## **ICART - 2022 Conference Proceedings**

# The behaviour of Passive Damping System in Tall **Buildings**

Nabeela P A Master's Student Department of Civil Engineering Mangalam College of Engineering Kottayam, Kerala, India

Aswathy Soman Assistant Professor Department of Civil Engineering Mangalam College of Engineering Kottayam, Kerala, India

Abstract-Inestimable elevated structure has been built everywhere on over the world and the number is expanding gradually. Major earthquakes of the last few decades lead to a great deal of interest in structural control systems, to weaken seismic hazards to lifeline structures. The need to build flexible and slender structures, that have relatively low damping properties has attracted engineers' attention to look for efficient and economical techniques to control the vibrational response of structures. This isn't just because of concern over the high thickness of population in the urban communities, business sector, and space sparing yet additionally to build up land marks. To guarantee the useful execution of tall structures, different plan adjustments are possible, extending from elective basic frameworks to the use of inactive and dynamic control gadgets. Thus to secure re functional performance of tall buildings, against wind and earthquake forces, it is important to keep the frequency of structural motion below the threshold. Nowadays various damping techniques have been implemented. The various techniques to achieve this can be classified into 2 categories i.e. Active Damping Techniques and Passive Damping Techniques. The best example of passive systems is the Tuned Mass Damper (TMD) and Tuned Liquid Damper (TLD). In this study, TMD and TLD are considered for comparative

Keywords— Damping, Tuned Mass Damper, Tuned Liquid Damper, Structural Motions

## 1. INTRODUCTION

Vibration means the mechanical oscillation about an equilibrium point. The oscillation may be periodic or nonperiodic. Vibration control is indispensable for machinery, space shuttle, aero plane, and a ship floating in the water. With the modernization of engineering, the vibration lessening technique has found a way into civil engineering and the infrastructure field. The seismic waves will make structures influence and waver in different manners depending upon the recurrence and heading of ground movement, and the tallness and development of the structure. Seismic movement can cause extreme motions of the give rise to even auxiliary structure which may disappointment. With the rapid economic development and advanced technology, civil structures such as high-rise buildings, towers, and long-span bridges are designed with additional flexibility, which increases their susceptibility to external excitation. Therefore, this flexible structure is open to being exposed to excessive levels of vibration under the actions of strong wind or earthquake. To protect such civil structures from significant damage, the response reduction of civil structures during dynamic loads such as severe earthquakes and strong winds has become a key topic in structural engineering.

To ensure the functional performance of tall buildings, against wind and earthquake forces, it is important to maintain the frequency of structural motion below the threshold. Several methods have been developed to minimize the structural response due to lateral excitations. The various techniques to achieve this can be classified into 2 categories i.e. Active Damping Techniques and Passive Damping Techniques. Active dampers use an external power source to create an additional force between the damper and structure and this type of supplying energy to the system is known as negative damping. Passive damping means energy dissipation within the structure. The force that is exerted on a structure due to external loads is channeled through the passive system and then it is dissipated through the



Fig 1: Tuned Mass Damper

damping device. The advantage is that there is no power source demanded in the operation of the system and so is environmentally friendly. The best example is the addition of a supplementary mass system to increase the level of damping (E.g. TMD, TLD).

## 2. TUNED MASS DAMPER (TMDs)

## A. Definition

TMDs being used, are usually oil dampers, thick and viscoelastic dampers. Normally, a TMD consists of an inertial mass added on to the structure area with the most extreme movement, by and large close to the top, through spring and damping element, ordinarily gooey and viscoelastic dampers. Tuned mass dampers (TMD) as an energy dissipating device can be used to increase the overall damping of the main structure and correspondingly reduce the building vibrations induced by wind. A Tuned Mass Damper (TMD) is a passive damping system that makes use of a secondary mass attached to the main structure normally through spring and dashpot,

**ICART - 2022 Conference Proceedings** 

which will reduce the dynamic response of the structure. The secondary mass system is designed to have the natural frequency, which is grounded on its mass and stiffness, tuned to that of the frequency of the primary structure. When that particular frequency of the structure gets agitated, the TMD will resonate out of phase along with the structural motion and minimizes its response. Then, the excess amount of energy, which is built up in the structure can be transferred to a secondary mass and is dissipated by the dashpot due to relative motion between them at a later time.

