Herschel-Bulkley fluid in an inclined tube. Alsaedi et al.[5], considered couple stress fluid in their investigations. Subba Reddy et at.[6], studied the peristaltic motion of a power-law fluid in an asymmetric channel. Hayat and Ali [7], Srinivas and Kothandapani [8], Nadeem and Akram [9], and Pandey and Tripathi [10], have considered Jeffrey fluid in their study.

Ravi kumar [11], considered Jeffry fluid in their analysis related to peristaltic pumping. Vajravelu et al.[12], Kothandapani [13] considered Jeffrey fluid, and Hina et al.[14] considered third grade fluid in their work.

2. FORMULATION OF THE PROBLEM

We consider the problem of peristaltic transport of a Jeffrey fluid in a circular cylindrical tube to study the effects of wall thickness and constantly added flux per unit volume.

The geometry of the wall surface is described as

$$\bar{H}(\bar{Z},\bar{t}) = a + b^{-d/a}\sin\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t}) \qquad (1)$$

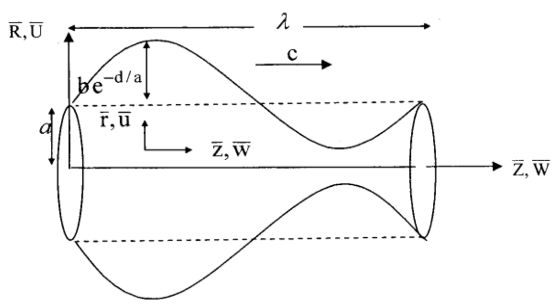


Figure1. Physical Model

where a is the mean radius of the tube, b is the wave amplitude, d is the tube wall thickness, λ is the wavelength of the peristaltic wave, c is the wave propagation velocity and $\bar{t}$ is the time. Here the flow is completely symmetrical about the axial coordinate $\bar{Z}$.

We choose the cylindrical coordinate system $(\bar{R},\bar{Z})$, where the $\bar{Z}$ -axis lies along the centerline of the tube, and $\bar{R}$ is the distance measured radially. Let $\bar{U}$ and $\bar{W}$ be the velocity components in the radial and axial directions respectively.

In the fixed frame of reference $(\bar{R},\bar{Z})$, the flow is unsteady. However, in a coordinate frame moving with the wave speed c, $(\bar{r},\bar{z})$ is stationary.

The transformation from fixed frame to wave frame is given by

$$\bar{z} = \bar{Z} - c\bar{t}\,,\ \bar{r} = \bar{R}\,,\ \bar{w}(\bar{r},\bar{z}) = \bar{W}(\bar{R},\bar{Z} - c\bar{t}) - c \qquad (2)$$

$$\bar{u}(\bar{r},\bar{z}) = \bar{U}(\bar{R},\bar{Z} - c\bar{t}) \qquad (3)$$

where $\bar{u}$ and $\bar{w}$ are the dimensional velocity components in the direction of $\bar{r}$ and $\bar{w}$ respectively.

The governing equations in the wave frame are given as

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\frac{\bar{r}\bar{u}}{1+\lambda_1}\right) + \frac{\partial \bar{w}}{\partial \bar{z}} = \bar{q} \qquad (4)$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + (\bar{w}+c)\frac{\partial \bar{u}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{r}} + \mu\left(\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\frac{\bar{r}\bar{u}}{1+\lambda_1}\right)\right) + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) + \rho\bar{q}\bar{u} \qquad (5)$$

$$\rho\left(\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + (\bar{w}+c)\frac{\partial \bar{w}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{r}} + \mu\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\frac{\bar{r}}{1+\lambda_1}\frac{\partial \bar{w}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) + \rho\bar{q}\left(\bar{w}+c\right) \qquad (6)$$

where $\bar{P}$ is the pressure, μ is the coefficient of viscosity, ρ is the density and $\bar{q}$ is the constantly added mass flux per unit volume, λ_1 is the Jeffrey parameter.

Now, we introduce the following non-dimensional variables

$$r = \frac{\bar{r}}{a},\ R = \frac{\bar{R}}{a},\ z = \frac{\bar{z}}{\lambda},\ Z = \frac{\bar{Z}}{\lambda},\ U = \frac{\lambda\bar{U}}{ac},\ u = \frac{\lambda\bar{u}}{ac},$$

$$W = \frac{\bar{W}}{c},\ w = \frac{\bar{w}}{c},\ P = \frac{a^2\bar{P}}{\lambda\mu c},\ q = \frac{\bar{q}\lambda}{c}$$

And defining the modified Reynolds number and wave number as

$$\mathrm{Re}^* = \left(\frac{\rho ca}{\mu}\right)\left(\frac{a}{\lambda}\right),\ \delta = \frac{a}{\lambda}$$

$$H = \frac{\bar{H}}{a} = 1 + \varphi e^{-\varepsilon}\sin\left[2\pi z\right] \qquad (7)$$

where φ is the amplitude ratio when thickness ratio tends to zero, $\varphi = \dfrac{b}{a} \le 1$, ε is thickness ratio and q is the additional parameter which is equal to the ratio between period time of relaxation and contraction to additional time of secretion to flow region in the small intestine.

