The Effect of Permeability on the Squeeze Film Lubrication with a Couple Stress Fluid in Human Synovial Hip Joint

Ahmad Mawlood.Abdulhadi
Mathematics Department
College of Science, Baghdad University
Baghdad,Iraq

Albert Elya Yousif
Medical Engineering Department
College of Engineering, Al-Nahrian University
Baghdad,Iraq

Enas Yehea Abdullah
Mathematics Department
College of Science, Baghdad University
Baghdad,Iraq

***Abstract* - On the basis of the Stokes micro continuum fluid theory ,a theoretical analysis of effect of permeability and couple stress on squeeze film characteristics in human synovial hip joint is presented .To take into account the couple stress effects due to the lubricant containing additives or suspended particles ,the modified Reynold's equation governing the fluid film pressure was derived.The modified Reynold's equationis is solved analytically, and closed form expressions for the squeeze film pressure, and load carry capacity were presented. The influence couple stresses and permeability on the squeeze film characteristics were discussed. It has been found that the effect of couple stresses increased the pressure distribution , load carry capacity and squeeze film time, and decreased the friction coefficient, as compared to the Newtonian lubricant case . The effect of permeability on the healthy hip joint was found to increase the pressure distribution , ,load carry capacity , squeeze film time and decreased the coefficient of friction as compared to diseased**

Keywords – Permeability; Couple stress fluid; Articular cartilage; Synovial fluid, Micro continuum theory; Hip joint .

Nomenclature

W Load carrying capacity

W^* Dimensionless load carrying $W^* = \dfrac{W.h_o^2}{\mu.R^3(dh/dt)}$

p Film pressure

p^* Dimensionless film pressure

r, z Radial and axial coordinates

u, v Velocity component

t Time

t^* Dimensionless time , $h_0 t/\mu R^2$

F Friction force

F^* Dimensionless friction force

h Film thickness, $h_m + \dfrac{r^2}{2R}$

h_0 Film thickness at $t = 0$

hm Minimum film thickness

β The ratio of the microstructure size to the pore size

r^* Dimensionless radial coordinates, r/R

h^* Dimensionless film thickness, h/h_0

h_m^* Dimensionless minimum film thickness, h_m/h_0

l Characteristic length of the additives, $(\eta/\mu)^{1/2}$

I- INTRODUCTION

The natural process of lubrication of the human synovial joint is to lower friction and reduce wear . The hip joint is one of the most important joint in the human body. It allows us to walk, run, and jump. It bears our body' and the hip joint is also one of our most flexible joints and allows a greater range of motion than all other joints in the body .The squeeze film phenomenon is observed in several engineering application such as gears ,bearings ,machine tools, dampers and human joints [1] .It arises when two lubricating surface move towards each other in normal direction so pressures are generated and in normal circumstances surface contact does not occur until a long time has elapsed .Squeeze film lubrication is capable of carrying the heavy loads during the walking process even though the velocity was very low at this time [2].The hip joint is a spherical joint between the femoral head and acetabulum in the pelvis .It is a synovial joint ,since it is wrapped in a capsule that contains the synovial fluid ,a biological lubricant that acts also like a shock absorber . The hip bone is formed by three bones; ilium, ischium and pubis. At birth, these three component bones are separated by hyaline cartilage. They join each other in a Y- shaped portion of cartilage in the acetabulum .By the end of puberty the three bones will have grown together ,as shown in Fig.(1). The synovial fluid ,the inner lining of capsule ,the synovial member secretes a viscous non-Newtonian fluid called synovial fluid . It is believed

to be the dialysate of blood plasma with the addition of long chain hyaluronate molecules(hyaluronic acid). The thin film of synovial fluid that covers the surfaces of the inner layer of the joint capsule and articular cartilage helps to keep the joint surfaces lubricated and reduces friction [3].The fluid nourishment for the hyaline cartilage covering the articular surfaces ,as fluid moves in and out of the cartilage as compression is applied ,then released. The composition of synovial fluid also contains hyaluronic acid component of synovial fluid is responsible for the viscosity of the fluid and is essential for joint lubrication Hyaluronate reduces the friction between the synovial fluid in the capsule and the articular surface .Lubricin is the component of synovial fluid thought to be responsible for cartilage on cartilage lubrication [6] .Changes in the concentration of hyaluronate or lubricin in the synovial fluid will affect the overall lubrication and the amount of frication that is present. Many experiments have confirmed that articular coefficient of friction in synovial joints are far lower than created with manufactured lubricants. The lower the coefficient of friction is the lower is the resistance to movement .Normal synovial fluid appears as a clear, pale yellow fluid present in small amounts at all synovial joints. The synovial fluid ,like all viscous substances ,resists shear loads The viscosity of the fluid varies with the joint velocity or rate of shear ; that is ,it becomes less viscous at high rates.Thus ,synovial fluid is referred to as thixortropic when the bony components of a joint are moving .