A TMD is an inertial mass attached to the structure position with maximum motion (generally near the top), through a duly tuned spring and damping element, Generally viscous and viscoelastic dampers are used. TMDs give a frequencydependent hysteresis which increases damping in the frame structure attached to it to reduce its motion. The robustness is determined by their dynamic characteristics, stroke, and the amount of added mass they use. The additional damping introduced by the TMD is mainly dependent on the ratio of the damper mass to the effective mass of the building in a particular mode of vibration. TMDs weight is varied between 0.25% - 1.0% of the structure's weight in the fundamental mode (typically around one-third). Regularly, dividing limitations won't license customary TMD designs, requiring the establishment of optional arrangements including multistage pendulums, modified pendulums, and frameworks with precisely guided slide tables, hydrostatic heading, and covered elastic direction. Curl springs or variable solidness pneumatic springs generally give firmness to the tuning of TMDs. Although TMDs are constantly feasible, far superior reactions have been noted using different damper setups (MDCs) which comprise of a few dampers set in corresponding with proper characteristic frequencies around the control tuning recurrence.

The frequency of a TMD is tuned to a particular structural frequency when that frequency is excited the TMD will resonate out of phase with frame motion and it will reduce the response. Frequently for better response control, multipledamper configurations (MDCs) consist of several dampers placed in parallel with distributed natural frequencies around the control tuning frequency are used. A multiple mass damper can significantly increase the equivalent damping introduced to the system for the same total mass.

# B. Real life structures provided with TMDs

Description of the several building structures that are equipped with Tuned Mass Dampers (TMDs) follows.

## A. Chiba Port Tower, Japan

Chiba Port Tower, a steel structure of 125 m tall and having a rhombus-formed arrangement with a side length of 15 m (completed in 1986) was the main pinnacle in Japan to be outfitted with a TMD. The timespan in the first and second method of vibrations are 2.25 s and 0.51 s, separately for the x axis, and 2.7 s and 0.57 s for the y heading collectively. Damping for the critical mode was figured at 0.5%. For an advanced method of vibration damping proportions relative to frequencies were expected in the examination. The application of the TMD was to build damping of the primary mode for both the x and y bearings. The mass proportion of the damper regarding the modular mass of the principal mode was around 1/120 in the x axis and 1/80 in the y heading; periods in the x and y bearings of 2.24 s and 2.72 s, collectively; and a damper damping proportion of 15%. Decreases of around 30 to 40% in the relocation of the top level and 30% in the pinnacle twisting minutes are normal.

# **B.** Citicrop Centre, New York

Another leading application of TMDs has been in the Citicorp Building in New York. The elevation of the structure is 278 m with a major time of around 6.5 s and a damping proportion of 1% along with the two tomahawks. The framework, estimating 9.14 x 9.14 x 3.05 m, comprises of a 410-ton solid square upheld on a progression of twelve 60-cm lengths across the water powered weight offset heading with two spring damping instruments, one for the north-south movement and another one for the east-west movement, was introduced in the 63rd floor in 1978. The framework lessens the breeze-activated reaction of the Citicorp working by 40% in both the north-south and east-west bearings, at the same time (Wiesner 1979). The damper framework begins naturally at whatever point the flat quickening surpasses 0.003g for two successive cycles and will accordingly close itself down when the structure speeding up doesn't surpass 0.00075g in either hub over a 30-minute span length.

# C. Taipei 101, Taiwan

Taipei 101, is a steel-propped assembly and it is the third tallest structure on the planet. A circle formed TMD of weight 660 tons and width of 5.5 m has been established between 88th to 92nd floor of the structure. This structure is a case of a pendulum type Tuned Mass Damper. The tremendous circle was suspended by four arrangements of links, and the dynamic vitality is dissipated by 8 waterpowered cylinders each having a length of 2m. The damper can reduce 40% of the pinnacle development. Another two tuned mass dampers, each measuring 6 metric tons sit at the tip of the tower.



Fig 2: Taipei Tuned Mass Damper

# D. John Hancock Tower, Boston

Perhaps the earliest application of this sort was introduced in June 1977 in the 244 m tall (60 stories) John Hancock Tower in Boston. Two TMDs were established at the furthest edges of the 58th floor, at a dispersing of 67 m, to check the effect just as the torsional movement because of the state of the

structure. Each damper estimated about 5.2x5.2x1 m and was primarily a steel box loaded up with lead, weighing 300 tons, connected to the edge of the building by solid springs. The lead-filled weight slides to and fro on a hydrostatic bearing consisting of a slight layer of oil constrained through openings in the steel plate. At whatever point the level quickening exceeds 0.003g for two sequential cycles, the framework is naturally switching on. This framework is relied upon to reduce the influence of the structure by 40 to half.

