

The External Field-Like Effect to the Evolution of Public Opinion in the Stauffer-Galam Model

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Abstract— In this work, we performed numerical simulation to investigate the effect of an “external field” to the evolution of public opinion in the Stauffer-Galam or two-dimensional Sznajd opinion model. Based on our results, the critical point $d = d_c$ depended on the probability p , where d and p are the initial density of spin up and the probability agents following the influence of the external field for such as a mass media. For typical value $p=1$, the critical point occurred at $d_c \approx 0.409$ which causes the system reach a fixed point (complete consensus), i.e. all spin up or all spin down with the same probability. Moreover, we found a linear relation between the critical point d_c and the probability p i.e., $d_c \sim \alpha p$, with $\alpha \approx -0.09$. We also found a power law relation between the average relaxation time $\langle \tau \rangle$ and the size of lattice L , i.e., $\langle \tau \rangle \sim L^\beta$, with $\beta \approx 2.60$.

Keywords— Sociophysics; opinion dynamics; phase transition; critical point; consensus

I. INTRODUCTION

Sociophysics is an interdisciplinary science that applies concepts and rules of statistical physics to understand the social-political behavior [1,2,3]. One of the most interesting topics in sociophysics is opinion dynamics. Several models have been proposed to explain the social-politics phenomena [1,2,3,4]. Statistical physics features such as order-disorder phase transition, scaling, and universality attract huge interest in sociophysics [1]. The opinions in opinion dynamics are usually represented by discrete numbers such as Ising number or Ising spin $S = \pm 1$. These two numbers represent two distinct opinions such as “YES” and “NO”, “PRO” and “CONTRA” and so on.

One of the interested opinion dynamics models is Stauffer-Galam or two-dimensional Sznajd model [5]. The original or one-dimensional Sznajd model is an Ising-like model having two simple microscopic dynamics. The first is two adjacent agents (cluster agents) S_i and S_{i+1} that can persuade their two neighboring agents S_{i-1} and S_{i+2} to follow them if the cluster agents have the same opinion (social validation). The final state of this dynamics is homogenous with $m = \pm 1$ (ferromagnetic). The second is that two adjacent agents have

different opinion, then their two neighboring agents take the opposite state or opinion $S_{i-1} = S_{i+1}$ and $S_i = S_{i+2}$. The final state of this dynamics is antiparallel with $m = 0$ (antiferromagnetic). In addition, there is no phase transition in the original Sznajd model [6,7].

In the standard Stauffer model, the final state of system is homogenous or complete consensus, e.g., all the agents at the end have the same opinion. In the social system, this condition is understood as a complete consensus. The Stauffer-Galam model a two-dimensional version of one-dimensional Sznajd model which still follows the original rules as exhibited in Fig. 1.

In the Stauffer standard model, phase transition occurs when the probability of spin up $d = d_c = 0.5$, wherein system reaches complete consensus with all spins up or all spins down have the same probability. This phase transition separates two system states, e.g., for the probability spin up $d > 0.5$ or $d < 0.5$, the final state of system ends with all spins up or all spins down for a large population N . There is no situation to achieve an antiferromagnetic state or stalemate situation as in the original Sznajd model [5]. Other formulations are considered based on the Stauffer model such as opinion reputation [8] and agent diffusion [9].

To make more realistic, where the public opinion no longer leads to a fixed point, various social parameters are introduced in the Stauffer-Galam model such as independence [10], anticonformity which is defined in two or three dimension of lattice [11], and anticonformity which is defined on complete graph with two different configurations [12]. The previous work [13] also considered the effect of mass media based on the Stauffer rule only (this mass media acts like external field on Ising model), e.g., probability p agents follow the mass media if four neighbor agents are chose to have different opinion. It was obtained that phase transition occurred at $p \gtrsim 0.18$ although the density of spin up is very small $d \approx 0.01$. Several works that considered the mass media effect on any opinion dynamics are also found in [14,15,16,17].