Equations of motion in the dimensionless form have been reduced to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{ru}{1+\lambda_1}\right) + \frac{\partial w}{\partial z} = q \tag{8}$$

$$\frac{\partial P}{\partial r} = 0 \tag{9}$$

$$\mathrm{Re}^*\left(u\frac{\partial w}{\partial r} + (w+1)\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \left(\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w}{\partial r}\right)\right) + \gamma(w+1) \tag{10}$$

where γ is the additional Reynolds number which is equal to the ratio between additional force and viscosity force, $\gamma = q\,\mathrm{Re}^*$.

The instantaneous volume flow rate in the laboratory frame is given by

$$\overline{Q} = 2\pi \int_0^{\overline{H}} \overline{W}\overline{R}\,d\overline{R} \tag{11}$$

The rate of volume flow in the wave frame is given by

$$\overline{F} = 2\pi \int_0^{\overline{H}} \overline{w}\,\overline{r}\,d\overline{r} \tag{12}$$

If we substitute equations (2) and (3) into equation (11) and make use of (12), we find that the two rates of volume flow are related through

$$\overline{Q} = \overline{F} + \pi c \overline{H}^2 \tag{13}$$

The time-mean flow over a period $T = \dfrac{\lambda}{c}$ at a fixed $\overline{Z}$ -position is defined as

$$\widehat{Q} = \frac{1}{T}\int_0^T \overline{Q}\,d\bar{t} = \overline{F} + \pi c a^2\left(1 + \frac{\varphi^2 e^{-2\varepsilon}}{2}\right) \tag{14}$$

$$\widehat{Q} = \overline{F} + \pi c a^2\left(1 + \frac{\varphi^2 e^{-2\varepsilon}}{2}\right) \tag{15}$$

The dimensionless time-mean volume flow rates Θ and F respectively in the fixed and wave frame as

$$\Theta = \frac{\widehat{Q}}{2\pi c a^2} \quad \text{and} \quad F = \frac{\overline{F}}{2\pi c a^2}$$

and from Eq. (15) we can have

$$\Theta = F + \frac{1}{2}\left(1 + \frac{\varphi^2 e^{-2\varepsilon}}{2}\right) \tag{16}$$

$$\text{where} \quad F = \int_0^H rw\,dr \tag{17}$$

On the symmetry plane $(\bar{r} = 0)$ the normal velocity component and the slope of the velocity profile are zero. Also, the tangential component of the fluid velocity on the wall is usually determined from the mechanical property of the wall. Choosing the simple condition here that the wall has only transverse displacements at all times in the fixed frame, then the tangential component of fluid velocity in the wave frame on the wall is

$$\overline{W} = -c \quad \text{at } \bar{r} = \overline{H}(\bar{z}) \tag{18}$$

Using the concept of differentiation under the integral sign of (12) and make use of (18) and (4),

We have

$$\bar{u} = -c\frac{d\overline{H}}{d\bar{z}} + \frac{\bar{q}\overline{H}}{2} \quad \text{at} \quad \bar{r} = \overline{H}(\bar{z}) \tag{19}$$

The dimensionless boundary conditions in the wave frame are

$$\frac{\partial w}{\partial r} = 0 \quad u = 0 \qquad \text{at } r = 0 \tag{20}$$

$$w = -1 \quad u = -\frac{dH}{dz} + \frac{qH}{2} \qquad \text{at } r = H \tag{21}$$

3. NUMERICAL SOLUTION

We use perturbation method to find the solution. For this, we expand u, w, P, F in terms of the modified Reynolds numbers (Re^*) of first order is considered as

$$u = u_0 + \mathrm{Re}^* u_1 + ...$$

$$w = w_0 + \mathrm{Re}^* w_1 + ...$$

$$P = P_0 + \mathrm{Re}^* P_1 + ...$$

$$F = F_0 + \mathrm{Re}^* F_1 + ... \tag{22}$$

Substituting Eq.(22) into Eqs. 8-10, and 20 -21 we get the zeroth order and first order problems as mentioned below

3.1 ZEROTH-ORDER PROBLEM

The governing equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r u_0}{1+\lambda_1}\right) + \frac{\partial w_0}{\partial z} = q \tag{23}$$

$$\frac{\partial P_0}{\partial r} = 0 \tag{24}$$

$$\frac{\partial P_0}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w_0}{\partial r}\right) \tag{25}$$