## E. Buri Al Arab, Dubai

The world's tallest hotel Burj Al Arab is equipped with 11 TMDs have been established at different locations to control the wind-induced vibration.

#### F. Atc Tower in New Delhi, India

A 50-ton Tuned Mass damper has been established just beneath the ATC floor at 90m level.

## G. Statue of Unity, India

It is the world's tallest statue, the Statue of Unity (182m high) two 200-ton Tuned mass dampers have been installed at the shoulder level.

#### C. Equations

One TMD is effective in reducing the dynamic response of only a single vibration mode of the structure. Although a structure has many vibration modes, in reality, the basic properties of TMD can be discussed using a simplified 2-DOF model consisting of the main structure and the TMD system

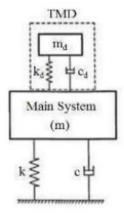


Fig 3: 2-DOF modeling of the main structure and tuned mass damper system

Let us describe the following parameters to be used in the following discussion

Natural frequency of TMD, 
$$\omega_d = \sqrt{\frac{k_d}{m_d}}$$
 (1)

The damping ratio of TMD, 
$$\xi_d = \frac{c_d}{2m_d\omega_d}$$
 (2)

The natural frequency of the main structure,  $\omega$ 

$$=\sqrt{\frac{\mathbf{k}}{\mathbf{m}}}\tag{3}$$

The damping ratio of the main structure,  $\xi = \overline{2m\omega}$ 

Mass ratio, 
$$\mu = \frac{m_d}{m}$$
 (5)

Frequency (tuning) ratio, 
$$\gamma = \frac{\omega_d}{\omega}$$
 (6)

When  $\xi_d = 0$ , a 2-DOF system allows 2 uncoupled vibration modes, and when  $\xi_d = \infty$  the 2-DOF system becomes a 1-DOF vibration system. Steady-state dynamic response subjected to harmonic excitation can be acquired analytically. It is usually called the resonant curve or dynamic magnification factor (DMF) curve plotted against the angular frequency of the harmonic excitation. There noticed that DMF curves cross two fixed points independent of the damping ratio  $\xi_d$ . Den Hartog defined the optimum TMD by letting the two fixed points have the same value and be as high as possible in the DMF curve. The physical meaning of this is to obtain a flat DMF curve at the resonant frequency, and as a result, suppress the dynamic response of the main building most effectively. From this definition, the optimum frequency ratio  $\gamma_{opt}$  and the optimum damping ratio  $\xi_{dopt}$  of TMD are obtained by Den Hartog as a function of mass ratio u, i.e

$$\gamma_{\text{opt}} = \frac{1}{1+u} \tag{7}$$

$$\xi_{\text{dopt}} = \frac{1}{2} \sqrt{\frac{3\mu/2}{1+3\mu}}$$
 (8)

$$\Delta \xi_{eq} = \frac{1}{2} \sqrt{\frac{\mu/2}{1 + \mu/2}}$$
 (9)

Multiple Tuned Mass Dampers (MTMDs) comprise more than one TMD whose frequencies are distributed around the natural frequency of the controlled mode of the main structure. The natural frequencies of MTMD are distributed uniformly around their average natural frequency, equal to the fundamental natural frequency of the primary structure.

The natural frequency of the j<sup>th</sup> TMD is expressed as:

$$\omega_{j} = \omega_{T} \left[ 1 + \left( j - \frac{n+1}{2} \right) \right] \frac{\beta}{n-1}$$
 (10)

where, n is the total number of MTMDs and  $\beta$  is the nondimensional frequency spacing of the MTMD, given as

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \tag{11}$$

If  $k_d$  is the constant stiffness of each TMD, then the mass of the  $j^{th}$  TMD is expressed as:

$$m_{dj} = \frac{k_d}{\omega_j^2} \tag{12}$$

The mass ratio is the ratio of the total MTMD mass to the total mass of the main structure and it is expressed as

$$\mu = \frac{\sum_{j=1}^{n} m_{dj}}{m} \tag{13}$$

where m denotes the total mass of the primary structure.