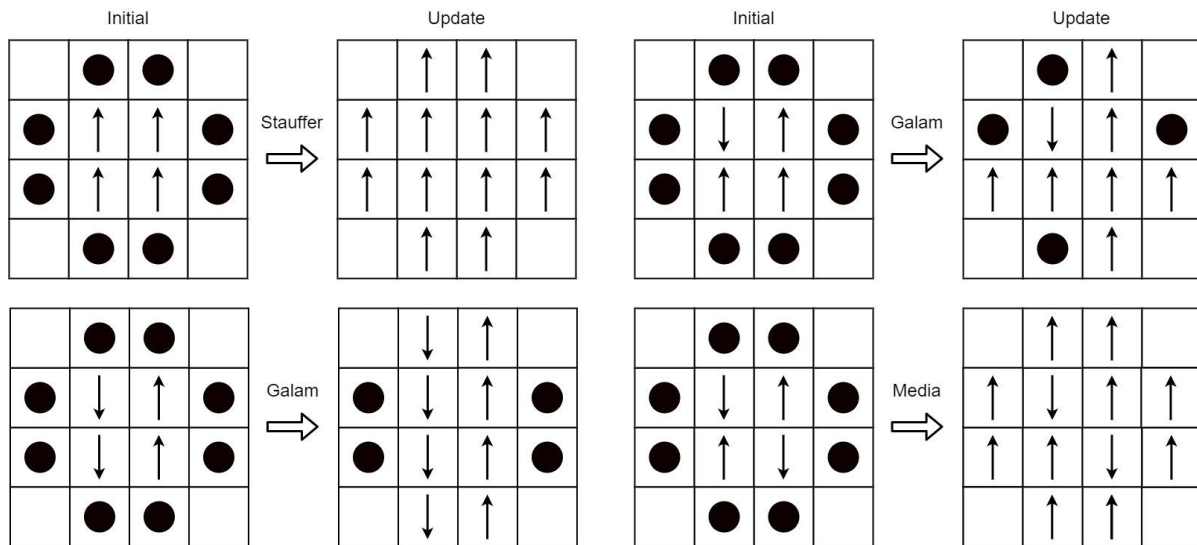


Fig. 1. : Illustration of Stauffer-Galam model defined on a regular lattice size of $L \times L$. From left to right, the first panel is Stauffer rule, the second and third are Galam rules, and the fourth is the mass media rule.

In this work, we consider the effect of an external field, such as mass media to the evolution of the public opinion based on the Stauffer-Galam model. The different formulation of this model with the previous work [13] is that we involved the Galam rules as exhibited in Fig. 1. From our numerical simulation, we found that the critical point depends on the probability p , for example, for typical $p = 1$, the critical point occurred at the probability spin up $d_c \approx 0.409$. This critical point separates two homogenous states with all spins up and all spins down. Moreover, we found a linear relation between the critical point d_c and the probability p , which is $d_c \sim \alpha p$, with $\alpha \approx 0.09$. We also found that the average relaxation time $\langle \tau \rangle$ and lattice size L have a power-law relation $\langle \tau \rangle \sim L^\beta$, with $\beta \approx 2.6$ for all values of $p \in [0, 1]$.

II. MODEL

The Stauffer-Galam model or two-dimensional Sznajd model are defined on a square lattice $N = L \times L$ with a periodic boundary condition, where L is a lattice size length as exhibited in Fig. 1. Each of spin or agent occupies a lattice node that has two possible states or opinions $S = \pm 1$.

In this model, we consider two microscopic rules. The first rule is that four agents can persuade eight nearest neighbors if all of four agents have the same state or opinion (Stauffer rule). The second rule is two or three agents can persuade two or four nearest neighbors if the agents have the same opinion (Galam rule). The next Galam rule is that each of two agents have the same opinion, then four nearest neighbors will follow the agents' state. The chosen agents or cluster agents have a stronger influence to the nearest neighbors compared to the mass media, therefore the effect of the mass media has an impact if each of cluster agents has a different state or opinion, e.g., with a probability p , eight nearest neighbors follow the opinion of the media mass. The algorithm of the model is as follows:

- 1) Four agents (cluster agent) are chosen randomly.
- 2) If the cluster agents have the same opinion, eight nearest neighbors will follow the opinion of the cluster agents (Stauffer rule).
- 3) If two (three) agents have the same opinion, two (four) of their neighbor agents will follow the opinion of the cluster agents (Galam rule).
- 4) If each two of cluster agents have the same opinion, four of their neighbor agents will follow the opinion of the cluster agents (Galam rule).
- 5) If the rules (2)-(4) are not fulfilled, then each of neighboring agent acts according to the mass media rule, e.g., with probability p , agent follow the mass media's opinion.