The boundary conditions are given by

$$\frac{\partial w_0}{\partial r} = 0\ , \quad u_0 = 0 \qquad \text{at } r = 0 \tag{26}$$

$$w_0 = -1\ , \quad u_0 = -\frac{dH}{dz} + \frac{qH}{2} \qquad \text{at } r = H \tag{27}$$

On solving Eqs. (23) - (25) subject to boundary conditions given by Eqs. (26) and (27), we get the solution of the zeroth order problem as

$$w_0 = c_1(z) + c_2(z) r^2 \tag{28}$$

$$u_0 = (1+\lambda_1)\left[(q - c_1')\frac{r}{2} - c_2'\frac{r^3}{4}\right] \tag{29}$$

$$\frac{dP_0}{dz} = \frac{-1}{1+\lambda_1}\left[\frac{16F_0}{H^4} + \frac{8}{H^2}\right] \tag{30}$$

where

$$c_1(z) = \frac{4F_0}{H^2} + 1, \quad c_2(z) = -\left(\frac{4F_0}{H^4} + \frac{2}{H^2}\right) \tag{31}$$

$$c_i'(z) = \frac{d}{dz}(c_i(z)), i = 1,2.$$

3.2 FIRST-ORDER PROBLEM

The governing equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r u_1}{1+\lambda_1}\right) + \frac{\partial w_1}{\partial z} = 0 \tag{32}$$

$$\frac{\partial P_1}{\partial r} = 0 \tag{33}$$

$$u_0\frac{\partial w_0}{\partial r} + (w_0+1)\frac{\partial w_0}{\partial z} = -\frac{\partial P_1}{\partial z} + \left(\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w_1}{\partial r}\right)\right) + q(w_0+1) \tag{34}$$

The boundary conditions are given by

$$\frac{\partial w_1}{\partial r} = 0, \quad u_1 = 0 \qquad \text{at } r = 0 \tag{35}$$

$$w_1 = 0, \quad u_1 = 0 \qquad \text{at } r = H \tag{36}$$

On solving the Eqs (32) - (34) together with boundary conditions given by Eqs (35) and (36)
we get the first order solution as

$$w_1 = \frac{A}{2}\left(r^2 - H^2\right) + \frac{B}{2}\left(r^4 - H^4\right) + \frac{C}{9}\left(r^6 - H^6\right) \tag{37}$$

where

$$A=(1+\lambda_1)\begin{bmatrix}\frac{-8F_1}{(1+\lambda_1)H^4}+\\ \frac{8F_0{}^2H'}{3H^4}\left(\frac{1}{H}(1+\lambda_1)+2(\lambda_1-2)\right)\\ +\frac{2F_0H'}{H^2}\left(\frac{1}{H}(1+\lambda_1)+\frac{4}{3}(\lambda_1-3)\right)\\ +\frac{H'}{3}\left(\frac{1}{H}(1+\lambda_1)-4\right)\\ +\lambda_1 q\left(\frac{2F_0}{3H}+H\right)\end{bmatrix} \tag{38}$$

$$B=(1+\lambda_1)\begin{bmatrix}\frac{F_0{}^2H'}{H^7}(8-4\lambda_1)+\frac{F_0H'}{H^5}(6-2\lambda_1)\\ +\frac{H'}{H^3}-\lambda_1 q\left(\frac{F_0}{2H^4}+\frac{1}{4H^2}\right)\end{bmatrix} \tag{39}$$

$$C=-(1+\lambda_1)^2\left[\frac{8F_0{}^2H'}{H^9}+\frac{6F_0H'}{H^7}+\frac{H'}{H^5}\right] \tag{40}$$

The pressure gradient, at this order, is given as

$$\frac{dP_1}{dz}=\frac{2}{3}(1+\lambda_1)\left(\frac{8F_0{}^2H'}{H^5}+\frac{6F_0H'}{H^3}+\frac{H'}{H}\right)+ \frac{2}{3}\begin{pmatrix}\lambda_1 q\left(\frac{2F_0}{H^2}+1\right)+\frac{16F_0{}^2H'}{H^5}(\lambda_1-2)+\\ \frac{8F_0H'}{H^3}(\lambda_1-3)-\frac{4H'}{H^5}\end{pmatrix}+ 2\left(\frac{16F_0{}^2H'}{H^5}+\frac{8F_0H'}{H^3}+\frac{2F_0q}{H^2}-q\right)-\frac{16F_1}{H^4(1+\lambda_1)} \tag{41}$$