The tuning ratio is the ratio of the average frequency of the MTMD to the fundamental frequency of the main structure, expressed as

$$f = \frac{\sum_{j=1}^{n} m_{dj}}{m}$$
(14)

It is to be noted that as the stiffness and normalized damper force of all the TMD are constant and only mass is varying, the friction force adds up. Thus, the non-dimensional frequency spacing  $\beta$  controls the distribution of the frequency of the TMD units.

# D. Design of Tuned Mass Damper

The design of a damped TMD for an un-damped structure includes the following steps:

- Establish the allowable values of displacement of the primary mass and the TMD for the design loading.
- Determine the mass ratios required to satisfy these motion constraints. Select the largest value of m.
- Determine *f* :
- Compute  $\omega_d$ :
- Compute  $k_d$ .
- Compute  $c_d$
- Determine Pendulum Length (L):

# 3. TUNED LIQUID DAMPER (TLDs)

## A. Definition

TLDs can be broadly classified into Tuned Sloshing Dampers (TSDs), Tuned Liquid Column Dampers (TLCDs), and controllable TLDs. TSDs are rectangular or circular tanks partially filled with liquid (usually water) and are typically located either at the terrace level or immediately below it. When the tank is excited through structural motion, the fluid in the tank begins to slosh, providing inertial forces into the structure, out of phase with the structural motion, thereby reducing the movement. Tuned Liquid Column Dampers (TLCDs) dissipate structural vibration due to the motion of the liquid in the tube as a result of gravity action and by the loss of hydraulic pressure due to the orifice established inside the container. Controllable TLDs are used to increase the efficiency of the damper when the forces that act on the structure are spread over a band of frequencies. This is achieved by active or semi-active control devices such as controlling the angle of baffles provided in the tank or by utilizing propellers powered by a motor.

The optimum performance of TLD is obtained by tuning its fundamental sloshing frequency to the structure's natural frequency which causes a large amount of sloshing & wave breaking. A TLD can be classified as shallow water type and deep water type based on the height of water in the tank. If the ratio of the height of water against the length of the tank in the direction of excitation is less than 0.15, it is a shallow water type else a deep water type TLD. Shallow water type TLD's have a large damping effect for a small amount of externally excited vibrations, but it is hard to analyse the same for large amplitude vibrations since the sloshing of water in a tank exhibits nonlinear behaviour. For deep water type TLDs, the sloshing can be defined by a linear behaviour for large amplitude external forces.

Studies relating to the effectiveness of TMDs are plentiful, however, the use of TLDs to control vibration is an area still undergoing various modifications. The performance of TLDs against wind-induced lateral loads was studied more precisely, while studies related to seismic performance were found to be scarce.

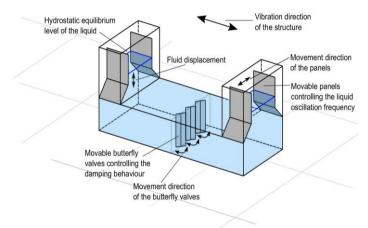


Fig 4: picture model of Tuned Liquid Damper

# B. Equations

While developing a mathematical model, some assumptions are made for the easiness of the analysis. The assumptions are, that the mass of columns and flexibility of the slab are neglected and the beam column joint is assumed to be rigid. By these assumptions, the chance of lateral deformation is due to only the rigid beam /slab. This type of model is called a shear building model. This shear building idealization is required for the mathematical formulation of vibration problems. Consider a spring mass system under the effect of viscous damping and subjected to a harmonic force of F sin ot.

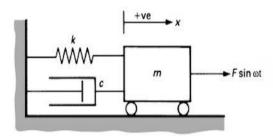


Fig 5: Damped Harmonic Oscillator

**ICART - 2022 Conference Proceedings** 

The governing equation of motion can be written as

$$m\ddot{x} + c\dot{x} + kx = F\sin\omega t \tag{15}$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m}\sin\omega t \tag{16}$$

where m, c, and k represent the mass, damping coefficient, and stiffness of the structure. The general solution x(t) can be divided into complimentary solution xc(t) and particular solution xp(t). Assuming the system is underdamped, i.e.  $\rho < 1$ , the solution is obtained in the following form

$$x(t) = \sqrt{A^{2} + B^{2}} e^{-\rho\omega_{,t}} \sin(\omega_{,t} t - \phi_{,c}) + \frac{\frac{P}{k}}{\sqrt{(1 - \beta^{2})^{2} + (2\rho\beta)^{2}}} \sin(\omega t - \phi_{,p})$$
(17)