The public opinion in this system can be treated as the magnetization m as follows:

$$m = \frac{1}{N} \sum S_i, \quad (1)$$

with S_i and $N = L \times L$ are the i^{th} spin and the total spin in the system respectively. We consider a relaxation time τ , a time in which all agents or spins at the end have the same opinion [9]. The relaxation time can be used to identify the critical system, for example, there is a certain of spin up density to make the relaxation time to be maximum.

III. RESULTS AND DISCUSSION

We performed numerical simulation of the Stauffer-Galam model in a presence of an external field which is defined in two dimension lattice with a total population $N = L \times L$ with periodic boundary condition. From our simulation, the evolution of public opinion m to the time t (Monte Carlo sweep) is given in Fig. 2. It can be seen that, even for the case that spin up density d is smaller than 0.5 i.e. $d = 0.40$, there are several samples that achieves the homogenous state (ferromagnetic) with $m = 1$, for a large of lattice size $L = 120$

and the mass media probability $p = 0.81$. This indicates that the critical point is no longer at the density of spin up $d = d_c = 0.5$ as at the Stauffer standard model, that the system will complete consensus with all spin up for $d > 0.5$ and all spins down for $d < 0.5$ for a large limit population, but it depends on the mass media probability p .

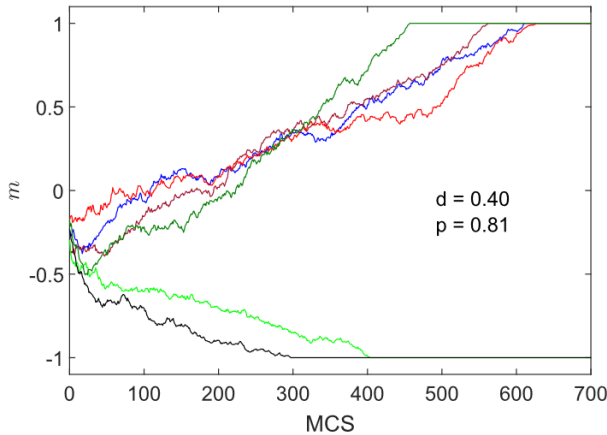


Fig. 2. Evolution of public opinion m with a presence of an influence of mass media, with probability $p = 0.81$ agents follow the mass media's opinion. One can see that, even the probability of spin up $d < 0.5$ several samples show that the system achieves a complete consensus with (all spins up). Lattice size is $L = 120$.

To investigate the dependence of critical density of spin up d_c to the probability p , we consider the relaxation time τ . For $p = 0$, the maximum relaxation time occurs at $d = d_c \approx 0.5$ that confirms the Stauffer-Galam standard model [4]. For typical values $p = 0.5$ and $p = 1$ the critical point shifts to $d_c \approx 0.642$ and $d_c \approx 0.409$, respectively, based on the fitting curve as shown in Fig. 3.

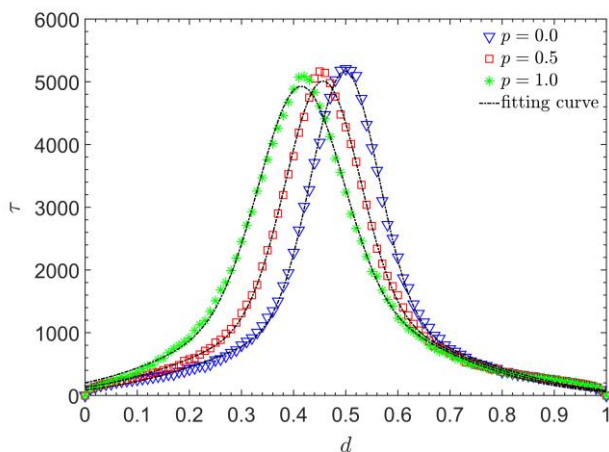


Fig. 3. Relaxation time τ versus the spin up density d for typical values p . Data is obtained from average over 500 samples for each point, with lattice size $L = 20$.

We consider an average magnetization $\langle m \rangle$ that shows the ferromagnetic state with $\langle m \rangle = 1$ when the probability spin up is varied $0 \leq d \leq 1$ as (Fig. 4). There is no probability p that

makes all samples achieve the homogenous with $\langle m \rangle = 1$. Each point of data is obtained from average over 500 simulations and the lattice size $L = 40$. The critical condition can be analyzed more clearly for a large limit population N . For small N , for example $L = 40$ (Fig. 4), we can use a sigmoidal fitting curve to find the critical point d_c as follows [15]:

$$\langle m \rangle = -1 + \frac{2}{1 + 10^{(d_c - d)b}}, \quad (2)$$

where d_c , d and b are the critical point of spin-up density, the spin-up density, and the slope of curve, respectively.

