Telescoping Decomposition Method for Solving Second Order Nonlinear Differential Equations

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***Abstract* - In this paper we present a reliable algorithm for solving first and second order nonlinear differential equations. The Telescoping Decomposition Method (TDM) is a new iterative method to obtain numerical and analytical solutions for first order nonlinear differential equations. We aim to extend the works of Mohammed Al-Refaiet al (2008) and make progress beyond the achievements made so far in this regard. We have modified the function $f(t,u,u_t)$ to solve second order nonlinear differential equations. The method is a modified form of Adomian Decomposition Method (ADM) where the computation of the Adomian Polynomial is a difficult task in most cases. The TDM is easier to apply when compared to ADM and offers better convergent to the exact solution while it is not the case in ADM.**

Keywords: Telescoping Decomposition Method, Adomian Decomposition Method, Adomian Polynomial and Nonlinear Operators.

1. INTRODUCTION

Differential equations appear in various application problems in the physical sciences and engineering. Most of the differential equations coming from real life application are nonlinear and the seek of analytical solutions post a real challenge to mathematics. The Adomian decomposition methodG. Adomian (1994), G.Adomian (1988) and G. Adomian (1984) has been applied to a wide class of stochastic and deterministic problems in physics, biology and chemical reactions. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly. It is well known by many authors that the decomposition method decomposes the linear term u(x,t) into an infinite sum of components $u_n(x,t)$ defined by

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \quad (1.1)$$

And identifies the nonlinear term F(u) by the decomposition series

$$F(u)=\sum_{n=0}^{\infty}A_n(x,t) \quad (1.2)$$

Where A_n are the so called Adomian polynomials G. Adomian (1994), G. Adomian (1988) and G. Adomian (1984) formally introduced formulas that can generate Adomianpolynomials for all forms of nonlinearity.
Recently, a great deal of work has been done by Abdul-Majid Wazwaz (1999,2000), G. Adomian (1986) and V. Seng et al (1996) among others to develop a practical method for the calculation of Adomian polynomials A_n. The concern was to develop and calculate Adomian polynomials in a practical way without any need for formulas introduced by Adomian, G. Adomian (1986) and V. Seng, et al (1996) require a huge size of calculations and employ several formulas identical in spirit to that used by Adomian. It is worth nothing that calculating Adomian polynomials is difficult for large n, and the formular given by Adomian,G.Adomian(1986), G. Adomian(1992) and manjak(2008) cannot be applied if f is a function of several variables, such as

$$f(u,u',u'')$$

Also, the ADM is shown to be divergent for certain problems Hosseini and Nasabzadeh (2006). The objective of this paper is to introduce the Telescoping decomposition method (TDM) for solving first and second nonlinear initial value problem. We will use the idea of the ADM for the linear part and introduce a new way of computing the nonlinear part by avoiding the calculation of the Adomian polynomials using the formulas discussed in Mohammed et al (2008). In section 2, we present the expansion procedure of the TDM.

2.TELESCOPING DECOMPOSITION METHOD EXPANSION PROCEDURE

We consider the initial value problem of the form

$$u_t = f(t,u,u_t), t\in\Omega \ldots \quad (2.1)$$

$$u(0)=u_0 \quad (2.2)$$

Where $\Omega=[0,T]$ is compact subset of R. By integrating the equation we have

$$u(t) = u(0) + \int_0^t f(\tau, u(\tau), u_\tau(\tau))d\tau \ \ldots \quad (2.3)$$

We consider a solution of the form

$$u(x,t) = \sum_{n=0}^{\infty} u^n(t).$$

Where $u^n(t)$ has to determine sequentially upon the following

$$u^0 = u_0, u^1(t) = \int_0^t f(\tau, u^0(\tau), u_\tau^0(\tau))d\tau \ldots \qquad (2.4)$$

$$u^2(t) = \int_0^t f(\tau, \sum_{k=0}^{1} u^k(\tau), \sum_{k=0}^{1} u_\tau^k(\tau))d\tau - \int_0^t f(\tau, u^0(\tau), u_\tau^0(\tau))d\tau.$$

$$u^3(t) = \int_0^t f(\tau, \sum_{k=0}^{2} u^k(\tau), \sum_{k=0}^{2} u_\tau^k(\tau))d\tau - \int_0^t f(\tau, \sum_{k=0}^{1} u^k(\tau), \sum_{k=0}^{1} u_\tau^k(\tau))d\tau.$$

$$u^4(t) = \int_0^t f(\tau, \sum_{k=0}^{3} u^k(\tau), \sum_{k=0}^{3} u_\tau^k(\tau))d\tau - \int_0^t f(\tau, \sum_{k=0}^{2} u^k(\tau), \sum_{k=0}^{2} u_\tau^k(\tau))d\tau.$$

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$$u^n(t) = \int_0^t f(\tau, \sum_{k=0}^{n-1} u^k(\tau), \sum_{k=0}^{n-1} u_\tau^k(\tau))d\tau - \int_0^t f(\tau, \sum_{k=0}^{n-2} u^k(\tau), \sum_{k=0}^{n-2} u_\tau^k(\tau))d\tau. \quad (2.5)$$