In the above equation,  $\omega_n$  and  $\omega_d$  refer to the frequency of the system and damper while  $\beta$  and  $\Phi$  represent frequency ratio and phase angle respectively. A. Modelling of Wave Sloshing in TLD Considering a 2-dimensional wave fluid as shown in Fig. 6 (X-O\_Z plane), liquid depth be h, and z=0 be the still liquid surface.  $\eta$  represents the free surface elevation, which is a function of location x and time t. L and H express wave length and wave height respectively. The wave amplitude is assumed to be extremely small so that the wave motions can be regarded as linear.

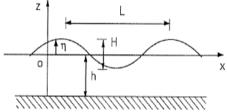


Fig 6: Schematic Diagram Showing Wave Motion in TLD

# a. Modelling of Wave Sloshing in TLD

Considering a 2-dimensional wave fluid as shown in Fig. 6 (X-O\_Z plane), liquid depth is h, and z=0 is the still liquid surface.  $\eta$  represents the free surface elevation, which is a function of location x and time t. L and H express wave length and wave height respectively. The wave amplitude is assumed to be extremely small so that the wave motions can be regarded as linear.

Liquid motion is assumed to be inviscid, irrational, and incompressible. The velocity potential  $\Phi$ , therefore exists and is satisfied with the Laplace equation, i.e.

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{18}$$

At the bottom of the tank

$$w = \frac{\partial \phi}{\partial z} = 0 \quad (z = -h) \tag{19}$$

On the free surface  $z = \eta(x,t)$ , there are two kinds of boundary conditions; one is the dynamic boundary condition

$$p = p0 = 0 \ (z = \eta)$$
 (20)

and the other is the kinematic boundary condition

$$\Phi(x,z,t) = -\frac{gH}{2\omega} \frac{\cosh(k(z+h))}{\cosh(kh)} \cos(kx - \omega t)$$
(21)

Let  $\eta$  take the form of

$$\eta = \frac{H}{2}\sin(kx - \omega t) \tag{22}$$

the dispersion equation

$$\omega^2 = gk \tanh(kh) \tag{23}$$

Therefore, the velocity potential,  $\Phi$  can be expressed as

$$\frac{D\eta}{Dt} = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} = w \quad (z = \eta)$$
(24)

The profile of  $\Phi$  along the z direction is a function of  $\cosh(k(z+h))$ 

b. Linear Natural Frequency of Liquid Sloshing in a Rectangular Tank

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2n-1}{2a} \pi g \tanh(\frac{2n-1}{2a} \pi h)} \quad (n = 1, 2, ...) (25)$$

## 4. REVIEW ON RESULTS

#### A. Tuned Mass Damper

From the study of Uba Gul Khan .al.[1] the following results are obtained from the modeling and analysis of a 60m tall building having 15 stories with a square arrangement of 20x20m. He studies the 3D model considered 60 m tall building with 15 stories with floor to floor height of 4 m using ETABS software. The building has a square plan of 20x20 m. In this study, both the building and the damper have been modeled as linear. Four numbers of identical models were created, the first model is the base model (uncontrolled) and the remaining three models (controlled) have TMDs with mass ratios of 0.01, 0.02, and 0.04. Linear time history analysis of the structure has been done using the acceleration data of the Bhuj/Kutch 2001 earthquake. The present study focused on the ability of TMD to reduce earthquake-induced structural vibration and to compare the building response with the effect of variation in mass ratio and damping ratio of TMD. From this study, it can be concluded that. The acceleration of the structure in the fundamental mode is reduced by 17.67%, 23.72%, and 28.83% for the mass ratios of 0.01, 0.02, and 0.04 respectively.

- 1) The effective damping of the building in the fundamental mode is increased to 8.01%, 9.27%, and 11.09% for the mass ratios of 0.01, 0.02, and 0.04 respectively.
- 2) The maximum story displacement of the building is lowered by 19.8%, 25.1%, and 29.1% for the mass ratios of 0.01, 0.02, and 0.04 respectively.
- 3) The effective damping of the building increases and the dynamic response of the building reduces as the mass ratio of

the TMD is increased. The TMD becomes robust with an increasing mass ratio. Hence an optimal mass ratio of the TMD can be found to reduce the building responses substantially there by giving the desired level of human comfort, safety, and economy to the structure.