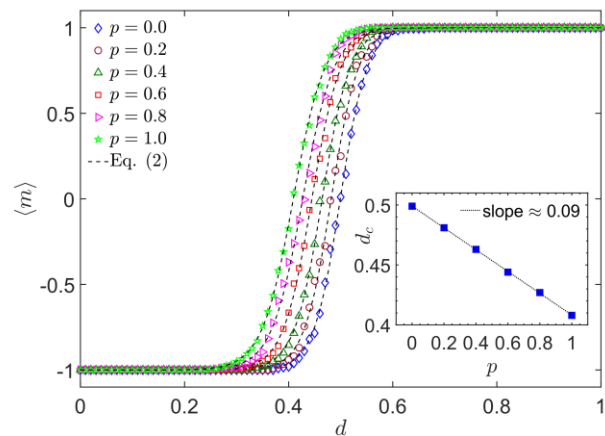


Fig. 4. The average of final public opinion for the spin up density $0 \leq d \leq 1$ for lattice size $L = 40$. The critical point which makes system achieves a complete consensus with all spins up or all spins down with the same probability, can be obtained by sigmoidal fitting curve. The inset for the critical point d_c versus the probability p with slope ≈ -0.09 .

For a maximum probability $p = 1$, the critical point occurs at $d = d_c \approx 0.409$ that confirms the maximum relaxation time versus the density of spin up d in Fig. 3. This means that the system will achieve a complete consensus with all spins up for $d > 0.409$ and all spin down for $d < 0.409$ for a large limit population. The critical point d_c for each p based on Fig. 4 are given in Table 1.

Table 1: The critical points and the constant b for each of probability value p based on (2), for a lattice size $L = 40$.

Variable	Probability of mass media p					
	0.0	0.2	0.4	0.6	0.8	1.0
d_c	0.499	0.481	0.464	0.444	0.427	0.409
b	17.11	16.76	16.33	15.82	14.58	14.44

We test the evolution of public opinion m for $d = 0.41 > d_c$, the probability $p = 1$ and a large size $L = 250$ and we found that all samples achieve the homogenous state with all spin up ($m = 1$) (Fig. 5). The screenshot of the simulation is shown in Fig. 6

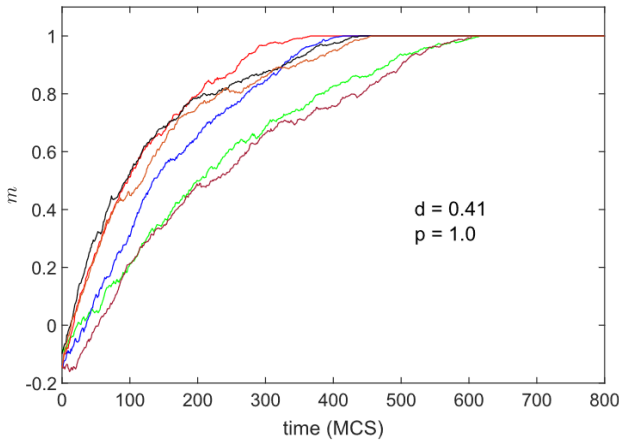


Fig. 5. The evolution of public opinion m with density of spin up $d = 0.415 > d_c$ and probability $p = 1$, agents follow the mass media's opinion. On can see that all samples achieve a homogenous state with $m = 1$. The size of lattice is $L = 250$.