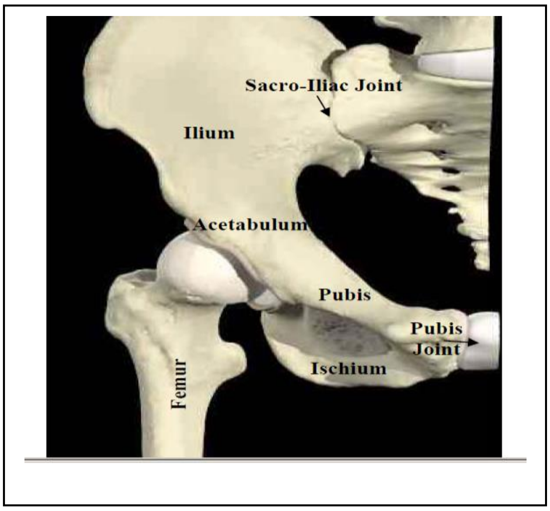


Fig.1 shows the nature human hip joint

2. ANALYSIS

On the basis of the Stockes [6] microcontinuum theory the continuity and Momentum equations of the flow filed with couples stresses are :

$$\nabla.\vec{V} = 0 \quad \text{...} (1)$$

$$\rho.\frac{D\vec{V}}{Dt} = -\nabla p + \mu\nabla^2\vec{V} - \eta\nabla^4\vec{V} \quad \text{.................................} (2)$$

Where $\vec{V}$ the fluid velocity vector ρ is the density ,p is the pressure, μ is the viscosity and η is the material constant responsible for the couple stress fluid property . In this theoretical study .Synovial fluid as the lubricant is blended with long chain polymers and can be considered as Stokes couple stress fluid .The fluid film is assumed to be thin ,and body force and body couples are assumed to be absent .Then the governing equations of the lubrication system in polar coordinates reduce to

$$\frac{\partial p}{\partial r} = \mu\frac{\partial^2 p}{\partial z^2} - \eta\frac{\partial^2 p}{\partial z^4} \quad \text{...} (3)$$

$$\frac{\partial p}{\partial z} = 0 \quad \text{...} (4)$$

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad \text{..} (5)$$

The ratio ($\frac{\eta}{\mu}$) is a dimensional square length and hence characterizes the chain length

$$l = \sqrt{\frac{\eta}{\mu}} \quad \text{..} (6)$$

where u and v are the velocity components in the r and z directions. The flow of cou-ple stress fluid in a porous matrix is given by the modified Darcy law, which accounts for the polar effects.

Where

$$\vec{q}^{\,*} = (u^*, w^*), \beta = \{(\eta/\mu)/\phi\} = \frac{l^2}{\phi}. \text{ and } \vec{q}^{\,*} = \frac{-\phi}{\mu(1-\beta)}\nabla p^* \quad \text{...........} (7)$$

ϕ is the permeability of the porous matrix . The parameter (β) represents the ratio of the microstructure size to the pore size. If $(\eta/\mu)^{1/2} \approx \sqrt{\phi}, \beta \approx 1$ then the microstructure additives present in the lubricant block the pores in the porous layer and the redues the Darcy flow through the porous matrix . When the microstructure size is very small compared to the pore size ,i.e. $\beta \ll 1$,the additives percolate into the porous matrix .In the limit as $\beta \to 0^+$,the bearing conditions tend to the case of Newtonian flow in the porous matrix [5]. The pressure p* in the porous region ,due to continuity ,satisfies the Laplace equation

Integrating equation (8) with respect to (z) and using the boundary condition of solid bearing ($z = H_\circ$ at $z = 0$) where $H_\circ$ is the porous layer thickness.

$$\frac{\partial^2 p*}{\partial r^2} + \frac{\partial^2 p*}{\partial z^2} = 0 \quad (8)$$

$$\frac{\partial^2 p^*}{\partial z^2} = -\int_{-H_\circ}^{0} \frac{\partial^2 p^*}{\partial r^2} dz \quad (9)$$

Equation (7) reduces to:-

$$\frac{\partial p^*}{\partial z} = H_\circ \frac{\partial^2 p^*}{\partial r^2} \quad (10)$$

$$w^* = \frac{\varphi H_\circ}{\mu(1-\beta)} \frac{\partial^2 p}{\partial r^2} \quad (11)$$

The relevant boundary conditions for the velocity compoonents are:

(i) at the porous plane surface z = 0

$$u(r,0) = \frac{\partial^2 u(r,0)}{\partial z^2} = 0, w(r,0) = 0 \quad (12)$$

(ii) at the sphere surface z = h

$$u(r,h) = \frac{\partial^2 u(r,h)}{\partial z^2} = 0, w(r,h) = \frac{\partial h}{\partial t} \quad (13)$$

The film thickness in the region r << R ,can be approximated by

$$h = h_m + \frac{r^2}{2R} \quad (14)$$

Where h_m denotes the minimum film thickness. Solving equation (3) with the above boundary condition one can obtain the expression for u as

$$u(r,z) = \frac{1}{2\mu} \frac{\partial p}{\partial r} \{z^2 - hz + 2l^2 [1 - \frac{\cosh(\frac{2z-h}{2l})}{\cosh(\frac{h}{2l})}]\} \quad (15)$$

Now integrating the continuity equation (5)with respect to y with the boundary conditions of u(r,z).Then the modified Reynolds equation governing the film pressure is derived as

$$12\mu \frac{\partial h}{\partial t} = \frac{\partial^2 p}{\partial r^2} \{\frac{12\phi H_\circ}{(1-\beta)} + f(h,l)\} \quad (16)$$

