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# The Harvest of Energy Detection Adjunct Spectrum Sensing is Analyzed using ROC Curves

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***Abstract*—The rapid usage of wireless-communications in personal,commercial and governmental capacities, efficient spectrum utilization has become a prime topic of interest. Most of the licensed bands suffer from under-utilization and less spectral occupancy of spectrum. Cognitive radio technology promising solution to the problem of low spectral occupancy and inefficient utilization of the licensed radio spectrum. A prime constituent of the cognitive radio technology is spectrum sensing. Energy detection (ED) is one of the popular spectrum sensing technique for cognitive radio. In this paper show that performance of the ED technique was evaluated by use of ROC curves over AWGN channels. Simulation results show that the detection probability increase significantly when SNR increase. It is also observed that the signal can be detected even in very low SNR region by increasing no of samples (N)when the noise power is perfectly know.**

***Index Terms*—Cognitive Radio (CR),Energy detection (ED), Probability of detection ($P_D$), Probability of false alarm ($P_{FA}$), Threshold selection, Receiver operating characteristics (ROC).**

## I. INTRODUCTION

In last few decades a booming growth is experienced in the Wireless Communication [1], due to increase in the wireless device count, the radio spectrum is becoming increasingly congested. Devices based on wireless standards and technologies will remain increasing in future, which in turn will lead to spectrum scarcity in wireless communication. The limited availability of spectrum has become a bottleneck in the fulfillment of the consumers demand. The Federal Communication Commission (FCC) [2] report has shown that spectrum scarcity is mostly due to under-utilization of licensed spectrum. The licensed bands are exclusive usage band which provides protection against interference from other radio systems. It is observed that around 90-95% of the licensed radio spectrum is not in use at any location at any given time. The under-utilization of licensed spectrum has led to the problem of artificial spectrum scarcity.

In order to overcome the inefficient spectrum utilization and to meet the increasing demand has led to the coining of new concept “Cognitive Radio”. The Cognitive Radio is a technology which efficiently utilizes the licensed spectrum without causing any harm to the licensed users. It searches the licensed frequency bands for unused spectrum, and uses them efficiently. The unused licensed spectrum is also known as ‘white spaces’ [1, 3].

A main component of the CR technology is spectrum sensing. Spectrum sensing allows CRs to be aware of the surroundings environments by determining which frequencies are in use. A number of various methods are proposed for identifying the presence signal in transmissions. The prominent spectrum sensing techniques used are energy detection (ED), matched filter detection (MFD), cyclo-stationary detection. A comparative study of these schemes reveals that energy detection (ED) is the most extensively used spectrum sensing scheme because it's not required any a priori knowledge about the primary user (PU) characteristics. It`s straightforward to implement, and has low computational complexity while being excellent for detecting independent & identically distributed (IID) primary user (PU) signals.

Sensing the performance of the energy detector is specified by the following general metrics: The probability of detection ($P_D$), the probability of false alarm ($P_{FA}$), the probability of missed detection ($P_M$). In essence, the energy detection based spectrum sensing method should record a high probability of detection, low miss detection probability and low probability of false alarm. The receiver performance is quantified by depicting the receiver operating characteristics (ROC) curves. ROC graphs are employed to show trade-offs between detection probability and false alarm rates, thus allowing the determination of an optimal threshold. To plot ROC curves, one parameter is varied while the other is fixed.

This paper is organized as follows: In section II explains the energy detection (ED) system model & derivation ofprobability of detection ($P_D$) and probability of false alarm ($P_{FA}$) under the AWGN channel. In Section III describes the simulation result and comments. Finally,in Section IV concludes the entire research work carried out.

## II. COGNITIVE RADIO

The system model for energy detection which is used to identify the presence or absence of primary signal is shown below in Fig. the received signal x(t) is filtered by a band pass filter (BPF), followed by a square law device. The output signal of band-pass filter used to limit the noisepower and to normalize the noise variance with bandwidth W is squared and integrated over the observation time T. Eventually, the output of the integrator (also called decision statistic) u, is compared with a threshold λ, to decide whether a scanned band is vacant (*H*0) or occupied (*H*1).The decision statistic for ED technique is given as:

$$v = \sum_{k=1}^{N} x^2(k) \qquad (1)$$

Analytically, determining the sample signal x(t) is reduced to an identificationproblem, formalized as an hypothesis test; H0 and H1. H0 implies an absence of the signal, whereas H1 denotes presence of the signal.

