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The Coloring of Hyper Graph

(1)A. Ramesh Kumar,
1. Head,
Department of Mathematics,
Srimad Andavan Arts & Science College,
Trichy-05.

(2) G. Kavitha
2. Asst. Professor,
Kongunadu College of Engineering and Technology,
Tholurpatti, Thottiam

***Abstract*: In this paper, we are introducing the Hardness of Hyper-Graph coloring and uniform coloring of the vertices and Kneser Graph $KG_{n,s}$**

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1. INTRODUCTION

A hypergraph H is an ordered pair $H=(V,E)$, where V is a finite nonempty set and E is a collection of distinct nonempty subsets of V. H has dimension r if all edges have at most r vertices. If all edges have size exactly r, H is called r-uniform. Thus, a 2-uniform hypergraph is just a graph. A set $U \subseteq V(H)$ is called independent if U spans no edges of H. The maximal size of an independent se tin H is called the independence number of H and is denoted by $\alpha(H)$. A k-coloring of H is a mapping $c:V(H)\rightarrow\{1,......k\}$ such that no edge of H has all vertices of the same color. Equivalently, a k-coloring of H is a partition of the vertex set $V(H)$ into k independent sets. The chromatic number of H, denoted by $\chi(H)$ is the minimal k, for which H admits a k-coloring

In this paper we consider an algorithmic problem of coloring r-uniform hypergraphs, for given and fixed value of $r \geq 2$. The special case $r=2$ is relatively well studied and many results have been obtained in both positive and negative directions. .

NP-hard to determine whether a 3-uniform hypergraph is 2-colorable. Additional results on complexity of hypergraph coloring[1]. These hardness results give rise to attempts of developing algorithms for approximate uniform hypergraph coloring, aiming to use a small but possibly nonoptimal number of colors. The first nontrivial case of approximately coloring 2-colorable hypergraphs. Both papers arrived independently to practically identical results. They presented an algorithm for coloring a 2-colorable r-uniform hypergraph in $O(n^{1-1/r})$ colors, using an idea closely related to the basic idea [3] . Another result of the above mentioned two papers is an algorithm for coloring 3-uniform 2-colorable hypergraphs in $\widetilde{O}(n^{2/9})$ colors. The latter algorithm exploits the semidefinite programming approach, much in the spirit of coloring algorithm[3] . Not much is known about general approximate coloring algorithms for r-uniform hypergraphs (that is, when the chromatic number of a hypergraph is not given in advance). Recently it presented an approximation algorithm for this problem with performance ratio $O(n/\log^{(r-1)}n)^2)$, where $\log^{(r)}n$ denotes the r-fold iterated logarithm.

In Section 2 we describe a construction which enables to derive immediately results on hardness of approximating the chromatic number of r-uniform hypergraphs for any $r \geq 3$ from the corresponding graph results. Thus we get that unless $NP \subseteq ZPP$, for any fixed $r \geq 3$, it is impossible to approximate the chromatic number of r-uniform hypergraphs on n vertices in polynomial time within a factor of $n^{1-\varepsilon}$, for any $\varepsilon > 0$. It should be noted that Hofmeister and Lefmann obtained independently the same hardness result[7].

In Section 3 we present an approximation algorithm for coloring r-uniform hypergraphs on n vertices, whose performance guarantee is $O(n(\log\log n)^2/(\log n)^2)$, thus matching the approximation ratio of Wigderson's algorithm[3].

2. HARDNESSOF APPROXIMATION

Several results on hardness of calculating exactly the chromatic number of r-uniform hypergraphs have been known previously. It showed that it is NP-complete to decide whether a given 3-uniform hypergraph H is 2-colorable]. that it is NP-complete to decide k-colorability of r-uniform hypergraphs for all k, $r \geq 3$, even when restricted to linear hypergraphs[2]. A polynomial transformation from k-chromatic graphs to k-chromatic r-uniform hypergraphs. Finally, it showed in [4] that, unless $P=NP$, it is impossible to decide in polynomial time 2-colorability of r-uniform hypergraphs for any $r \geq 3$.

