The Clique Neighbourhood Domination Number in Graphs

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Abstract **- A dominating set $D \subseteq V(G)$ of graph G=(V,E) is a *clique neighbourhood dominating set* (cln-set) of G, if D is a clique dominating set of G and dominating set of N(G). The *clique neighbourhood domination number* is the minimum cardinality taken over all clique neighbourhood dominating sets of G and is denoted by $\gamma_{cln}(G)$. In this paper, $\gamma_{cln}(G)$ are obtained for some standard graphs.**

Key words: Domination number, Clique domination number, Neighbourhood domination number, Clique Neighbourhood Domination number.

1. INTRODUCTION

In this paper, G=(V,E) is a finite, undirected, simple, connected graph. In general the graph has p vertices and q edges. Terms not defined here are used in the sense of Harary[1]. The complement $\overline{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. Degree of a vertex v is denoted by d(v). The maximum(minimum) degree of a graph G is denoted by $\Delta(G)(\delta(G))$. A vertex v is said to be isolated vertex if d(v)=0.

A set $D \subseteq V(G)$ of a graph G=(V,E) is a *dominating set* of G, if every vertex in V\D is adjacent to some vertex in D. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. This concept was introduced by Ore in [6].

The concept of clique domination number was introduced by Cozzers and Kelleher in [2], in which a set $D \subseteq V(G)$ is said to a *dominating clique*, if the induced subgraph <D> is a complete graph. The c*lique domination number* $\gamma_{cl}(G)$ of G is the minimum cardinality of a dominating clique.

In [3],S.V. Siva Rama Raju, I.H. Nagaraja Rao introduced the concept of global neighbourhood domination number as follows: A set $D \subseteq V(G)$ is called a *global neighbourhood dominating set(gnd-set)*, if G is a dominating set for both G and N(G), where N(G) is the neighbourhood graph of G. The *global neighbourhood domination number* $\gamma_{gn}(G)$ is the minimum cardinality of a global neighbourhood dominating set of G.

In this paper, we introduced the clique neighbourhood domination by combining the concept of clique domination and global neighbourhood domination for a connected graph. The characteristics was studied and the exact value of the parameter was found for some standard graphs.

2. MAIN RESULTS

Definition 2.1

A dominating set $D \subseteq V(G)$ of graph G=(V,E) is a *clique neighbourhood dominating set* (cln-set) of G, if D is a clique dominating set of G and dominating set of N(G). The *clique neighbourhood domination number* is the minimum cardinality taken over all clique neighbourhood dominating sets of G and is denoted by $\gamma_{cln}(G)$.

Example :2.2

G_1:

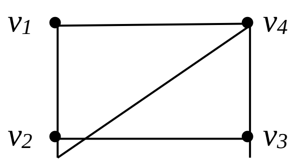


Figure 2.1

N(G_1):

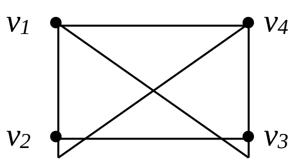


Figure 2.2

For the graph G_1, figure 2.1&2.2, the vertex set D={v_2} is the γ_{cln} - set and hence $\gamma_{cln}(G_1) = 1$. And also the clique domination, global domination is 1. $\gamma(G_1) = \gamma_{cln}(G_1) = \gamma_g(G_1) = 1$

Theorem: 2.3

For the complete graph K_n,$\gamma_{cln}(K_n) = \begin{cases} 2, n = 2 \\ 1, n \geq 3 \end{cases}$

Proof:

Let G be a complete graph K_n with atleast 1 vertex.

Case (i): n=2

Since the vertex set V(G) itself is a , γ_{cln}-set of G and hence $\gamma_{cln}(G) = |V(G)|$ which proves the result

Case(ii): $n \geq 3$

Let $u \in V(G)$ be the maximum degree in G then the set $S = \{u\}$ forms a clique neighbourhood set of G.