Where the function $f(h,l)$ is given:

$$f(h,l) = h^3 - 12l^2 h + 24l^3[\tanh(\frac{h}{2l})] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Introducing the dimensionless variables and parameters to get:

$$p^* = \frac{ph_o^2}{\mu R^2(\frac{\partial h}{\partial t})}, \; \mathrm{h}^* = \frac{h_m}{h_o}, \; \mathrm{r}^* = \frac{r}{R}, \; \mathrm{l}^* = \frac{\mathrm{l}}{h_o}, \; \beta = \frac{h_o}{R}, \; \phi^* = \frac{\phi}{\mathrm{h}_o{}^2} \ldots..(18)$$

$$-12\cos r^* = \frac{\partial}{\partial r^*}[\frac{12\phi^* H_\circ}{(1-\beta)} + f(h^*, l^*)\frac{\partial p^*}{\partial r^*}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Introducing the dimensionless Reynolds equation .is expressed as where

$$h^* = h_m^* + \frac{r^{*2}}{2\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The boundary conditions for the fluid film are:

$$p^* = 0 \text{ at } \mathrm{r}^* = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$$\frac{dp^*}{dr^*} = 0 \text{ at } \mathrm{r}^* = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Integrating the Reynolds equation with respect to r^* to the above conditions, the squeeze film pressure is obtained as:

$$p^* = \frac{-6(r^{2*} - 1)}{(h^{3*} - 12l^{2*}h^* + 24l^{3*}\tanh[\frac{h^*}{2l*}] + \frac{12\phi^* H^*}{(1-\beta)})} \ldots\ldots\ldots\ldots\ldots (23)$$

3. SQUEEZE FILM CHARACTERISTICS

The load carrying capacity is obtained by integrating the film pressure acting on the sphere

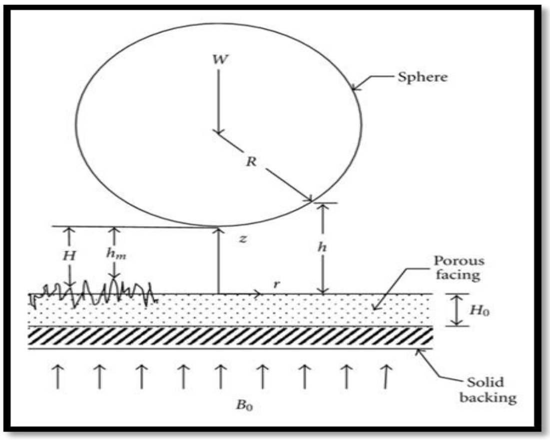


Fig.2 physical configuration of a sphere approaching a porous flat plate

$$W = 2\pi\int_0^R prdr \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

$$W^* = \frac{W.h._o{}^2}{\mu..R^3(dh/dt)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

$$W^* = 2\pi R\int_0^1 p^* r^* .dr^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

Although the value of the dimensionless load –carrying capacity in equation. (25)cannot to be calculated by direct integration ,it could be numerically evaluated by the method of power series then equation (26) becomes :-

$$W^* = \frac{6\pi R}{\left\{h^{3*} - 12l^{2*}h^* + 24l^{3*}\tanh\left[\frac{h^*}{2l^*}\right] + \frac{12\phi^* H^*}{(1-\beta)}\right\}} (27)$$

The time of approach can be obtained by integrating equation (27) with the initial condition , $h_m = (t = 0)$ to $h_\circ$. Performing the integration and expressing in a dimensionless form one can obtain the dimensionless time of approach .Let the response time be:

$$t^* = \frac{Wh^2 t}{\mu R^4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28)$$

Then the time- height relationship is obtained from the equation (25)

$$\frac{dh^*{}_m}{dt^*} = 6\pi R.\frac{1}{\int_0^1 f^*(h^*{}_m, r^*, \beta, l)dr^*} \ldots\ldots\ldots\ldots\ldots\ldots(29)$$

Dimensionless friction force $F^* = \frac{Fh}{\mu uR}$ therefore , the equation of friction force in a dimensionless form is [8]:-

$$F^* = \int_0^1 \left(\frac{1}{h^*} + \frac{h^*}{2}\frac{\partial p^*}{\partial r^*}\right)dr^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

Substituting for $\frac{\partial p^*}{\partial r^*}$ in the equation (23) And comparing with the result in equation (29) to get after an integration.

$$F^* = \frac{R}{h^*} - \frac{3Rh}{(h^{*3} - 12l^2h^* + 24l^3\tanh[\frac{h^*}{2l}] + \frac{12\phi^* H^*}{(1-\beta)})} \ldots\ldots.(31)$$

The non- dimensional coefficient of friction is given by

$$C_f = \frac{F^*}{W^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

Substituting for (F*) and (W*) from equation (30) and equation(27)

$$C_f = \frac{R}{6R\pi h^*} - \frac{3l^* h^*}{6R\pi h^*(h^{3*} - 12l^{2*}h^* + 24l^{3*}\tanh[\frac{h^*}{2l^*}]} \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

4. RESULTS AND DISCUSSION

In the present paper, the effect of permeability and couple stress on the squeeze film characteristics between a sphere approaching a flat plate is theoretically examined. The effect of couple stress on the performance of the sphere approaching a flat plate is observed with the aid of dimensionless parameters. The effect of permeability on the squeeze film characteristics is observed through the dimension parameters .The dimensionless ratio of $(\eta/\mu)^{1/2}$ may be identified as chain length of the polar additives in the lubricant . The numerical computation of all the results are performed, choosing the parametric values listed in table (1) and for various for the parameters (l, ϕ, h)

TABLE 1. Typical numerical values of the parameters involved [1,4,6]