However, until recently, there has been no result showing that it is also hard to approximate the chromatic number of r-uniform hypergraphs, where $r \geq 3$. For the graph case (r = 2) in[4] , using the result [6] that if NP does not have efficient randomized algorithms, then there is no polynomial time algorithm for approximating the chromatic number of an n vertex graph within a factor of $n^{1-\varepsilon}$, for any fixed $\varepsilon > 0$. In this section we present a construction for reducing the approximate graph coloring problem to approximate coloring of r -uniform hypergraphs[5], for any $r \geq 3$. Let $r \geq 3$ be a fixed uniformity number. Suppose we are given a graph $G = (V, E)$ on $|V| = n \geq r$ vertices with chromatic number $\chi(G) = k$. Define an r-uniform hypergraph $H = (V, F)$ in the following way. The vertex set of H is identical to that of G. For every edge $e \in E$ and for every $(r-2)$ subset $V_0 \subseteq V \setminus e$ we include the edge $e \cup V_0$ in the edge set F of H. If F (H) contains multiple edges, we leave only one copy of each edge. The obtained hypergraph H is r -uniform on n vertices. Now we claim that $k/(r-1) \leq \chi(H) \leq k$. Indeed, a k -coloring of G is also a k-coloring of H, implying the upper bound on χ(H). To prove the lower bound, let $f : V \to \{1, \ldots\ldots k\}$ be a k'-coloring of H. Let G_0 be a subgraph of G, whose vertex set is V and whose edge set is composed of all these edges of G that are monochromatic under f . It is easy to see that the degree of every vertex v ∈ V in G_0 is at most $(r-2)$. Thus G_0 is $(r-1)$-colorable. We infer that the edge set $E(G)$ of G can be partitioned into two subsets $E(G) \setminus E(G_0)$ and $E(G_0)$ such that the first subset forms a k' -colorable graph, while the second one is $(r-1)$-colorable. Then G is $k'(r-1)$ -colorable, as we can label each vertex

by a pair whose first coordinate is its color in a k' -coloration of the first subgraph, and the second coordinate comes from an $(r-1)$-coloration of the second subgraph. Therefore G and H as defined above have the same number of vertices, and their chromatic numbers have the same order. Applying now the result[4], we get the following theorem.

Theorem 2.1. Let r 3 be fixed. If $NP \subseteq ZPP$, it is impossible to approximate the chromatic number of r-uniform hypergraphs on n vertices within a factor of $n^{1-\varepsilon}$ for any fixed $\varepsilon > 0$ in time polynomial in n.

We prove that coloring a 3-uniform 2-colorable hypergraph with c colors is NP -hard for any constant c. The best known algorithm [8] colors such a graph using $O(n^{1/5})$ colors. t

This is the first hardness result for approximately-coloring a 3-uniform hypergraph that is colorable with a constant number of colors. For $k \geq 4$ such a result has been shown by [14], who also discussed the inherent difference between the $k = 3$ case and $k \geq 4$.

Our proof presents a new connection between the Long-Code and the Kneser graph, and relies on the high chromatic numbers of the Kneser graph[10] and the Schrijver graph[9]. We prove a certain maximization variant of the Kneser conjecture, namely that any coloring of the Kneser graph by fewer colors than its chromatic number, has ‘many’ non-monochromatic edges.

A hypergraph $H = (V, E)$ with vertices V and edges $E \subseteq 2V$ is 3-uniform if every edge in E has exactly 3 vertices. A legal χ-coloring of a hypergraph H is a function $f : V \to [\chi]$ such that no edge of H is monochromatic. The chromatic number of H is the minimal χ for which such a coloring exists.

The best algorithms for these problems require a polynomial number of colors: for example the best approximate coloring algorithm for 2-colorable 3-uniform hypergraphs requires $O(n^{1/5})$ colors [8], and the best coloring algorithm for 3-colorable graphs, requires $\tilde{O}(n^{3/14})$ colors .

3. UNIFORM HYPERGRAPH COLORING

Definition 3.1

3-uniform: if each edge contains exactly 3 vertices, |e|=3.

Definition 3.2

2 Colorable, or has property B:

if there exists a red-blue coloring of the vertices, with no monochromatic hyper-edge

Theorem 3.3

Given a 3- uniform hypergraph, deciding whether $\chi = 2$ or $\chi \geq c$ is NP-hard,

Proof

The property of being 2-colorable is well studied in combinatorics and is also referred to as ‘property B’. Nevertheless, prior to this work no hardness of approximation result was known for 3-uniform hypergraphs, and in fact it wasn’t even known if it is NP -hard to color a 3-uniform 2-colorable hypergraph with 3 colors. For 4-uniform hypergraphs and upwards] were able to show such a separation, i.e., they showed that it is NP-hard to color a 2-colorable 4-uniform hypergraph with any constant number of colors . In their work, an inherent difference between the case $k \geq 4$ and the case $k = 2,3$ was raised; this was considered evidence that the case $k = 3$ might be harder to understand. This difference has to do with the corresponding maximization problem called ‘Set-Splitting’, which is the problem of 2-coloring a k-uniform hypergraph while maximizing the number of non-monochromatic hyperedges.

