The Cycle non Split Domination Number of Fuzzy Graphs

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Abstract – A dominating set D of a fuzzy graph G=(σ,µ) is a cycle non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',µ') is a cycle. The cycle non split domination number γcD(G) of G is the minimum fuzzy cardinality of a cycle non split dominating set. In this paper we study a cycle non split dominating sets of fuzzy graphs and investigate the relationship of γcD(G) with other known parameters of G.

Keywords – Fuzzy graphs, Fuzzy domination, Split fuzzy domination number, Non Split fuzzy domination number, cycle non split domination number.

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I. INTRODUCTION

Kulli V.R. et al. introduced the concept of split domination and non-split domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles, and connectedness [10]. A. Somasundram and S. Somasundram discussed domination in fuzzy graphs [11]. Mahyoub Q.M. and Sonar N.D. discussed the split domination number of fuzzy graphs [6]. Ponnappan C.Y and et. al. discussed the strong non split domination number of fuzzy graphs [9]. In this paper we discuss the cycle non split domination number of fuzzy graph and obtained the relationship with other known parameters of G.

II. PRELIMINARIES

Definition: 2.1 [2]
Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by γ(G).

Definition: 2.2 [3]
A dominating set D of a graph G=(V,E) is a split dominating set if the induced subgraph <V-D> is disconnected. The split domination number γs(G) of a graph G is the minimum cardinality of a split dominating set.

Definition: 2.3 [3]
A dominating set D of a graph G=(V,E) is a cycle non split dominating set if the induced subgraph <V-D> is a cycle. The cycle non split domination number γc(G) of G is the minimum fuzzy cardinality of a cycle non split dominating set. In this paper we study a cycle non split dominating sets of fuzzy graphs and investigate the relationship of γc(G) with other known parameters of G.

Definition: 2.4 [4]
A dominating set D of a graph G=(V,E) is a cycle non split dominating set if the induced subgraph <V-D> is a cycle. The cycle non split domination number γc(G) of a graph G is the minimum cardinality of a cycle non split dominating set.

Definition: 2.5 [4]
A dominating set D of a graph G=(V,E) is a path in non split dominating set if the induced subgraph <V-D> is a path. The path non split domination number γp(G) of a graph G is the minimum cardinality of a path non split dominating set.

Definition: 2.6 [10]
Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=(σ,µ) is a set with two functions σ :V→[0,1] and µ : E→[0,1] such that µ(σ(u,v))≤σ(u)σ(v) for all u,v∈V.

Definition: 2.7 [11]
Let G=(σ,µ) be a fuzzy graph on V and V⊆V. Define σ1 on V1 by σ1(u)=σ(u) for all u∈V1 and µ1 on the collection E1 of two element subsets of V1 by µ1(u,v)=µ(u,v) for all u,v∈V1. Then (σ1,µ1) is called the fuzzy subgraph of G induced by V1 and is denoted by <V1>.

Definition: 2.8 [11]
The fuzzy subgraph H=(V1,σ1,µ1) is said to be a spanning fuzzy subgraph of G=(V,σ,µ) if σ1(u)=σ(u) for all u∈V1 and µ1(u,v)≤µ(u,v) for all u,v∈V. Let G=(V,σ,µ) be a fuzzy graph and µ1 be any fuzzy subset of µ, i.e., σ1(u)≤σ(u) for all u.
Definition : 2.9 [11]
Let \( G=(\sigma,\mu) \) be a fuzzy graph on \( V \). Let \( u,v \in V \). We say that \( u \) dominates \( v \) in \( G \) if \( \mu(u,v) = \sigma(u) \wedge \sigma(v) \). A subset \( D \) of \( V \) is called a dominating set in \( G \) if for every \( v \in D \), there exists \( u \in D \) such that \( u \) dominates \( v \). The minimum fuzzy cardinality of a dominating set in \( G \) is called the domination number of \( G \) and is denoted by \( \gamma(G) \) or \( \gamma \).

Definition : 2.10 [6]
A dominating set \( D \) of a fuzzy graph \( G=(\sigma,\mu) \) is a split dominating set if the induced fuzzy subgraph \( H=(V - D, \sigma', \mu') \) is disconnected.

The split domination number \( \gamma_s(G) \) of \( G \) is the minimum fuzzy cardinality of a split dominating set.

Definition : 2.11 [6]
A dominating set \( D \) of a fuzzy graph \( G=(\sigma,\mu) \) is a non split dominating set if the induced fuzzy subgraph \( H=(V - D, \sigma', \mu') \) is connected.

The non split domination number \( \gamma_{ns}(G) \) of \( G \) is the minimum fuzzy cardinality of a non split dominating set.

Definition : 2.12
A dominating set \( D \) of a fuzzy graph \( G=(\sigma,\mu) \) is a cycle non split dominating set if the induced fuzzy subgraph \( H=(V - D, \sigma', \mu') \) is a cycle.