Parameters	*Numerical values*	*Units*
Film thickness	1.5- 4	*(μm)*
Permeability of the cartilage (ϕ) healthy to diseased cartilage	6×10^{-17} 1.5×10^{-18}	*(m^2)*
Dimensionless couple stress length (l^*)	0.1-0.5	-------
Thickness layer of cartilage (H)	3-7	*(m)*
Radius of curvature	0.1-1	*(m)*
Viscosity of synovial fluid	$10^{-2} – 10^{-5}$	*(Pa.s)*

4.1. Squeeze film pressure

"Fig. 3" illustrates the dimensionless, film pressure (p*) generated by squeeze film action with dimensionless redius (r*)For different values of couple stress length Parameter l* .The solid line is for the a the Newtonian case (l^*= 0) .It is observed that couple stress effects are predominant for high values of l* .As the couple stress parameters l* decreases ,film pressure built up tends towards the Newtonian case (l^*= 0). For high values of l*, the hydrodynamic film pressure is substantiallyhigher. The percentage rate of increase in pressure distribution was approximately 90% at (r^* = 0, l^* = 0.6), while we fine the percentage rate of decrease in pressure distribution was approximately 15% at (r^* = 0, l^* =0). "Fig. 4" presents the variation of dimensionless film pressure (p^*) with dimensionless redius (r*) with different values of film thickness.It is observed increase film thickness effect on decrease the squeeze film pressure in cases,(squeeze lubrication and hydrodynamic lubrication) while it is found the effect of decrease film thickness "Fig.5" describes the variation of dimensionless film pressure (p^*) with dimensionless redius (r^*) with different values of permeability it was found value of permeability in healthy articular cartilage tend to increases in pressure.Comparing with casa disease arthrosis such decreases in film pressure.

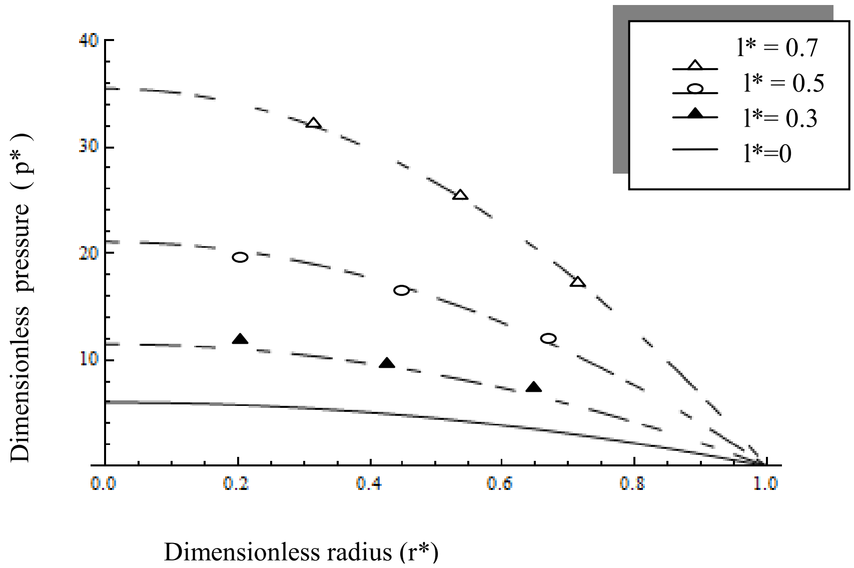


Fig. shows the variation of dimensionless pressure (p^*) with dimensionless radial(r^*) for different couple stress length parameters (l^*) ,(h^*= 1 and Φ=$10^{-18}$$m^2$)

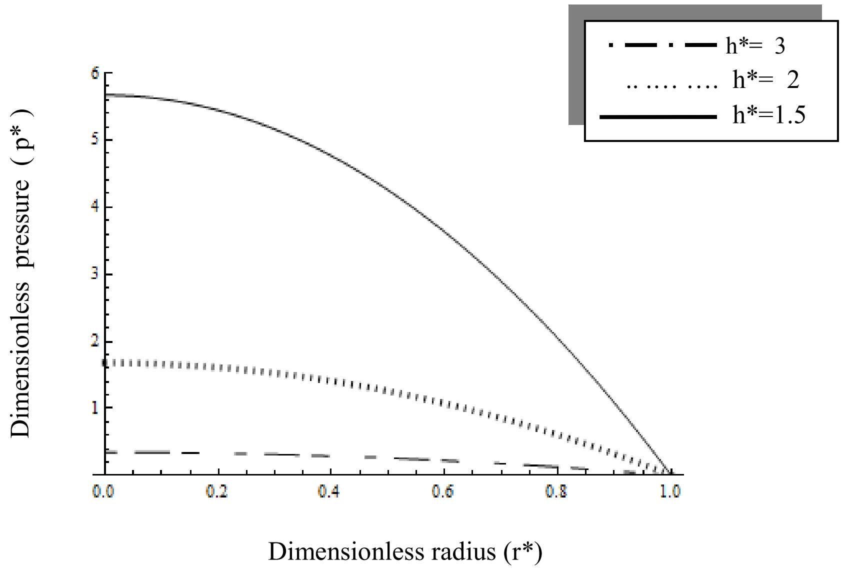


Fig. 4 shows the variation of(dimensionless pressure ,p) with (dimensionless radius , r^*) for different (film thickness parameters, h^*)