This is described as:

$$x(k) = n(k) \qquad H_0 \qquad (2)$$
$$= h\, s(k) + n(k) \qquad H_1\,,\quad k = 1, \ldots, N$$

Where , x(k) is the received signal sample to be analyzed at each instant k, n(k) represents the (AWGN) additive white Gaussian noise with zero mean and variance $\sigma_n^2$, s(k) represents the PU transmitted signal sample which is to be detected and h is the channel gain between the primary signal transmitter and the detector. N denotes the number of samples of observed signal of bandwidth W for T seconds, mathematically given by N = 2TW = 2m, where mrepresents the time-bandwidth product. This is a binary signal detection problem in which cognitive radio (CR) has to decide between two Hypothesis, $H_0$ (band vacant) and $H_1$ (band occupied).

The performance of ED measured by two metrics sensitivity (probability of detection) and specificity (Probability of false alarm): probability of false alarm ($P_{fa}$), which denotes the probability that the detection algorithm falsely decides that PU is present in the considered frequency band when it actually is unavailable, & probability of detection ($P_d$), which represents the probability of correctly detecting the PU signal in the scanned frequency band [4]. Probability of false alarm($P_{fa}$) and probability of detection ($P_d$) are mathematically represented by:

$$P_{fa} = P_r(\text{signal is detected}/H_0) = P_r(v > \lambda/H_0)$$
$$= \int_{\lambda}^{\infty} f(v/H_0)dv \qquad (3)$$

$$P_d = P_r(\text{signal is detected}/H_1) = P_r(v > \lambda/H_1)$$
$$= \int_{\lambda}^{\infty} f(v/H_1)dv \qquad (4)$$

Where,*f*(v|Hi) represents the pdf of test statistic under hypothesis $H_i$with i= 0, 1. So our target at maximizing $P_D$ while minimizing $P_{FA}$. ROC graphs are employed to display trade-offs between detection probability($P_D$) and false alarm rates($P_{FA}$). ROC curve mainly depends on the threshold(λ). The threshold determines all performance metrics, $P_D$, $P_{FA}$ and $P_{MD}$.When the threshold (λ) increases (or decreases), both $P_F$and $P_D$decrease (and increase) [5].Thus threshold (λ) selection can be seen as a vital problem to balance the two contrary objectives (i.e., increase $P_d$while decrease$P_f$or vice-versa).The detector`s threshold value is determined either from specific $P_D$ or from specific $P_{FA}$.The appraisal treating of the detection threshold from the $P_{FA}$ is called CFAR (Constant false alarm rate). In CFAR doesn't require the channel SNR information to be known [6]. The appraisal treating of the detection threshold from the $P_D$ is called CDR (Constant detection rate). From the result of equations the derivation of the threshold is very identical for CFAR & CDR. In this paper, we consider CFAR method is taken for detection threshold (λ) selection.

As discussed in energy detection system model, the probability of false alarm & detection depend on the probability of density function (PDF) of the test statistic under $H_0$& $H_1$ respectively. In the following, we explain the exact derivation of PFA and PD under the AWGN channel.

### *A. Exact derivation of Probability of false alarm ($P_{FA}$) under AWGN:*

The additive white Gaussian noise (AWGN) is a channel model where the onlyimpairment in communication is noise; with a constant spectral density. These models do not account for channel impairments.

From eq$^n$ (2) under hypothesis $H_0$, x(k) = n(k) ~ N(0, $\sigma_n^2$), where n(k) is assumed to Gaussian noise with zero mean and variance $\sigma_n^2$. The test statistic v is the sum of square of N Gaussian random variables (RVs), each with zero mean & variance $\sigma_n^2$. So, test statistic v normalized with $\sigma_n^2$ is known as a central chi-square distribution with N degrees of freedom [7]. It is represented as:

$$H_0\!: \ \frac{1}{\sigma_n^2} v = \sum_{k=1}^{N} \left(\frac{1}{\sigma_n^2} n(k)\right)^2$$

$$= \sum_{k=1}^{N} (z(k))^2 \ \text{ where } z(k) \sim N(0,1) \quad (5)$$

$$\sim \chi_N^2 \qquad (6)$$

Used these Equation into (3) show that $f\left(\frac{1}{\sigma_n^2} v/H_0\right) = \chi_N^2$ ,