Hence $\gamma_{cln}(G) \leq |S| = 1$ …(1)

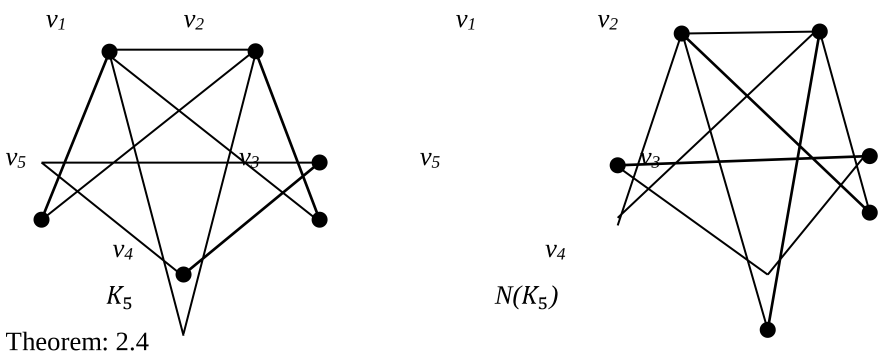
Let S be the γ_{cln}-set of G. The domination set in N(G) must contain atleast one vertex in N(G). Hence the γ_{cln}-set has atleast one vertex.

$\gamma_{cln}(G) = |S| \geq 1$ …(2)

The result follows from (1) and (2) ■

Example

In the below example K_5, A vertex set $D = \{v_1\}$ is a minimum clique neighborhood dominating set. Therefore $\gamma_{cln}(K_5) = 1$.



Theorem: 2.4

For the complete bipartite graph $K_{m,n}$, $\gamma_{cln}(K_{m,n}) = 2$ for m, n≥2

Proof:

Let G be a complete bipartite graph with atleast 3 vertices and let the vertex set of G is $V(G) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$.

Let $u_i \in V(G)$ has the maximum degree in G and v_i be any vertex adjacent to u_i in G then the set $S = \{u_i, v_i\}$ forms a clique neighbourhood set of G.

Hence $\gamma_{cln}(G) \leq |S| = 2$ …(1)

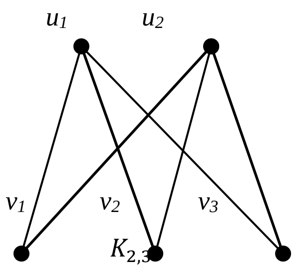
Let S be the γ_{cln}-set of G. Since N(G) contains two complete components, then for the domination of N(G), S must contain atleast one vertex from each component. Hence S has atleast two vertices.

Therefore $\gamma_{cln}(G) = |S| \geq 2$ … (2)

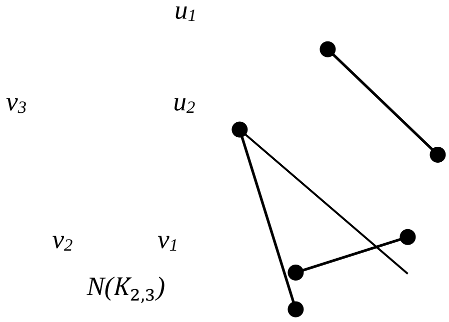
The result follows from (1) and (2).

■

Example



$K_{2,3}$



$N(K_{2,3})$

In the above example $K_{2,3}$, A vertex set $D = \{u_1, v_1\}$ is a minimum clique neighborhood dominating set. Therefore $\gamma_{cln}(K_{2,3}) = 2$

Theorem: 2.5

For star graph $K_{1,n}$, $\gamma_{cln}(K_{1,n}) = 2$ for $n \geq 2$.

Proof:

Let G be a star graph $K_{1,n}$ with atleast 3 vertices and $V(G) = \{u, v_1, v_2, \dots, v_n\}$.be the vertex set of G.

Let $u \in V(G)$ has the maximum degree in G, and v_i be any vertex adjacent to u in G then the set $S = \{u, v_i\}$ forms a clique neighbourhood set of G.