The expressions for the axial velocity component and the axial pressure gradient, up to first order, may be respectively written as

$$w=c_1(z)+c_2(z)r^2+\mathrm{Re}^*\begin{pmatrix}\frac{A}{2}(r^2-H^2)+\frac{B}{2}(r^4-H^4)\\ +\frac{C}{9}(r^6-H^6)\end{pmatrix} \tag{42}$$

$$\frac{dP}{dz}=\frac{-1}{1+\lambda_1}\left(\frac{16F_0}{H^4}+\frac{8}{H^2}\right)+ \mathrm{Re}^*\begin{bmatrix}\frac{2}{3}(1+\lambda_1)\left(\frac{8F_0{}^2H'}{H^5}+\frac{6F_0H'}{H^3}+\frac{H'}{H}\right)+\\ \frac{2}{3}\begin{pmatrix}\lambda_1 q\left(\frac{2F_0}{H^2}+1\right)+\frac{16F_0{}^2H'}{H^5}(\lambda_1-2)+\\ \frac{8F_0H'}{H^3}(\lambda_1-3)-\frac{4H'}{H^5}\end{pmatrix}\\ +2\left(\frac{16F_0{}^2H'}{H^5}+\frac{8F_0H'}{H^3}+\frac{2F_0q}{H^2}-q\right)-\frac{16F_1}{H^4(1+\lambda_1)}\end{bmatrix} \tag{43}$$

The above results can be expressed to first-order by defining $F=F_0+\mathrm{Re}^*F_1$ then substituting $F_0=F-\mathrm{Re}^*F_1$ into Eq. (42) and (43), neglecting the terms greater than $O(\mathrm{Re}^*)$, we find

$$w=\left[\left(\frac{4F}{H^2}+1\right)-\left(\frac{4F}{H^4}+\frac{2}{H^2}\right)r^2\right]+\mathrm{Re}^*\left[\begin{array}{l}\dfrac{\left(r^2-H^2\right)\left(1+\lambda_1\right)}{2}\left(\begin{array}{l}\dfrac{-8F_1}{\left(1+\lambda_1\right)H^4}+\\ \dfrac{8F^2H'}{3H^4}\left(\dfrac{1}{H}\left(1+\lambda_1\right)+2\left(\lambda_1-2\right)\right)+\\ \dfrac{2FH'}{H^2}\left(\dfrac{1}{H}\left(1+\lambda_1\right)+\dfrac{4}{3}\left(\lambda_1-3\right)\right)\\ +\dfrac{H'}{3}\left(\dfrac{1}{H}\left(1+\lambda_1\right)-4\right)+\\ \lambda_1 q\left(\dfrac{2F}{3H}+H\right)\end{array}\right)\\ +\dfrac{\left(r^4-H^4\right)\left(1+\lambda_1\right)}{2}\left(\begin{array}{l}\dfrac{F^2H'}{H^7}\left(8-4\lambda_1\right)+\dfrac{FH'}{H^5}\left(6-2\lambda_1\right)+\\ \dfrac{H'}{H^3}-\lambda_1 q\left(\dfrac{F}{2H^4}+\dfrac{1}{4H^2}\right)\end{array}\right)\\ -\dfrac{\left(r^6-H^6\right)\left(1+\lambda_1\right)^2}{9}\left(\dfrac{8F^2H'}{H^9}+\dfrac{6FH'}{H^7}+\dfrac{H'}{H^5}\right)\end{array}\right] \tag{44}$$

$$\frac{dP}{dz}=\frac{-1}{\left(1+\lambda_1\right)}\left(\frac{16F}{H^4}+\frac{8}{H^2}\right)+\mathrm{Re}^*\left[\begin{array}{l}\dfrac{2\lambda_1 q}{3}\left(\dfrac{2F}{H^2}+1\right)+\\ 2\left(\dfrac{2F_0 q}{H^2}-q\right)\end{array}\right] \tag{45}$$

The pressure rise ΔP_λ and friction force F_λ (at the wall) in the tube of length λ in their non-dimensional forms, are given by

$$\Delta P_\lambda=\int_0^1\left(\frac{dP}{dz}\right)dz \tag{46}$$

$$F_\lambda=\int_0^1 H^2\left(-\frac{dP}{dz}\right)dz \tag{47}$$

Substituting Eq (45) into Eq (46) and Eq (47) yields

$$\Delta P_\lambda=\frac{-8}{\left(1+\lambda_1\right)}\left(2FI_4+I_2\right)+2\gamma\left(2FI_2\left(\frac{\lambda_1}{3}+1\right)+\frac{\lambda_1}{3}\left(1+\varphi^2e^{-2\varepsilon}\right)-1\right) \tag{48}$$