$P_{FA}$ is represented as:

$$P_{fa} = P_r(v > \lambda/H_0)$$
$$= P_r\left(\frac{1}{\sigma_n^2} v > \frac{1}{\sigma_n^2} \lambda/H_0\right)$$
$$= \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} f\left(\frac{1}{\sigma_n^2} v/H_0\right) dv \qquad (7)$$
$$= Q_{\chi_N^2}\left(\frac{\lambda}{\sigma_n^2}\right) \qquad (8)$$

Where, $Q_{\chi^2_N}(\frac{\lambda}{\sigma_n^2})$ denoted the right-tail probability for a $\chi_N^2$ random variable (RVs) and λ represented the threshold. The threshold (λ) depends on the noise variance( $\sigma_n^2$ ).

Now the resulting probability density function (PDF) of $\chi_N^2$ random variable can be determined.

We early discuss $\chi_N^2$ random variables is a central chi-square distribution. So, according to general PDF equation of central chi-square distribution as:

$$f_X(x) = \frac{1}{\sigma^N 2^{N/2}\Gamma(\frac{N}{2})} x^{N/2-1} e^{-\frac{x}{2\sigma^2}} \text{ , x} \geq 0$$

Where in our case σ=1 re-write above equation as:

$$f_X(x) = \frac{1}{2^{N/2}\Gamma(\frac{N}{2})} x^{N/2-1} e^{-\frac{x}{2}} \qquad (9)$$

Now expressing $P_{FA}$ in the terms of probability density function (PDF) as:

$$P_{fa} = \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} f_X(x)\,\mathrm{dx}$$

$$P_{fa} = \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} \frac{x^{\frac{N}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{N}{2}}\Gamma(\frac{N}{2})}\,\mathrm{dx} \qquad (10)$$

Dividing and multiplying by $2^{\frac{N}{2}-1}$ above equation we get:

$$P_{fa} = \frac{1}{2\,\Gamma(\frac{N}{2})} \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} (\tfrac{x}{2})^{\frac{N}{2}-1} \mathrm{e}^{-\frac{x}{2}}\,\mathrm{dx}$$

Now putting $\frac{x}{2}$ =t, $\frac{\mathrm{dx}}{2} = \mathrm{dt}$ & $\frac{N}{2}$ =m, also changing the limits of above equations we get:

$$P_{fa} = \frac{1}{\Gamma(m)} \int_{\frac{\lambda}{2\sigma_n^2}}^{\infty} (t)^{m-1} \mathrm{e}^{-t}\,\mathrm{dt}$$

$$= \frac{\Gamma(\mathrm{m}, \frac{\lambda}{2\sigma_n^2})}{\Gamma(\mathrm{m})}$$

$$\cong F_m(\tfrac{\lambda}{2\sigma_n^2}) \qquad (11)$$

Where, $\Gamma(\alpha,\beta)$ is the incomplete Gamma function, it is defined by $\Gamma(\alpha,\beta) = \int_{\beta}^{\infty} x^{\alpha-1} e^{-x}\,\mathrm{dx}$ .

The energy detection (ED) threshold (λ) for CFAR is derived from equation (11) as [7]:

$$\lambda = 2\sigma_n^2 F_m^{-1}(P_{fa}) \qquad (12)$$

From equation threshold depend on the number of observed samples (N), noise variance ($\sigma_n^2$) & probability of detection ($P_{FA}$).So the value of threshold (λ) is not related to SNR.From equation $P_{FA}$ depends on two parameters: time-bandwidth product (m) and the threshold (λ).Typically, $P_{FA}$ is given a value between $10^{-1} - 10^{-2}$. Time-bandwidth product (m= 2WT) is between the range 1-25. For example, $P_{FA} < 10^{-2}$ is attained with m = 25 at $\lambda \geq 76$ [8-10]. Since λ varies from 0 to 1, $P_{FA}$ is easily computed using(11) for a given m.