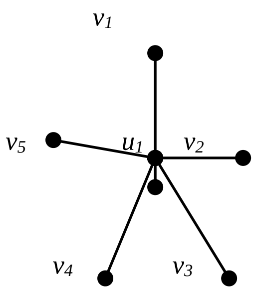
Hence $\gamma_{cln}(G) \leq |S| = 2$ …(1)

Let S be the γ_{cln}-set of G. The dominating set in N(G) must contain atleast one isolated vertex and one maximum degree vertex in N(G). Hence the clique neighbourhood set must contain atleast 2 vertices.

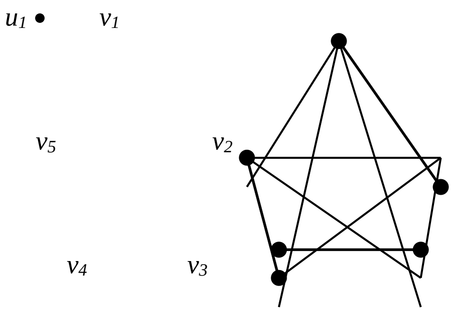
Therefore $\gamma_{cln}(G) = |S| \geq 2$ … (2)

The result follows from (1) and (2). ■

Example



$K_{1,5}$



$N(K_{1,5})$

In the above example $K_{1,5}$,

A vertex set $D = \{u_1, v_1\}$ is a minimum clique neighborhood dominating set, therefore $\gamma_{cln}(K_{1,5}) = 2$

Theorem: 2.6

For the Wheel W_n, $\gamma_{cln}(W_n) = 1, n \geq 2$.

Proof:

Let G be a wheel graph W_n with atleast 3 vertices and $V(G) = \{u, v_1, v_2, \dots, v_n\}$ be the vertex set of G.

Let $u \in V(G)$ has the maximum degree in G, then the set $S = \{u\}$ forms a clique neighbourhood set of G.

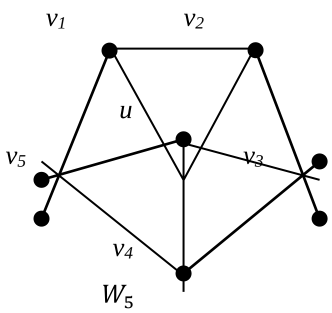
Hence $\gamma_{cln}(G) \leq |S| = 1$ …(1)

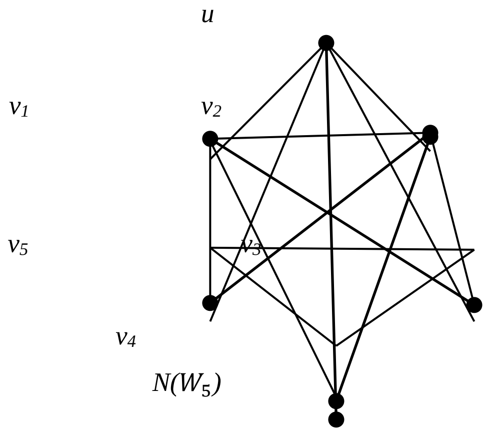
Let S be the γ_{cln}-set of G. The dominating set in N(G) must contain atleast one vertex in N(G) and hence the clique neighbourhood set has atleast 1 vertex.

Hence $\gamma_{cln}(G) = |S| \geq 1$ …(2)

The result follows from (1) and (2) ■

Example





W_5 $N(W_5)$

In the above example W5,

A vertex set *D = {u}* is a minimum clique neighborhood dominating set. Therefore $\gamma_{cln}(W_5) = 1$

Theorem: 2.7

For the fan F_n, $\gamma_{cln}(F_n) = 1$, for n≥2.

Proof:

Let G be a fan graph F_n with atleast 3 vertices and $V(G) = \{u, v_1, v_2, \dots, v_n\}$ be the vertex set of G.

Let $u \in V(G)$ has the maximum degree in G then the set $S = \{u\}$ forms a clique neighbourhood set of G.