$$F_\lambda=\frac{1}{\left(1+\lambda_1\right)}\left(16FI_2+8\right)-2\gamma\left(2F\left(\frac{\lambda_1}{3}+1\right)+\left(1+\varphi^2e^{-2\varepsilon}\right)\left(\frac{\lambda_1}{3}-1\right)\right) \tag{49}$$

where $I_2=\dfrac{1}{\left(1-\varphi^2e^{-2\varepsilon}\right)^{3/2}}$ & $I_4=\dfrac{2+3\varphi^2e^{-2\varepsilon}}{2\left(1-\varphi^2e^{-2\varepsilon}\right)^{7/2}}$ (50)

substituting Eq. (16) and Eq. (50) into Eq.(48) and Eq. (49) gives

$$\Delta P_\lambda=\frac{-8}{\left(1+\lambda_1\right)}\left(\begin{array}{l}\left(\Theta-\dfrac{1}{2}\left(1+\dfrac{\varphi^2e^{-2\varepsilon}}{2}\right)\right)\left(\dfrac{2+3\varphi^2e^{-2\varepsilon}}{\left(1-\varphi^2e^{-2\varepsilon}\right)^{7/2}}\right)+\\ \left(\dfrac{1}{\left(1-\varphi^2e^{-2\varepsilon}\right)^{3/2}}\right)\end{array}\right)+2\gamma\left(\begin{array}{l}2\left(\Theta-\dfrac{1}{2}\left(1+\dfrac{\varphi^2e^{-2\varepsilon}}{2}\right)\right)\left(\dfrac{1}{\left(1-\varphi^2e^{-2\varepsilon}\right)^{3/2}}\right)\left(\dfrac{\lambda_1}{3}+1\right)+\\ \dfrac{\lambda_1}{3}\left(1+\varphi^2e^{-2\varepsilon}\right)-1\end{array}\right) \tag{51}$$

$$F_\lambda=\frac{8}{\left(1+\lambda_1\right)}\left(\frac{2\left(\Theta-\dfrac{1}{2}\left(1+\dfrac{\varphi^2e^{-2\varepsilon}}{2}\right)\right)}{\left(1-\varphi^2e^{-2\varepsilon}\right)^{3/2}}+1\right)-2\gamma\left(\begin{array}{l}2\left(\Theta-\dfrac{1}{2}\left(1+\dfrac{\varphi^2e^{-2\varepsilon}}{2}\right)\right)\\ \left(\dfrac{\lambda_1}{3}+1\right)\\ +\left(1+\varphi^2e^{-2\varepsilon}\right)\left(\dfrac{\lambda_1}{3}-1\right)\end{array}\right) \tag{52}$$

The trapping limits are determined by calculating the ratio between minimum time-mean flow $\Theta_{\min}$ and maximum time-mean flow $\Theta^{(1)}{}_{\min}$ as a function of amplitude ratio φ as ε tends to zero. The minimum time-mean flow is obtained by Eq.(44) when we put $w = 0$ at $r = 0, H = 1+\varphi,$ and $H = 0$ solving it for Θ then we get

$$\Theta_{\min} = L_1 + \frac{1}{2}\left(1 + \frac{\varphi^2 e^{-2\varepsilon}}{2}\right) \qquad (53)$$

where $L_1 = \dfrac{\dfrac{H^2}{8}(4H-1)\gamma(1+\lambda_1)\lambda_1 + 1}{\dfrac{4}{H^2} + \gamma(1+\lambda_1)\lambda_1 \dfrac{(3-4H)}{12}}$

The maximum time-mean flow $\Theta^{(1)}{}_{\min}$ is obtained by putting $\Delta P_\lambda = 0$ in equation (51) and solving it for $\Theta^{(1)}{}_{\min}$

The absolute values of $\Theta_{\min}$ and $\Theta^{(1)}{}_{\min}$ must be real and satisfy the relation $0 \le \Theta_{\min} \le \Theta^{(1)}{}_{\min}$. Therefore trapping occurs such that $0 \le \Theta_{\min} \le \Theta^{(1)}{}_{\min} \le 1$ for all given values of γ, ε and φ.

4. RESULTS AND DISCUSSIONS

The effects of flow rate parameter Θ on pressure rise at $\varphi = 0.6, 0.2$ for different values of Jeffrey parameter (λ_1) is discussed through Fig.2 and Fig.3. From Fig.2(a) and Fig.2(b), it is observed that the pressure rise decreases with increasing thickness ratio (ε) in the peristaltic pumping case and its absolute value increases with increasing thickness ratio (ε) in augmented pumping case for $\varphi = 0.6$. From Fig.2(a) and Fig.2(b), it can be seen that the increases in the value of Jeffrey parameter (λ_1) decreases the value of ΔP_λ for a fixed values of thickness ratio. Similar phenomenon is observed from Fig.3 which is drawn for $\varphi = 0.2$

From Fig.2 and Fig.3, it is observed that pressure rise at $\varphi = 0.2$ is much smaller than the corresponding value at $\varphi = 0.6$ for certain values of thickness ratio (ε) with flow rate as a parameter.