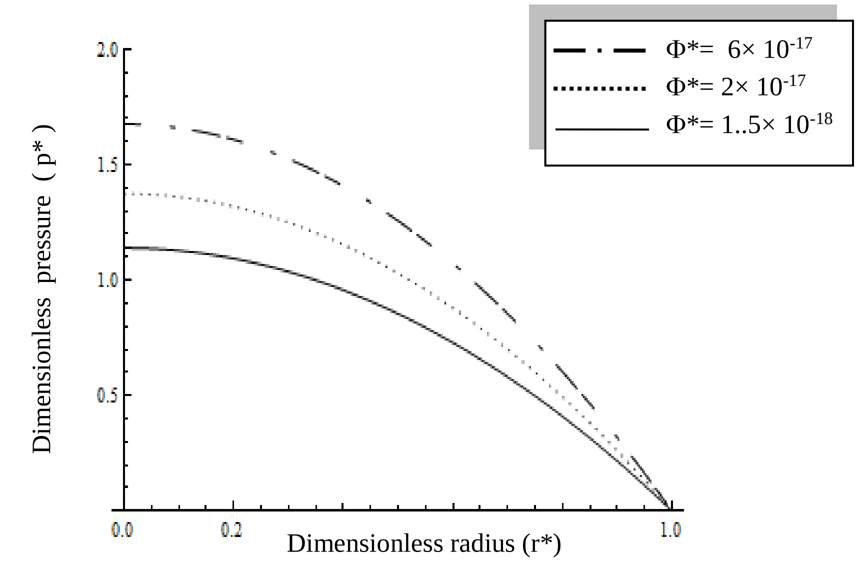


Fig.5 shows the variation of (dimensionless pressure ,p**) with (dimensionless radius (r*) for different (permeability dimensionless values of cartilage Φ*)

4.2 Load carrying capacity

The variation in dimensionless load capacity with dimensionless film thickness for different values of l^* and value of permeability parameter ($\phi^* = 1.5 \times 10^{-18}$) is depicted in "Figure .6",using equation (27) .It is observed that the increase in values of l^* increase *W* *as compared to the corresponding Newtonian case. The percentage rate of increase in load carry capacity is approximately 95.8% at case ($l^* = 0.7$, $h^*= 1$), as compared with Newtonian fluid ($l^* = 0$) approximately 25% . The variation in dimensionless load capacity with dimensionless film thickness for different values of permeability are depicted in "Figure .7"load carrying capacity increases for the decreasing values of permeability in healthy articular cartlage and decreas with decraes values of permeability in both cases(arthritis and ostoarthrosis) . From here it is clear the importance of the role of the permeability in bear increase in hip joint .The effect of permeability is sharply felt when the bearing operates at lower film thickness $h^* < 1.6$,At higher film thickness $h^* > 1.6$. The variances in di mensions, load capacity with dimensions couple stress for different values of film thickness are depicted in "Figure.8" , dimensionless load decrease with an increasing value film thickness (h^*) and thus decreasing in (*W* *) is more clearness for larger values of (h*) with fixed value of (l*)

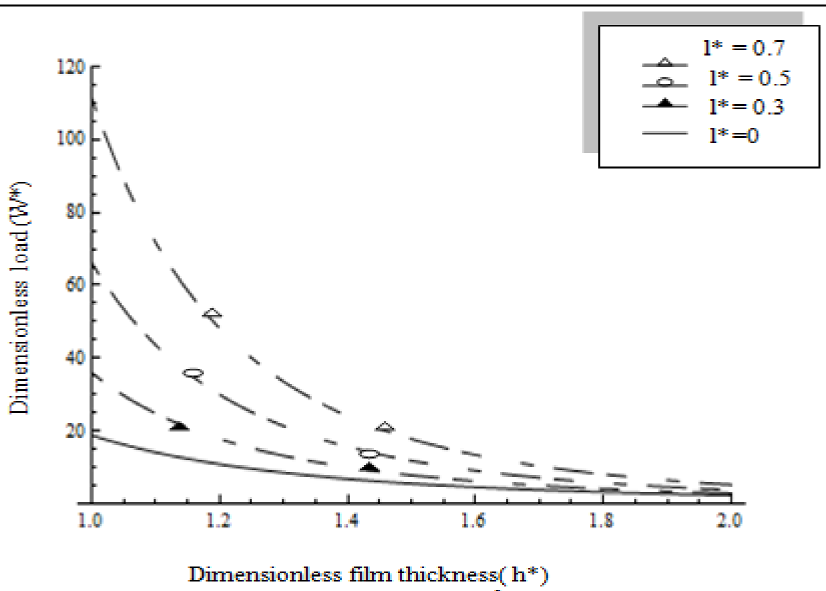


Fig.6 shows the variation of(dimensionless load ,W*) with (dimensionless film thickness , h*) for different(couple stress length parameters, l*)