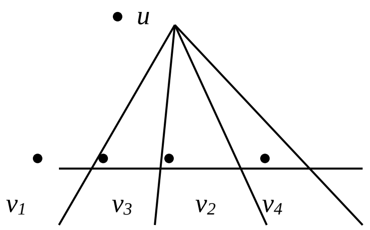
Hence $\gamma_{cln}(G) \leq |S| = 1$ …(1)

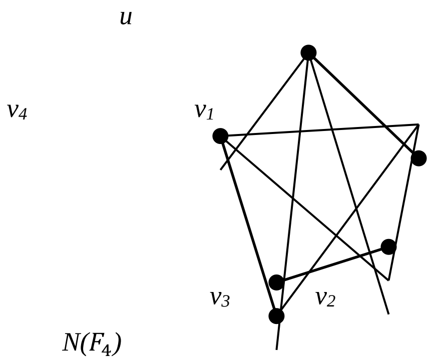
Let S be the γ_{cln}-set of G. The dominating set in N(G) must contain atleast one vertex in N(G) and hence the clique neighbourhood set has atleast 1 vertex.

Therefore $\gamma_{cln}(G) = |S| \geq 1$ …(2)

The result follows from (1) and (2) ■

Example





F_4 $N(F_4)$

In the above example F4, A vertex set *D = {u}* is a minimum clique neighborhood dominating set. Therefore $\gamma_{cln}(F_4) = 1$

Theorem: 2.8

For the banana tree $B_{n,n}$, $\gamma_{cln}(B_{n,n}) = 2$ for n≥2.

Proof:

Let G be a banana tree with atleast 6 vertices and $V(G) = \{u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ be the vertex set of G.

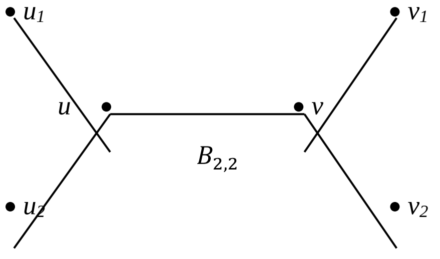
Let $u, v \in V(G)$ such that d(u)=d(v)= $\Delta(G)$ in G then the set $S = \{u, v\}$ forms a clique neighbourhood set of G.

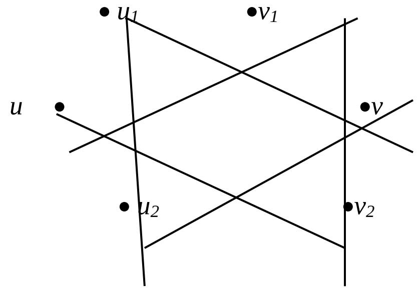
Hence $\gamma_{cln}(G) \leq |S| = 2$ …(1)

Let S be the γ_{cln}-set of G. Since N(G) contains two complete components, then for the domination of N(G) , S must contain atleast one vertex from each component. Hence S has atleast two vertices.

Therefore $\gamma_{cln}(G) = |S| \geq 2$ … (2)

The result follows from (1) and (2). ■





N($B_{2,2}$)

In the above example $B_{2,2}$, A vertex set *D* = *{u,v}* is a minimum clique neighborhood dominating set. Therefore $\gamma_{cln}(B_{2,2}) = 2$

Theorem: 2.9

For the book graph B_n, $\gamma_{cln}(B_n) = 2$ for n≥2.

Proof:

Let G be a book graph B_n with atleast 6 vertices and $V(G) = \{u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ be the vertex set of G.

Let $u, v \in V(G)$ such that d(u)=d(v)= $\Delta(G)$ in G then the set $S = \{u, v\}$ forms a clique neighbourhood set of G.