The cycle non split domination number \( \gamma_{cns}(G) \) is the minimum fuzzy cardinality of a cycle non split dominating set.

Definition : 2.13
A dominating set \( D \) of a fuzzy graph \( G=(\sigma,\mu) \) is a path non split dominating set if the induced fuzzy subgraph \( H=(V - D, \sigma', \mu') \) is a path.

The path non split domination number \( \gamma_{pns}(G) \) is the minimum fuzzy cardinality of a path non split dominating set.

The order \( p \) and size \( q \) of a fuzzy graph \( G=(\sigma,\mu) \) are defined to be \( p = \sum_{u \in V} \sigma(u) \) and \( q = \sum_{u \in V} \mu(u,v) \).

Definition : 2.15 [11]
An edge \( e = \{u,v\} \) of a fuzzy graph is called an effective edge if \( \mu(u,v) = \sigma(u) \wedge \sigma(v) \).

The effective degree of a vertex \( u \) is defined to be the sum of the weights of the effective edges incident at \( u \) and is denoted by \( \varepsilon E(u) \). \( \sum_{e \in E(u)} \sigma(v) \) is called the neighborhood degree of \( u \) and is denoted by \( dN(u) \). The minimum effective degree \( \delta_{E}(G) = \min \{ dE(u)|u \in V(G) \} \) and the maximum effective degree \( \Delta_{E}(G) = \max \{ dE(u)|u \in V(G) \} \).

Definition : 2.16 [11]
The complement of a fuzzy graph \( G \) denoted by \( \bar{G} \) is defined to be \( \bar{G} = (\sigma, \overline{\mu}) \) where \( \overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v) \).

Definition : 2.17 [11]
Let \( \sigma: V \rightarrow [0,1] \) be a fuzzy subset of \( V \). Then the complete fuzzy graph on \( \sigma \) is defined to be \( (\sigma,\mu) \) where \( \mu(u,v) = \sigma(u) \wedge \sigma(v) \) for all \( u \in E \) and is denoted by \( K_{\sigma} \).

Definition : 2.18 [11]
A fuzzy graph \( G=(\sigma,\mu) \) is said to be bipartite if the vertex \( V \) can be partitioned into two nonempty sets \( V_1 \) and \( V_2 \) such that \( \mu(v_1,v_2)=0 \) if \( v_1 \in V_1 \) and \( v_2 \in V_2 \). Further if \( \mu(u,v)=\sigma(u) \wedge \sigma(v) \) for all \( u \in V_1 \) and \( v \in V_2 \) then \( G \) is called a complete bipartite graph and is denoted by \( K_{\sigma_1,\sigma_2} \) where \( \sigma_1 \) and \( \sigma_2 \) are, respectively, the restrictions of \( \sigma \) to \( V_1 \) and \( V_2 \).

Definition : 2.19 [11]
A dominating set \( D \) of a fuzzy graph \( G \) is said to be a minimal dominating if no proper subset \( D' \) of \( D \) is dominating set of \( G \) such that \( |D'|<|D| \).

III. MAIN RESULTS

Proposition : 1
For any complete fuzzy graph \( K_{\sigma} \) then
\[ \gamma(G) = \gamma_{cns}(G) = \min \{ \sigma(u)| u \in V \} \]

Proposition : 2
For fuzzy bipartite graph \( K_{\sigma_1,\sigma_2} \),
\[ \gamma_{cns}(K_{\sigma_1,\sigma_2}) = \min \{ \sigma(u) + \min \{ \sigma(v) | u \in V_1 \} \} \]

Proposition : 3
For fuzzy wheel \( \gamma_{cns}(G)=\sigma(u) \) such that \( u \) is the spoke of the wheel.

Proposition : 4
\[ \gamma_{cns}(G \circ K_1) = \sum_{i} \sigma(u_i) \] and \( u_i \) is the pendant vertices of the corona and \( G \) contains at least one cycle.

Proposition : 5
The cycle non split dominating set exists for Petersen graph and Davidson graph.

Note :
The cycle non split dominating set does not exist for path, tree and fan.

Theorem : 1
For any fuzzy graph \( G=(\sigma,\mu) \), \( \gamma(G) \leq \gamma_{cns}(G) \leq \sigma \)

Proof
Let \( G=(\sigma,\mu) \) be a fuzzy graph. Let \( D \) be the minimum dominating set. \( D_{cns} \) is the fuzzy cycle non split dominating set. \( D_{cns} \) is also a dominating set but need not be a minimum fuzzy dominating set.