### *B. Exact derivation of Probability of detection ($P_D$) under AWGN:*

From equation (2) under hypothesis $H_1$, x(k) = s(k) + n(k) ~ N(A, $\sigma_n^2$ ).The test statistic v is the sum of square of N Gaussian random variables (RVs), each with mean A and variance $\sigma_n^2$ . So the distribution of the test statistic v follows a non-central chi-square distribution with N degrees of freedom.The probability density function (PDF) of non-central chi-square distribution as:

$$f_X(x) = \tfrac{1}{2}(\tfrac{x}{\gamma_c})^{\frac{N-2}{4}} e^{-\frac{(x+\gamma_c)}{2}} I_{\frac{N}{2}-1}(\sqrt{x\gamma_c}) \qquad (13)$$

Where, $I_n$ (y) is the n^th-order modified Bessel function.
$\gamma_c$ is the non-centrality parameter given as:

$$\gamma_c = \sum_{k=1}^{N} (\tfrac{A}{\sigma_n^2})^2 = \tfrac{NA^2}{\sigma_n^2} = N\gamma \qquad (14)$$

And γ is the known as signal to noise ratio (SNR).

Now expressing $P_D$ in the terms of probability density function (PDF) as:

$$P_d = \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} f_X(x)\,\mathrm{dx}$$

$$= \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} \tfrac{1}{2}(\tfrac{x}{\gamma_c})^{\frac{N-2}{4}} e^{-\frac{(x+\gamma_c)}{2}} I_{\frac{N}{2}-1}(\sqrt{x\gamma_c}) \qquad (15)$$

Now substituting x=$y^2$, $\gamma_c=a^2$,also changing the limits of above equations we get:

$$P_d = \int_{\sqrt{\frac{\lambda}{\sigma_n^2}}}^{\infty} \frac{y^m}{a^{m-1}} e^{-\frac{y^2+a^2}{2}} I_{m-1}(ay)\, dy$$

Now, we are using the definition of generalized Marcum Q-function incomplete gamma function as:

$$Q_\theta(\alpha,\beta) = \int_{\beta}^{\alpha} x \left(\tfrac{x}{\alpha}\right)^{\theta-1} e^{-\frac{x^2+\alpha^2}{2}} I_{\theta-1}(\alpha x)\, dx$$

Therefore,

$$P_d = Q_m\left(a, \sqrt{\tfrac{\lambda}{\sigma_n^2}}\right)$$

$$= Q_m\left(\sqrt{\gamma_c}, \sqrt{\tfrac{\lambda}{\sigma_n^2}}\right)$$

$$= Q_m\left(\sqrt{N\gamma}, \sqrt{\tfrac{\lambda}{\sigma_n^2}}\right) \qquad (16)$$

Where, m=N/2 is the time-bandwidth product, inferred to be an integer number.Using derivation of $P_D$ and $P_{FA}$ Receiver operating characteristics (ROC) shows the performance of the energy detector under the AWGN channel can be drawn.

## III. SIMULATION RESULT AND DISCUSSION

In our experiment we use the Lenovo laptop with i3 2.5 GHz processor with Windows 8.1 operating system. All simulations in this work are executed using MATLAB version R2013a. We used Monte Carlo (MC) method for simulations. The receiver performance is decided by depicting the receiver operating characteristics (ROC) curves. These curves enable examination of the association between sensitivity (probability of detection) and specificity (false alarm rate), for different thresholds, thus allowing the determination of the best threshold.With the draw of Roc curves, one parameter is varied while the other is certain.This enables the practices of different scenarios of interest.

The plot of Probability of detection ($P_D$) Vs SNR with varying values of probability of false alarm ($P_{FA}$) in AWGN is illustrated in figure 1.

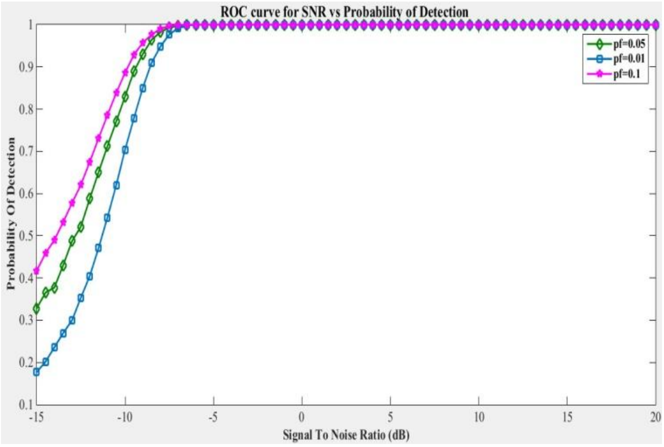

Figure 1. Probability of detection ($P_d$) vs SNR ROC curve for AWGN channel.