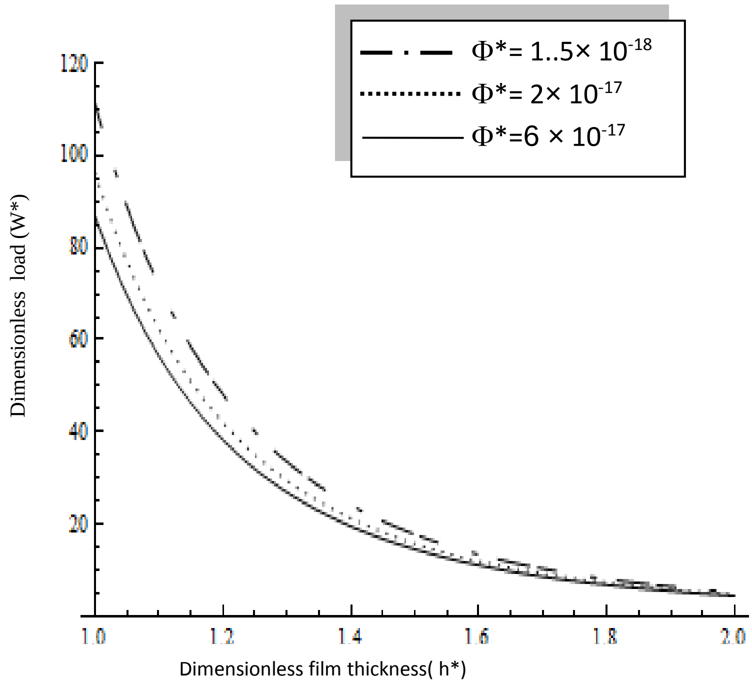


Fig.7 shows the variation of(dimensionless load carrying ,W^*) with (dimensionless film thickness ,h^*) for different(permeability parameters ,Φ^*)

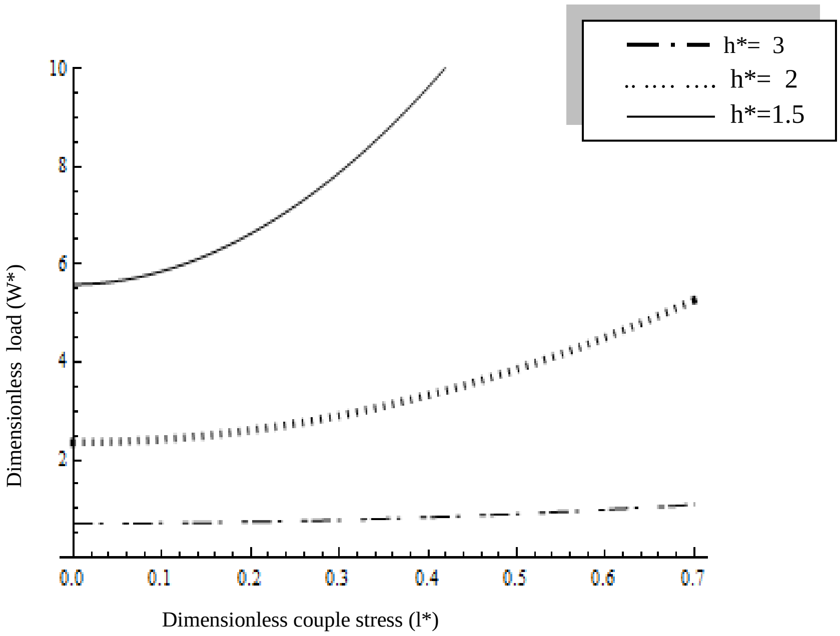


Fig.8 shows the variation of(dimensionless load ,W^*) with (dimensionless couple stress length ,l^*) for different(film thickness parameter h*)

4. 3 Squeeze Time-film thickness

The response time of the squeeze film is an important factor in describing squeeze film bearings. This is the time elapsed to reduce the initial film thickness to the minimum permissible squeeze film height. The variation of the (h_m) with the dimensionless response time (t *) for the different values of (l^*) is shown in "Fig.9" by solving equation (29) in computer program. It is seen that the presence of couple stresses provides an increase in the time of approach .These phenomena can be realized that since the couple stress effects yield a higher load- carrying capacity. The approaching time for the couple stress Fluid lubricant (l^* = 0.7) was about (91.6%), which was greater than the approaching time (58%) for the case of Newtonian lubricant (l^*=0). "Fig.10" depicts the variation of dimensionless response time (t*) with dimensionless minimum Film thickness for different values of film thickness. It is observed that film thickness has longer response time in case (elastohydrodynamic lubrication)compared to (hydrodynamic and squeeze lubrication), Fig. 11 depicts the variation of dimensionless for time as a function of dimensionless couple stress for different values of permeability It is observed that the effect of increasing permeability tend to decreasing time approach sphere to plate

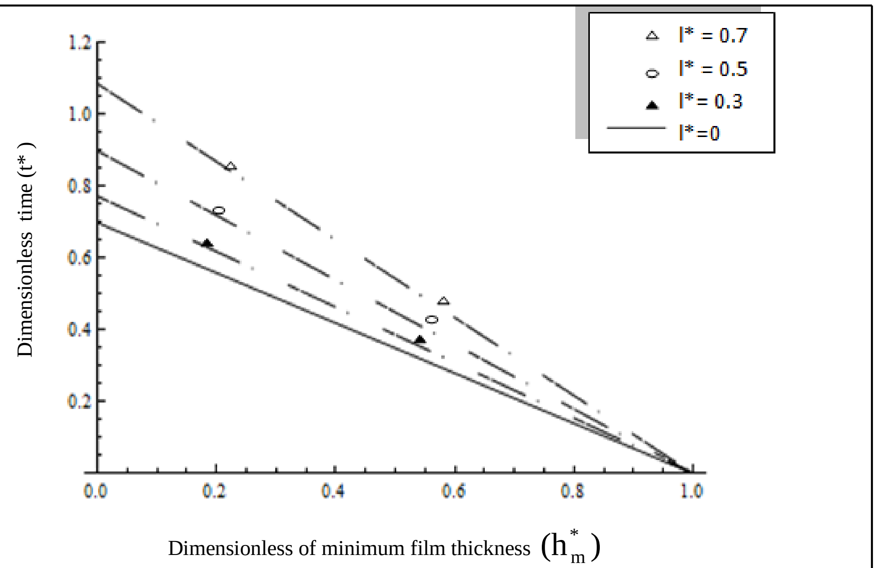


Fig.9 shows the variation of the(dimensionless squeeze time ,t *) with (dimensionless minimum film thickness ,h_m) for different couple stress length parameters,l*).