Therefore we get \( |D| \leq |D_{cns}| \) That is \( \gamma(G) \leq \gamma_{cns}(G) \).
Example : Fig. (i)

\[ D = \{ u_1, u_3 \} \]
\[ \gamma(G) = 0.6 \]
\[ D_{\text{cns}} = \{ u_1, u_2 \}, \gamma_{\text{cns}}(G) = 0.7 \]

**Theorem 1.1**

\[ \gamma(G) \leq \gamma_{\text{pns}}(G) < p. \]

**Proof**

Let \( G = (\sigma, \mu) \) be a fuzzy graph. \( D \) be the minimum fuzzy dominating set. Let \( D_s \) and \( D_{\text{cns}} \) be the minimum fuzzy split dominating set and minimum fuzzy cycle non-split dominating set of \( G \) respectively. The cardinality of fuzzy dominating set need not exceed either one of the minimum of cardinality of fuzzy split dominating set or fuzzy cycle non split dominating set.

Therefore \( |D| \leq \min \{|D_s|, |D_{\text{cns}}|\} \)

Hence \( \gamma(G) \leq \min \{\gamma_s(G), \gamma_{\text{cns}}(G)\} \)

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**Example :** Fig. (ii)

\[ u_1(0.5), u_2(0.4), u_3(0.3), u_4(0.6), u_5(0.3), u_6(0.2) \]

\[ D = \{ u_1, u_3 \} \]
\[ \gamma(G) = 0.6 \]
\[ \gamma_s(G) = 0.3, \gamma_{\text{cns}}(G) = 1.3 \]

**Theorem 2.1** \( \gamma(G) \leq \min \{\gamma_s(G), \gamma_{\text{pns}}(G)\} \)

**Proof**

Let \( G = (\sigma, \mu) \) be a fuzzy graph and \( H = (\sigma', \mu') \) be the fuzzy spanning sub graph of \( G \). \( D_{\text{cns}}(G) \) be the fuzzy minimum cycle non-split dominating set of \( G \). \( D_{\text{cns}}(H) \) is fuzzy cycle non-split dominating set of \( H \) but not minimum.

Therefore \( \gamma_{\text{cns}}(H) \geq \gamma_{\text{cns}}(G) \).

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**Example**:

Spanning fuzzy sub graph \( H \) of \( G \) (Fig (ii))

\[ u_1(0.5), u_2(0.4), u_3(0.3), u_4(0.6), u_5(0.3), u_6(0.2) \]

\[ D_{\text{cns}}(G) = 0.7, \gamma_{\text{cns}}(H) = 1.0 \]

**Theorem 3.1** \( \gamma_{\text{pns}}(H) \geq \gamma_{\text{pns}}(G) \).

**Proof**

Let \( G = (\sigma, \mu) \) be a complete fuzzy graph then

\( \gamma_{\text{cns}}(G) = \min \{\sigma(u)\} \), where \( u \) is the vertex having minimum cardinality.

Let \( G_i \) is subgraph of \( G \) induced by \( \langle V \rangle \) where \( u \) is the vertex of minimum cardinality, \( G_i \) has a vertex set \( V_i = \{V - u\} \)

\( \gamma_{\text{cns}}(G) \leq \gamma_{\text{cns}}(G_1) \leq \gamma_{\text{cns}}(G_2) \leq \ldots \leq \gamma_{\text{cns}}(G_n) \) provided the fuzzy graph \( G_n \) is a elementary cycle with three vertices.

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**Example** : Fig. (iii)

\[ u_1(0.1), u_2(0.2), u_3(0.3), u_4(0.4), u_5(0.1), u_6(0.2) \]

\[ \gamma(G) = \gamma_{\text{cns}}(G) = 0.1 \]

\( G \) is a fuzzy graph induced by \( \langle V \rangle \)

\( \gamma_{\text{cns}}(G_1) = \sigma(u_2) = 0.2 \)

\( \gamma_{\text{cns}}(G) \leq \gamma_{\text{cns}}(G_1) \).

**Theorem 5**

For any fuzzy graph without isolated vertices

\( \gamma_{\text{cns}}(G) \leq p/2. \)
Theorem 8.1. : $\gamma_{\text{cns}}$ set satisfies ore’s theorem.

Theorem 9 : For the domination number $\gamma_{\text{cns}}$, the following theorem gives a Nordhaus–Gaddum type result.

For any fuzzy graph $G$, $\gamma_{\text{cns}}(G) + \gamma_{\text{cns}}(\overline{G}) \leq 2p$.

Proof : Let $G$ be a connected fuzzy graph it may or may not contains a cycle.

Suppose $G$ contains a cycle then by theorem $\gamma_{\text{cns}}(G) \leq p$.

Also $\overline{G}$ may or may not contains a cycle. We have $\gamma_{\text{cns}}(\overline{G}) \leq p$ or $\gamma_{\text{cns}}(\overline{G}) = 0$

Vice versa. Hence the inequality is trivial.

Theorem 9.1 : $\gamma_{\text{cns}}(G) + \gamma_{\text{cns}}(\overline{G}) \leq 2p$.

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