Fig.4, shows that the pressure rise decreases with increasing additional Reyn
olds number γ in the pumping $(\Delta P_\lambda > 0)$, free pumping $(\Delta P_\lambda = 0)$ and co-pumping $(\Delta P_\lambda < 0)$ regions. The effect of Jeffrey parameter is observed from the Fig.4(a) and 4(b), which are drawn for $\lambda_1 = 0.5$ and 1.5 respectively . It can be seen that increase in Jeffrey parameter decreases the pressure rise for the fixed values of remaining parameters.

The effect of various parameters on friction force can be observed from Figs 5 - 9. Fig.5 and Fig.6 are drawn to study the effects of flow rate ΔF_λ on friction force at $\varphi = 0.6$ and $\varphi = 0.2$ respectively. From Fig.5(a), it is noticed that reflux occurs at $\Theta = 0.2, 0.4$ and the peristaltic pumping occurs at $\Theta = 0$ for all values of ε but the reflux occurs at all the values of $\Theta = 0, 0.2, 0.4$ can be observed from Fig.5(b). From these two, we can study the effect of Jeffrey parameter i.e. increase in the Jeffrey parameter effects the pumping phenomenon. From Fig.6, it is noticed that increase in flow rate increases the friction force for any fixed value of thickness ratio (ε).

It can be seen from Fig.6 that the friction force increases with an increase in the flow rate (Θ) for a given thickness ratio ε. From Figs.5 and 6 it can be concluded that the decreases in φ for a given flow rate (Θ), decreases the friction force (ΔF_λ).

The effects of additional Reynolds number ε on friction force for $\varphi = 0.6$ and $\varphi = 0.2$ is observed from Figs. 7 and 8. It is observed that the friction force increases with increasing γ at $\varphi = 0.6$ and $\varphi = 0.2$ For a given ε ,

ΔF_{λ} increases with an increase in Θ value is noticed from Figs.7 and 8. And also for a fixed Jeffrey parameter (λ_1) and additional Reynolds numbers (γ), the frictional force increases with decreasing φ. Also, it is observed that the increase in the Jeffrey parameter (λ_1) decreases the friction force for any given value of ε.

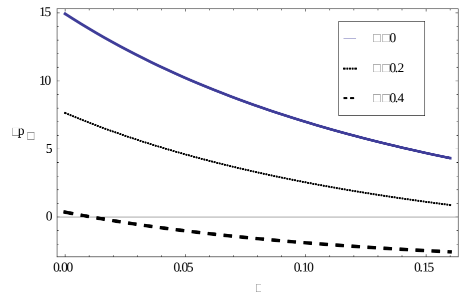
From Fig.9, it is observed that the time-mean flow ratio decreases with the increasing additional Reynolds number (γ) for a given amplitude ratio (φ). It is noticed from Fig.10, that the time mean flow ratio increases with increasing ε.

5. CONCLUSIONS

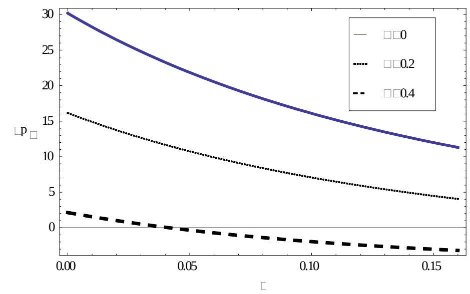
The influence of wall thickness and additional mass flux on the peristaltic pumping of non-Newtonian fluid is studied by considering Jeffrey fluid model.

The following are the conclusion drawn from this study.

1. The pressure rise decreases with increasing thickness ratio.
2. The pressure rise decreases with increasing Jeffrey parameter for any given thickness ratio and flow rate parameter.
3. Increase in the additional Reynolds number decreases the pressure rise.
4. The friction force increases with increasing thickness ratio for any given flow rate parameter.
5. The friction force increases with increasing additional Reynolds number for a fixed thickness ratio.
6. The trapping limit increases with increasing thickness ratio and the additional Reynolds number.