Here, the probability of false alarm ($P_{FA}$)is increased from 0.01 to 0.05 &N = 1000.From this plot, it is inferred that the performance of the energy detector improves with an increase in SNR and increase in probability of false alarm ($P_{FA}$) respectively. It is shown in table 1 &2.

Figure 2 shows the results for the performance metrics of ROC (Receiver operating Characteristics) plot of probability of detection ($P_d$) vs probability of false alarm ($P_{fa}$). Here we have taken probability of false alarm ($P_{fa}$) is 0-1 with increment of 0.05, N=1000 & the signal to noise ratio (SNR) at three different values -10dB,-12dB,-15dB. From this plot, it is inferred that the SNR (signal to noise ratio) increases the probability of detection ($P_d$) is increasing.

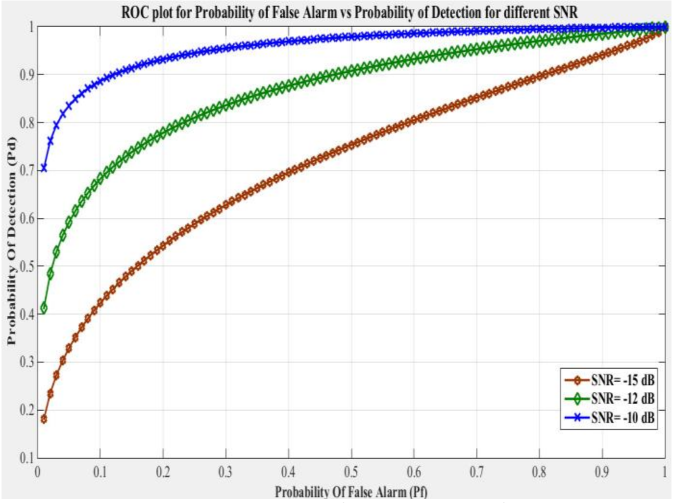

Figure 2.$P_d$vs$P_f$ROC curve for various value of SNR.

Table 1. Improvement in Probability of detection($P_D$) with increase in Signal to Noise Ratio(SNR) in Energy Detection Method for AWGN Channel.

| Probability of false alarm ($P_F$) | Probability of detection ($P_D$) (SNR= -15dB) | Probability of detection ($P_D$) (SNR= -10 dB) | Improvement (In times) |
|---|---|---|---|
| 0.01 | 0.1774 | 0.6987 | 2.9385 |
| 0.05 | 0.3211 | 0.8366 | 1.6054 |
| 0.1 | 0.4161 | 0.8862 | 1.1297 |

Table 2. Improvement in Probability of detection($P_D$) with increase in probability of false alarm ($P_{FA}$) in Energy Detection Method for AWGN Channel.

| SNR (in dB) | PFA=0.01 | PFA=0.05 | PFA=0.1 |
|---|---|---|---|
| -15 | 0.1774 | 0.3211 | 0.4161 |
| -14 | 0.2328 | 0.3867 | 0.4944 |
| -13 | 0.3078 | 0.4769 | 0.5757 |
| -12 | 0.4097 | 0.5955 | 0.6783 |
| -11 | 0.5454 | 0.7108 | 0.7856 |
| -10 | 0.6987 | 0.8366 | 0.8862 |
| -9 | 0.8465 | 0.9345 | 0.9537 |
| -8 | 0.9572 | 0.9815 | 0.9895 |
| -7 | 0.9916 | 0.998 | 0.9994 |
| -6 | 0.9996 | 0.9999 | 1 |

Figure 3 shows the results for the performance metrics of complementary ROC (Receiver operating Characteristics) plot of probability of miss-detection ($P_m$) vs probability of false alarm ($P_{fa}$). Here we have taken probability of false alarm ($P_{fa}$) is 0-1 with increment of 0.01, N=1000 & the signal to noise ratio (SNR) at three different values -10dB,-12dB,-15dB. From this plot, it is inferred that the SNR (signal to noise ratio) increases the probability of detection ($P_d$) is increasing &probability of miss-detection ($P_m$) is minimized at a fixed point of probability of false alarm ($P_{fa}$).