Hence $\gamma_{cln}(G) \leq |S| = 2$ …(1)

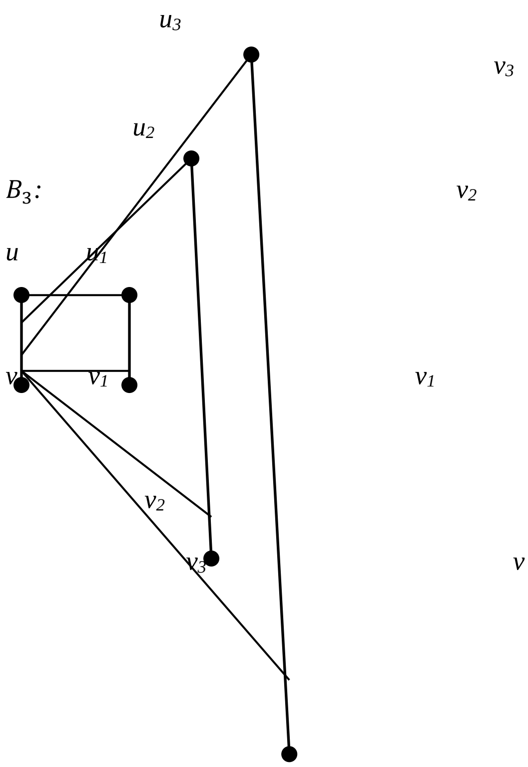
Let S be the γ_{cln}-set of G. Since N(G) contains two complete components, then for the domination of N(G) , S must contain atleast one vertex from each component. Hence S has atleast two vertices.

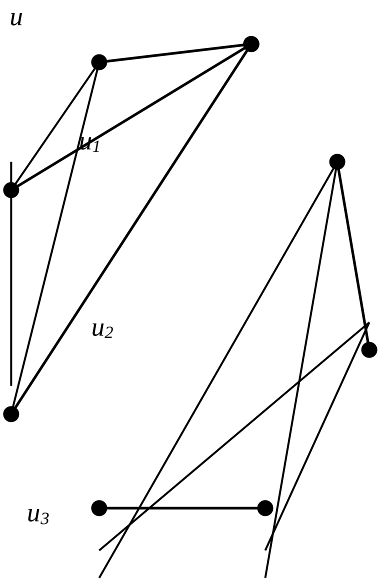
Therefore $\gamma_{cln}(G) = |S| \geq 2$ … (2)

From (1) and (2) the result follows. ■

Example

N(B_3)





In the above example B$_3$, A vertex set *D* = *{ u, v}* is a minimum clique neighborhood dominating set, therefore $\gamma_{cln}(B_3) = 2$

Theorem: 2.10

For the n-barbell graph, $\gamma_{cln}(G) = 2$, for n≥2.

Proof:

Let G be a n-barbell graph with atleast 6 vertices and $V(G) = \{u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set of G.

Let $u, v \in V(G)$ such that d(u)=d(v)= $\Delta(G)$ in G then the set $S = \{u, v\}$ forms a clique neighbourhood set of G.

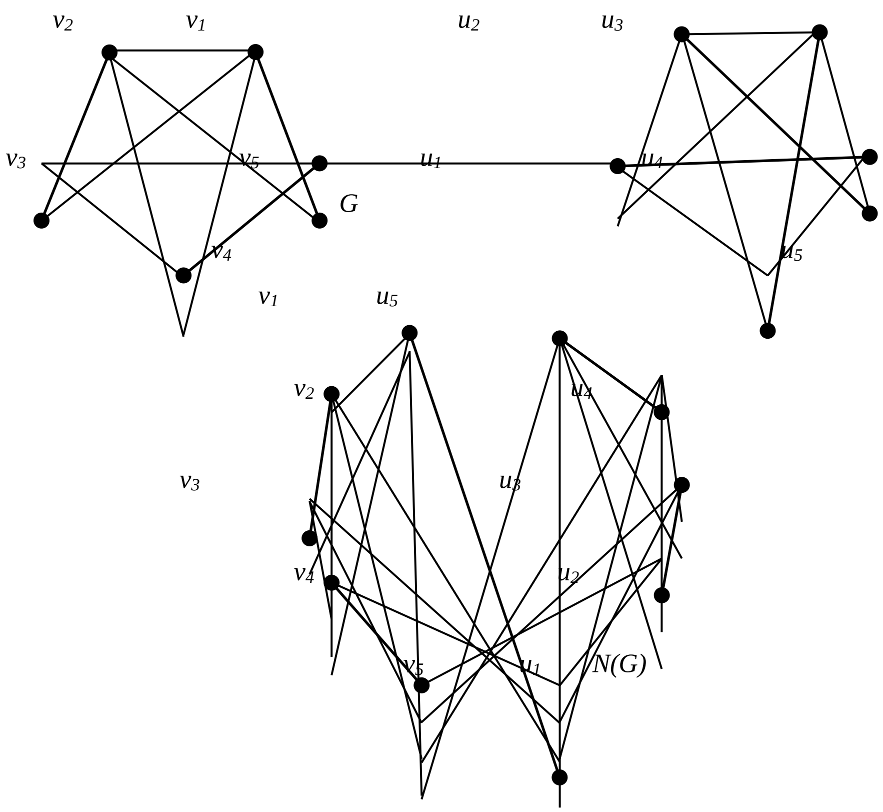
Hence $\gamma_{cln}(G) \leq |S| = 2$ …(1)