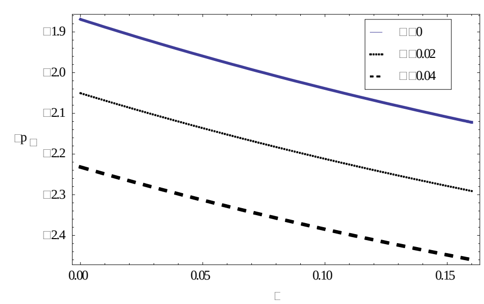


(a)

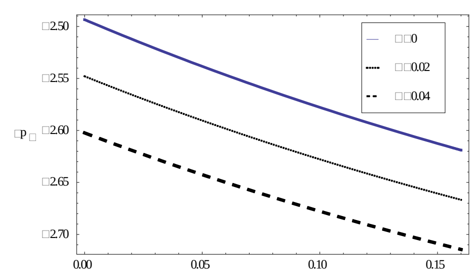


(b)

Fig. 2 *Variation of Pressure rise with thickness ratio for different values of* Θ with $\varphi = 0.6$ *and* $(a)\lambda_1 = 0.5, (b)\lambda_1 = 1.5$.

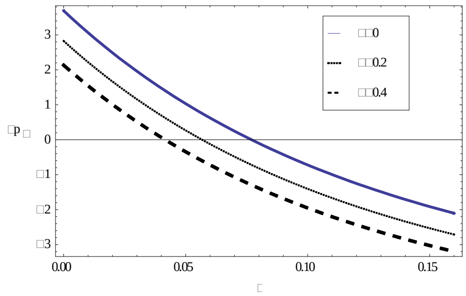


(a)

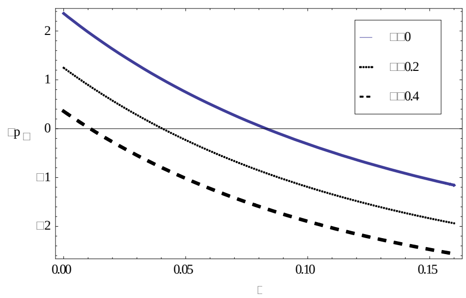


(b)

Fig. 3 *Variation of Friction force with thickness ratio for different values of* Θ with $\varphi = 0.2$ *and* $(a)\ \lambda_1 = 0.5 \quad (b)\ \lambda_1 = 1.5$.

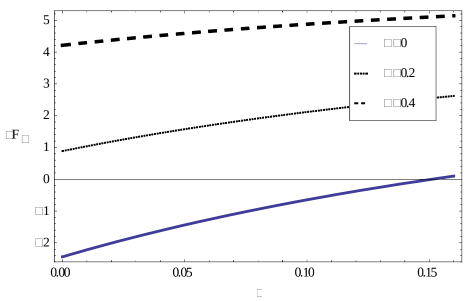


(a)

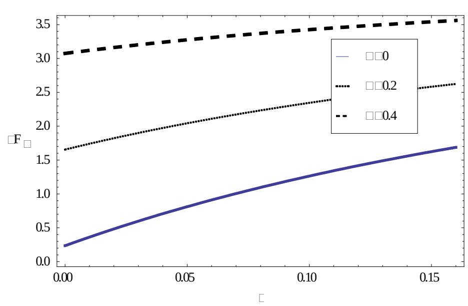


(b)

Fig. 4 Variation of Pressure rise with thickness ratio for different values of γ with $\varphi = 0.6$ *and* $(a)\lambda_1 = 0.5$ $(b)\lambda_1 = 1.5$.

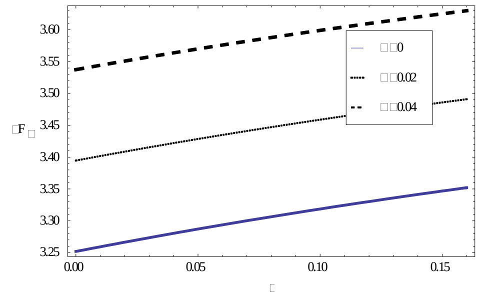


(a)

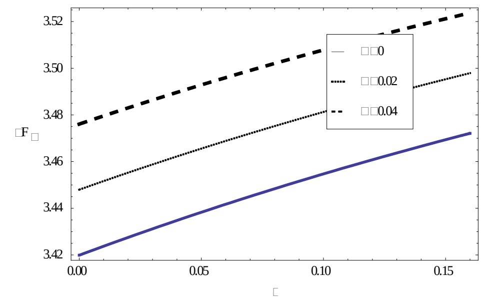


(b)

Fig. 5 Variation of Friction force with thickness ratio for different values of Θ with $\varphi = 0.6$ *and* $(a)\ \lambda_1 = 0.5$ $(b)\ \lambda_1 = 1.5$.