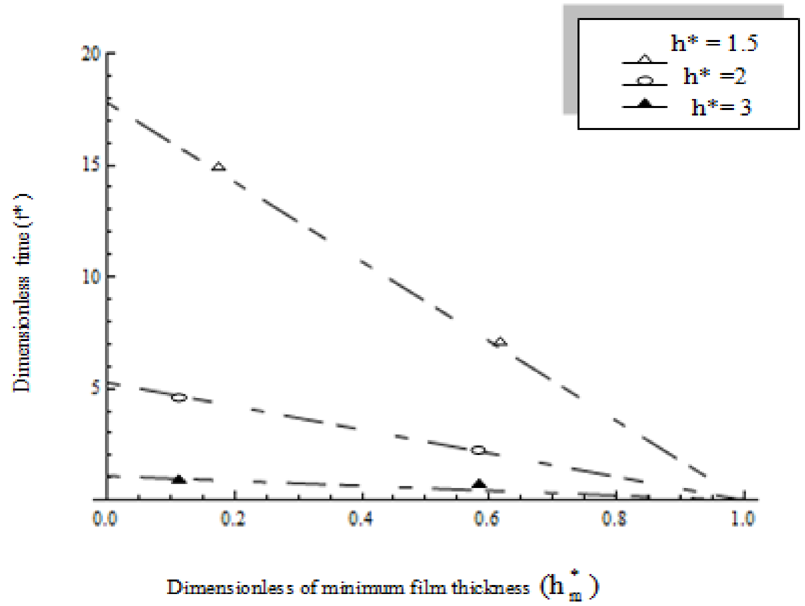


Fig. 10 shows the variation of the dimensionless squeeze time (t^*) with dimensionless minimum film thickness (h_m) for different film thickness parameters

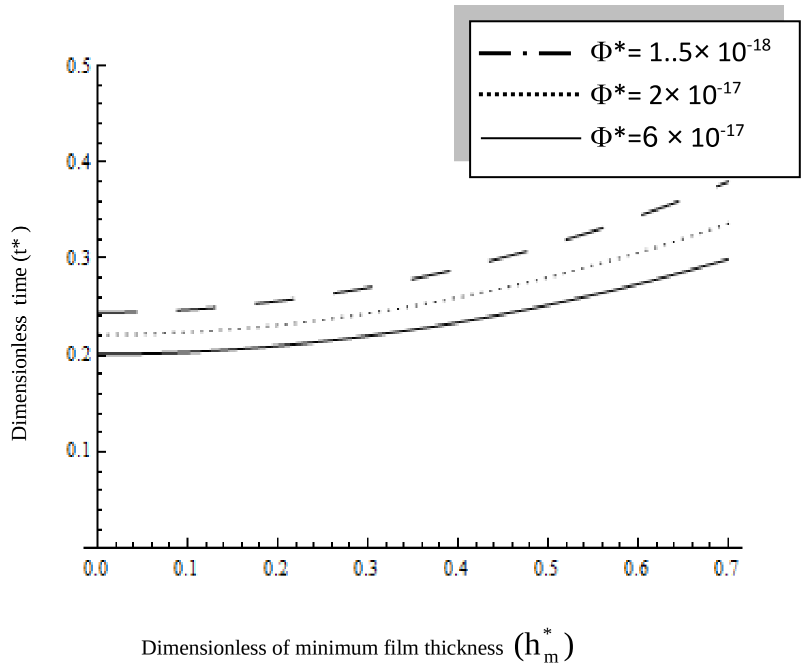


Fig. 11 shows the variation of the (dimensionless squeeze time, t^*) with dimensionless minimum film thickness ,h_m) for different film(thickness parameters,h*)

Coefficient of Friction

A decrease in coefficient of friction of the bearing was when the bearing lubricated with couple stress lubricant rather than that lubricated with Newtonian lubricant as shown in "Figure.12"after execute equation (32) in (Wolfram Mathematic 9), the all figures use ($\phi^*= 10^{-18}$) and ($h^*= 3$) . The percentage reduction rate in coefficient of friction at ($l^*= 0.7$) for couple stress fluid is approximately 57% as compared with that Newtonian lubricant 92.8% . The ariation in the coefficient of friction with film thickness for different values of permeability is depicted in "Figure. 13"When articular cartlige is healthy then vlue of permeability is higher of which to perform decreases in coefficient of friction ,invearsly in case diseased hip joint.It can be realized that since the permeability effects yield a higher pressure film and load carrying capacity .. The variation in coefficient of friction with couple stress for different values of film thickness is depicted in "Figure.14" coefficient of friction between two articular cartlige increases with decreases values of the film thickness.