Corollary

For any constants $k \geq 3$ and $c_2 > c_1 > 1$, coloring a k-uniform c_1-colorable hypergraph with c_2 colors is NP-hard; the case $k = 2$, however, remains wide open.

4. KNESAR GRAPH

In this section we define the Kneser graph and describe some of its important properties. For $n \geq 2s+1$, the Kneser graph $KG_{n,s}$ has the set

$$\binom{[n]}{s} \stackrel{def}{=} \{S \subseteq [n] \| S| = s\}$$

two vertices S_1, S_2 are adjacent iff $S_1 \cap S_2 = \phi$. In other words, each vertex corresponds to an s-set and two vertices are adjacent if their corresponding sets are disjoint. In this paper we are mainly interested in the case where s is smaller than n/2 by a constant. These graphs have the important property that the chromatic number is high although large independent sets exist. For a discussion of Kneser graphs and other combinatorial problems.

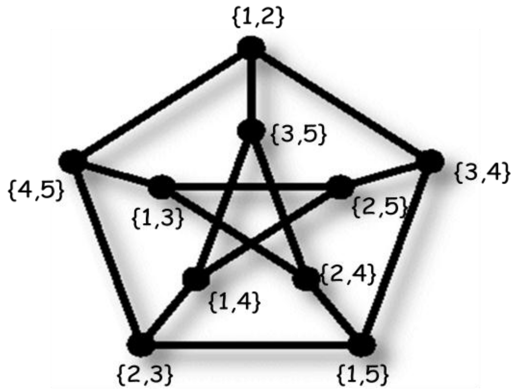
There exists a simple way to color this graph with $n-2s+2$ coloring. It conjectured that there is no way to color the graph with less colors, i.e., $\chi(KG_{n,s}) = n-2s+2$. Many other proofs and extensions are known and the latest and simplest one. Our goal in this section is to prove the following property of the Kneser graph: in any coloring of the vertices of the Kneser graph with less colors than its chromatic number, there must exist many monochromatic edges. The way we prove this is by considering certain induced subgraphs of the Kneser graph known as Schrijver graphs. The proof follows from the fact that these induced subgraphs have the same chromatic number as the whole Kneser graph.

Definition 4.1

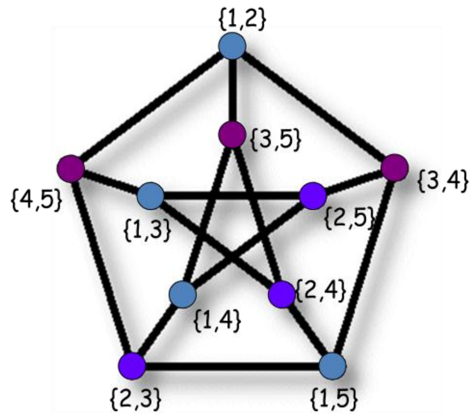
The Kneser Graph $KG_{n,s}$ is the graph whose vertices correspond to the k-element subsets of a set of s elements, and where two vertices are adjacent if and only if the two corresponding sets are disjoint. The chromatic number of the Kneser graph $KG_{n,s}$ is exactly

$$n-2s+2$$

Example



The above Kneser Graph of $KG_{5,2}$ in Petersen graph with 5 vertices and 2 subsets.



The subsets are $\{1,2,3,4.5\}$ and it satisfy of the Maximization variant: different for 3-unif and k>3.

i.e $\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,3\},\{2,4\},\{2,5\},$
$\{3,4\},\{3,5\},\{4,5\}$

CONCLUSION

We concludethat, the above informations sloving that the kneser Graph is given the approximation and Hardness of the hyper graph and also prove the applications of the Knesar graph. Some of Knesar graph G is label covering.Therefore without layers,the hypergraph is always 2-colourable.This is really the “Long-Code” Observation of the Knesar graph .

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