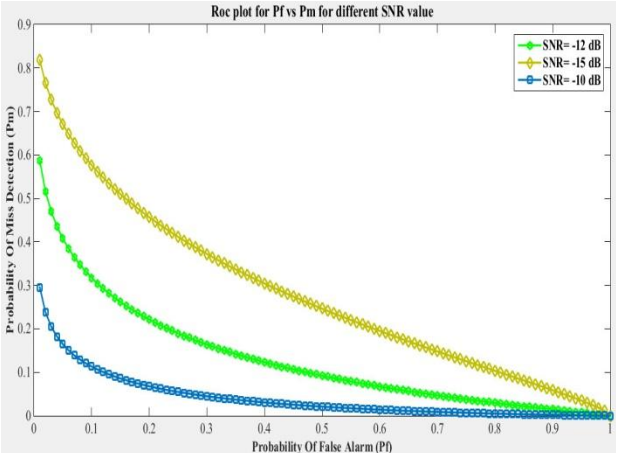

Figure 3.$P_m$vs $P_f$ ROC curve for various value of SNR.

Figure 4 shows that ROC plot of probability of detection ($P_D$) for different SNR and no of samples (N) under AWGN channel. Here we take the probability of false alarm ($P_{fa}$) is 0.1 & the no of samples (N) is changed to 1000, 1500, 2000. From this plot, it is inferred no of samples (N) increases the probability of detection ($P_d$) is increasing. It is shown in table 3.

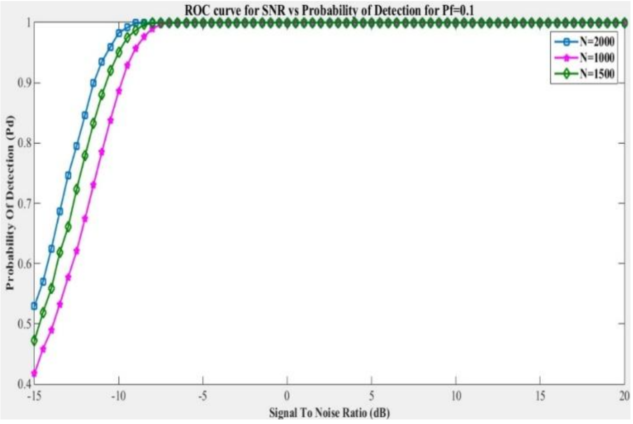

Figure 4. ROC curve for various value of SNR &N.

Table 3. Improvement in Probability of detection ($P_D$) with increase in no of samples (N) in Energy Detection Method for AWGN Channel.

| SNR | N=1000 | N=1500 | N=2000 |
|---|---|---|---|
| -15 | 0.4092 | 0.4804 | 0.5372 |
| -14 | 0.4885 | 0.5671 | 0.6298 |
| -13 | 0.5713 | 0.6754 | 0.7254 |
| -12 | 0.6728 | 0.7764 | 0.8519 |
| -11 | 0.7912 | 0.8794 | 0.9351 |
| -10 | 0.883 | 0.9557 | 0.9834 |
| -9 | 0.956 | 0.9901 | 0.9975 |
| -8 | 0.9909 | 0.9996 | 0.9997 |
| -7 | 0.9985 | 0.9999 | 1 |

## IV. CONCLUSIONS

In this paper, we discuss energy detection based spectrum sensing method. We derive exact close form expression of probability of detection and probability of false alarm under the AWGN channel. The operation of energy detection (ED) techniques has been harvested using receiver operating characteristics curves (ROC). A Separate ROC curves such as Probability of detection (Pd) vs SNR plots, Probability of detection ($P_d$) vs probability of false alarm ($P_{fa}$), probability of miss-detection ($P_m$) vs probability of false alarm ($P_{fa}$) has been plotted over AWGN channel. Probability of detection is depends on a SNR& no of samples (N). When the SNR is increased probability of miss-detection decreases & Probability of detection ($P_d$) increases. The Probability of false alarm ($P_{fa}$) is an effect on the probability of detection ($P_d$). So the probability of false alarm ($P_{fa}$) is increased probability of detection ($P_d$) increases.

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