# B. Tuned Liquid Damper

From the studies of Riju Kuriakose. al. [2], the following results are obtained from the modeling and analysis of 40 storied buildings in Kerala using Ansys Workbench. He designed TLDs for mass ratios ranging from 0.2 % to 1.6 %. Following conclusions could be made from the study.

- 1. From this study, it has been found that the TLD can be successfully used to control the vibration of the structure.
- 2. After carrying out the normal mode analysis of the structure with TLD tanks, it was found that the structural frequency increased with increasing mass ratios, making it less vulnerable to exciting forces.
- 3. An attempt was made to optimize TLD's mass ratio for the particular structure. It was found that 0.8 % would be the most optimum considering the structural, damping, and economic factors.
- 4. The reduction in amplitude was found as 28.73 %. 5. 8 tanks are proposed with 5 x  $4.25 \times 3$  m dimensions. The water depth in each tank is to be maintained at 0.985 m.

## 5. CONCLUSION

Current trends in the construction industry demand taller and lighter structures, which are more flexible and have low damping value. This increases failure possibilities and problems considering the serviceability point of view. Several techniques are available today to minimize the vibration of the structure.

Based on the present study and reviewed literature following conclusions can be drawn:

- 1. Response of the frame building reduces with the increase in the mass ratio of the single damper.
- 2. The non-uniform distribution of mass ratio is more effective than a single damper of the same mass ratio.
- 3. The uniform distribution of mass ratio is most effective in vibration control.

#### 6. REFERENCE

- [1] Ubair Gul Khan, Mirza Aamir Baig "Behaviour of Tall Buildings using Tuned Mass Dampers" International Journal of Engineering Research & Technology (IJERT), Vol. 9 Issue 09, September-2020
- [2] Riju Kuriakose, Lakshmi P. "Effectiveness of Tuned Liquid Dampers on High Rise Buildings in Kerala" International Journal of Engineering Research & Technology (IJERT), Vol. 5 Issue 09, September-2016
- [3] V R Sindhu Priya, Dr. Gopisiddappa, Subramanya K G "Influence of Tuned Mass Dampers on Vibration Control of Tall buildings" International Research Journal of Engineering and Technology (IRJET), Volume: 05 Issue: 06 | June-2018
- [4] Y.L. Xu, K.C.S. Kwok and B. SamaU "Control of wind-induced tall building vibration by tuned mass dampers" Journal of Wind Engineering and Industrial Aerodynamics, 40 (1992) 1-32
- [5] Kang Zhou, Jun-Wei Zhang, Qiu-Sheng Li "Control performance of active tuned mass damper for mitigating wind-induced vibrations of a 600-m-tall skyscraper" Journal of Building Engineering, 2021
- [6] Ashasi-Sorkhabi, A., Kristie, J., and Mercan, O., "Investigations of the use of multiple tuned liquid dampers in vibration control." Proc., Structures Congress, ASCE, Reston, VA, (2014), pp.1185–1196
- [7] Banerji P., Murudi M, Shah A.H. & Popplewell N., "Tuned liquid dampers for controlling earthquake response of structures", Earthquake Engineering Structural Dynamics, 29 (2000), pp. 587-602
- [8] Bauer H.F., "Oscillations of immiscible liquids in a rectangular container: A new damper for excited structures." Journal of Sound and Vibration, 93(1), (1984), pp. 117-133
- [9] Fujii K., Tamura Y., Sato T., Wakahara T., "Wind-induced vibration of tower and practical applications of Tuned Sloshing Damper." Journal of Wind Engineering and Industrial Aerodynamics, 33, (1990), pp. 263-272.
- [10] Fujino, Y., and Sun, L. M., "Vibration control by multiple tuned liquid dampers (MTLDs)." J. Struct. Eng., 10.1061/(ASCE)0733 -9445(1993)119:12(3482), (1993), pp. 3482–3502
- [11] Introduction to Structural Motion Control, Jerome J. Connor.
- [12] Seismic Analysis of Structures, T K Datta, IIT Delhi..