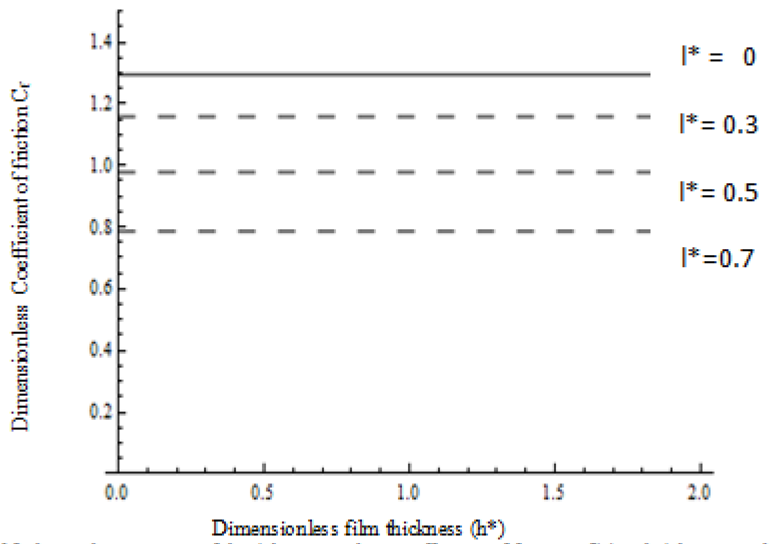


Fig.12 shows the variation of the (dimensionless coefficient of friction, C_f)with (dimensionless film thickness,h^*)for different(couple stress length, l^*)

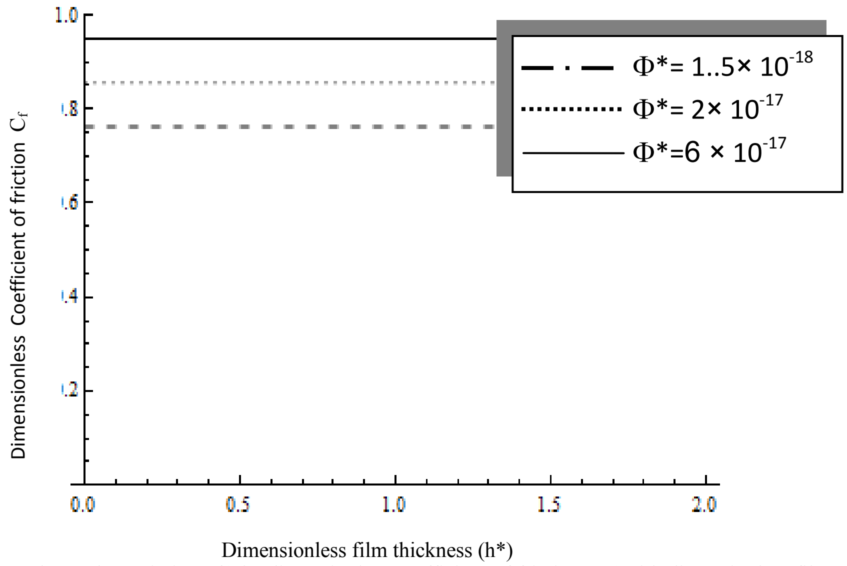
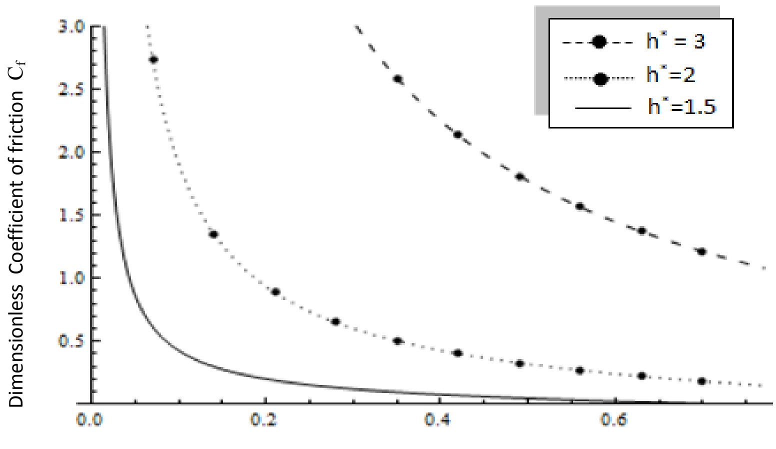


Fig.13shows the variation of(the dimensionless coefficient of friction, C_f)with dimensionless film thiclness, h^*)for different (permeability parameters,ϕ^*)



Dimensionless couple stress (l*)

Fig.14 shows the variation of the dimensionless coefficient of friction ,C_f) with couple stree (l*) for different film thickness.

5.CONCLUSIONS

The effects of couple stresses on the squeeze film between a sphere and a permeable flat plate are presented on the basis of Stockes micro continuum theory . The modified Reynolds equation, governing the squeeze film pressure is derived using the Stockes constitutive equation and is solving numerically using (Wolfram Mathematic 9) .According to the results obtained the following conclusions :

(1) The effect of couple stress is increase the film pressure ,load carrying capacity and time in side and decrease in cofficent of friction in other side significantly as compared to the Newtonian case.

(2) The effect of Permeability prmeters causes increase the film pressure ,load carrying capacity and time and decrease in coefficient of friction in healthy joint

(3) The effect of film thickness prmeters causes reduction in film pressure , load carrying capacity , time and coefficient of friction in squeeze lubrication ,and increase in film pressure ,load carrying capacity , time and coefficient of fric tion in elastohydrodynamic lubrication .

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