Let S be the γ_{cln}-set of G. Since N(G) contains two complete components, then for the domination of N(G) , S must contain atleast one vertex from each component. Hence S has atleast two vertices.

Therefore $\gamma_{cln}(G) = |S| \geq 2$ … (2)

From (1) and (2) the result follows. ■

Example



In the above example 5-barbell graph,

A vertex set D= {u_1, v_1} is a minimum clique neighborhood dominating set, therefore $\gamma_{cln}(G) = 2$

Theorem: 2.11

For the friendship graph, $\gamma_{cln}(C_3^m) = 1$, for m≥ 2.

Proof:

Let G be a friendship graph with atleast 5 vertices and $(G) = \{u, v_1, v_2, \ldots, v_n\}$ be the vertex set of G.

Let $u \in V(G)$ has the maximum degree in G then the set $S = \{u\}$ forms a clique neighbourhood set of G.

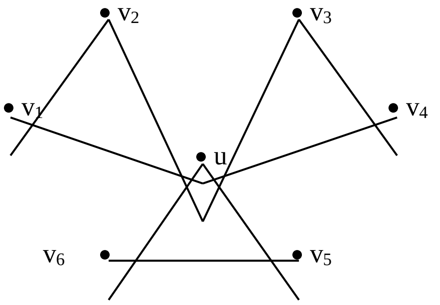
Hence $\gamma_{cln}(G) \leq |S| = 1$ …(1)

Let S be the γ_{cln}-set of G. The dominating set in N(G) must contain atleast one vertex in N(G) and hence the clique neighbourhood set has atleast 1 vertex.

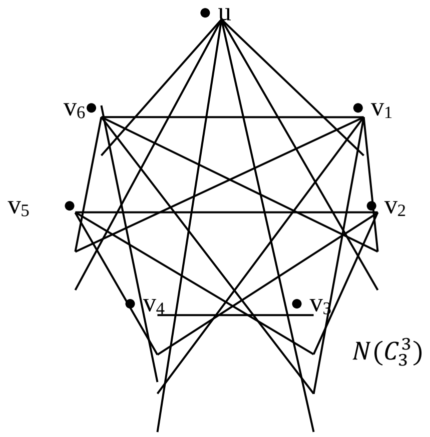
Therefore $\gamma_{cln}(G) = |S| \geq 1$ …(2)

The result follows from (1) and (2) ■

Example:



C_3^3



$N(C_3^3)$

In the above example C_3^3

The vertex set $S = \{u\}$ is a minimum clique neighbourhood dominating set.

Therefore $\gamma_{cln}(G) = 1$.

3. CONCLUSION

In this paper, we found the exact values of clique neighbourhood domination number for Complete graph, Complete bipartite graph, Star graph, Wheel graph, Fan graph, Banana tree, Book graph, n-barbell graph, Friendship graph.

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