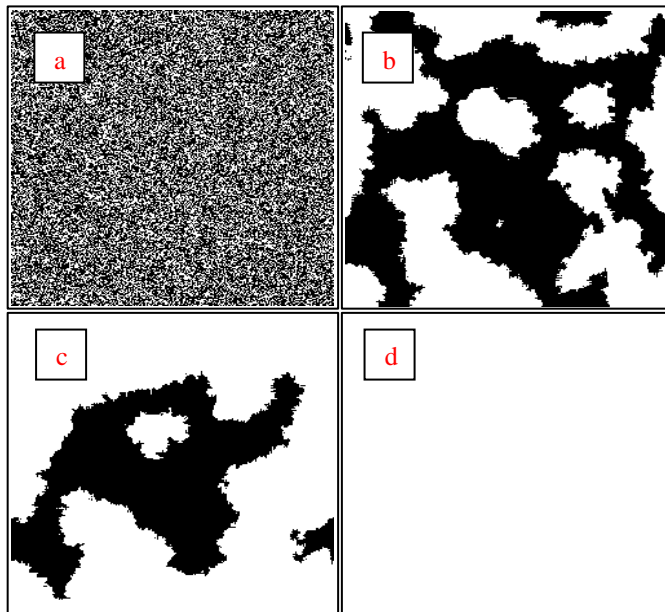


Fig. 6. Screenshot of the lattice sweeping to the homogenous state with all spin up (white) based on the Stauffer-Galam model with the presence of an external field. The density of spin up $d = 0.41$ and with probability $p = 1$, agents follow the mass media's opinion. From left, (a) $m = -0.18$ (MCS = 0), (b) $m = -0.01763$ (MCS = 50), (c) $m = 0.3747$ (MCS = 200), and (d) $m = 1$ (MCS = 659). The lattice size is $L = 250$.

From the inset graph in Fig. 4, we found a linear relation between the critical point d_c and the probability p namely $d_c \sim \alpha p$, where $\alpha \approx -0.09$. In addition, we found the relation between the average of the relaxation time τ and lattice size L , as exhibited in log-log plot scale in Fig. 7, satisfies the power law relation

$$\langle \tau \rangle \sim L^\beta,$$

where $\beta \approx 2.60$. This relation was also found in the previous work [9,13,19] with different value of β .

IV. CONCLUSSION

In this work, we study the Stauffer-Galam or two-dimensional Sznajd opinion dynamics model in Sociophysics which is defined o regular lattice $L \times L$ for a presence of an external field.

We performed numerical simulation to observe the mass media effect to the evolution of the public opinion. We found that the critical point is no longer at $d_c = 0.5$ such in the Stauffer standard model, but it depends on the probability p (agents were completely follow the opinion of the mass media). For typical value $p = 1$ the critical point occurred at $d_c \approx 0.409$. This critical point separated two different state with all spins up for $d_c > d$ and all spins down for $d < d_c$ for a large limit population.

the mass media effect in this formulation was lower than at the Stauffer standard model as in the previous work [13]. This is because the effect of the cluster spins to the evolution of the public opinion m is very high. Therefore, even if the probability mass media $p = 1$, several samples achieve the homogenous state with $m = -1$ for a large of the density of spin up d .

From our results, we found the linear relation between the critical point d_c with the probability mass media p , namely $d_c \sim \alpha p$, with $\alpha \approx -0.09$. In addition, we also found the power law relation between the average of relaxation time $\langle \tau \rangle$ and the lattice size L namely $\langle \tau \rangle \sim L^\beta$, with $\beta \approx 2.60$.

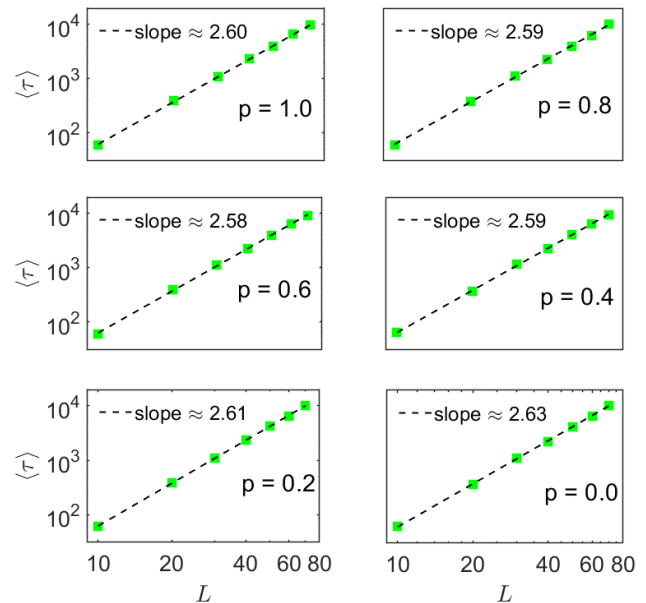


Fig. 7. The average of time relaxation versus lattice size L in log-log scale for typical values p with $d = 0.8$. The straight line is a fitting line that shows the slope between $\langle \tau \rangle$ with L which satisfies the power law relation $\langle \tau \rangle \sim L^\beta$, where $\beta \approx 2.60$.

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