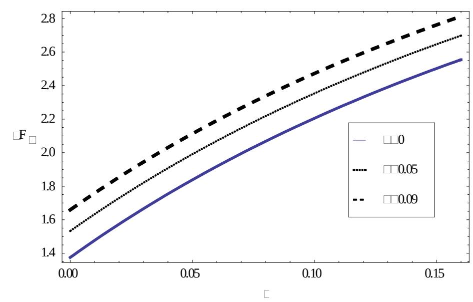


(a)

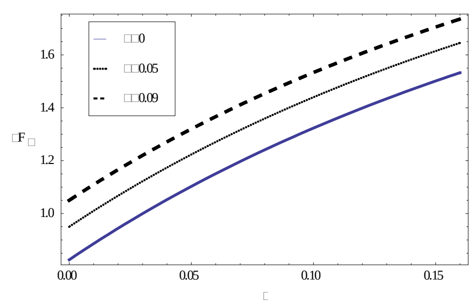


(b)

Fig. 6 Variation of Friction force with thickness ratio for different values of Θ with $\varphi = 0.2$ *and* $(a)\ \lambda_1 = 0.5$ $(b)\ \lambda_1 = 1.5$.

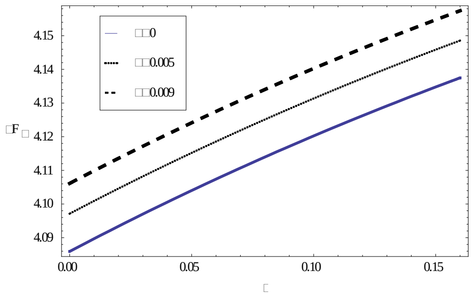


(a)

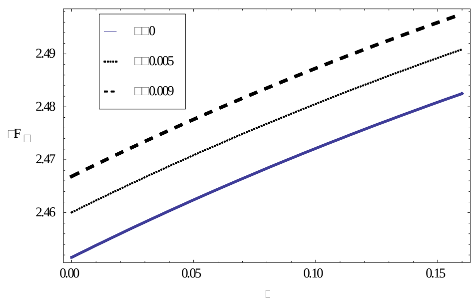


(b)

Fig. 7 Variation of Friction force with thickness ratio for different values of γ with $\varphi = 0.6$ *and* $(a)\ \lambda_1 = 0.5$ $(b)\ \lambda_1 = 1.5$.

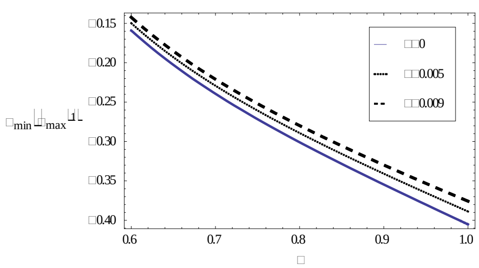


(a)

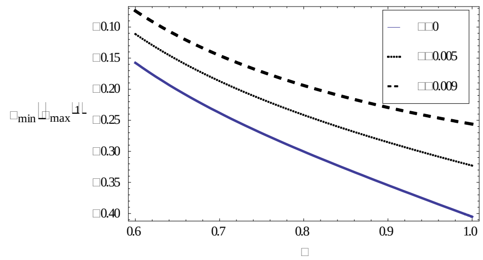


(b)

Fig. 8 *Variation of Friction force with thickness ratio for different values of* γ with $\varphi = 0.2$ *and* $(a)\ \lambda_1 = 0.5 \quad (b)\ \lambda_1 = 1.5.$

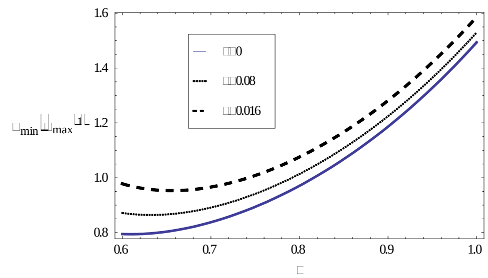


(a)

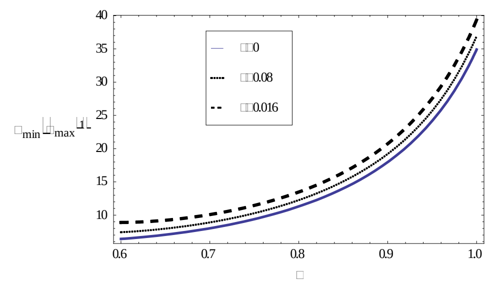


(b)

Fig. 9 *Variation of time mean flow ratio with amplitude ratio for different values of* γ with $\varphi = 0.2$, $\Theta = 0.4 \quad \varepsilon = 0.08$ *and* $(a)\ \lambda_1 = 0.5 \quad (b)\lambda_1 = 1.5.$



(a)



(b)

Fig. 10 *Variation of time mean flow ratio with amplitude ratio for different values of* ε with $\varphi = 0.4$, $\Theta = 0.4 \quad \gamma = 0.08$ *and* $(a)\ \lambda_1 = 0.5 \quad (b)\lambda_1 = 1.5.$

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