Adding these equations (2.4) through (2.5), we obtain

$$\sum_{k=0}^{n} u^k(t) = \int_0^t f(\tau, \sum_{k=0}^{n-1} u^k(\tau), \sum_{k=0}^{n-1} u_\tau^k(\tau))d\tau, n \geq 1 \quad (2.6)$$

We remind here that the choice of u^o in (2.4) is not unique, we can chose it to be any function of t and this depends on the nature of the problem in question as will be seen in our subsequent test problems. Also if the problem is a simple linear case, then the ADM and TDM will coincide and give the exact solution of the problem. However, the decomposition series (2.5) requires simple calculation and yields a series solution with excellent accelerated convergence of the linear differential equation.

3. THE TELECOPING DECOMPOSITION METHOD APPLIED TO FIRST-ORDER LINEAR AND NONLINEAR DIFFERENTIAL EQUATIONS.

Test problem 1

Consider the first order linear differential equation

$$u' + 2tu = 0, with, u(0) = 1 \text{chama(2007)} \quad (3.1)$$

From (2.3) we have that

$$u(t) = u(0) - \int_0^t (2u\tau)d\tau = 1 - \int_0^t (2u\tau)d\tau$$

$$u(t) = 1 - \int_0^t (2u\tau)d\tau$$

$$u^0 = 1$$

Take

$$u^1(t) = \int_0^t (2u^0\tau)d\tau = \int_0^t (2t)d\tau = -t^2$$

$$u^2(t) = \int_0^t 2\tau(u^0 + u^1)d\tau = \int_0^t (2\tau u^0)d\tau = -\int 2\tau(1-\tau^2)d\tau - (-t^2) = \frac{t^4}{2}$$

$$u^3(t) = \int_0^t 2\tau(u^0 + u^1 + u^2)d\tau - \int_0^t 2\tau(u^0 + u^1)d\tau = -\int 2\tau(1 - \tau^2 + \frac{\tau^4}{2})d\tau - (-t^2 + \frac{t^4}{2}) = \frac{t^6}{6}$$

$$u^4(t) = \int_0^t 2\tau(u^0 + u^1 + u^2 + u^3)d\tau - \int_0^t 2\tau(u^0 + u^1 + u^2)d\tau = -\int 2\tau(1 - \tau^2 + \frac{\tau^4}{2} - \frac{\tau^6}{6})d\tau - (-t^2 + \frac{t^4}{2} - \frac{t^6}{6}) = \frac{t^8}{24}.$$

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$$u^n(t) = (-1)^n \frac{t^{2n}}{n!}$$

Therefore,

$$u(t) = \sum_{n=0}^{\infty} u^n(t) = 1 - t^2 + \frac{t^4}{2} - \frac{t^6}{3!} = \frac{t^8}{4!} + \ldots.. = e^{=t^2}$$

Which is the exact solution of the problem in it closed form and in this case the ADM and the TDM coincides as earlier discussed.

Test Problem 2

Consider the following first –order nonlinear autonomous differential equation

$$u' = 2u - u^2, with, u(0) = 1$$

Applying TDM, we have that

$$u(t)=u(0)+\int(2u-u^2)dt=1+\int(2u-u^2)dt$$

Take

$$u^0=1$$

$$u^1=\int(2u^0-u^{0^2})dt=t$$

$$u^2=\int[2(u^0+u^1)-(u^0+u^1)^2]dt-\int(2u^0-u^{0^2})dt$$

$$=\int[2(1+t)-(1+t)^2]dt-\int dt=-\frac{t^2}{3}$$

$$u^3=\int[2(u^0+u^1+u^2)-(u^0+u^1+u^2)^2]dt-\int[2(u^0+u^1)-(u^0+u^1)^2]dt$$

$$u^3=\int[2(1+t-\frac{t^3}{3})-(1+t-\frac{t^2}{3})^2]dt-\int[2(1+t)-(1+t)^2]dt$$

$$=\frac{2t^5}{15}-\frac{t^7}{63}$$

$$u^4=\int[2(u^0+u^1+u^2+u^3)-(u^0+u^1+u^2+u^3)^2]dt-\int[2(u^0+u^1+u^2)-(u^0+u^1+u^2)^2]dt$$

$$u^4=\int[2(1+t-\frac{t^3}{3}+\frac{t^5}{15}-\frac{t^7}{63})-(1+t-\frac{t^3}{3}+\frac{2t^5}{15}-\frac{t^7}{63})^2]dt-\int[2(1+t-\frac{t^3}{3})-(1+t-\frac{t^3}{3})^2]dt$$

$$=-\frac{22t^7}{315}+\frac{11t^9}{2835}-\frac{134t^{11}}{51975}+\frac{4t^{13}}{12285}-\frac{t^{15}}{59535}+.........$$

Therefore

$$u(t)=1+t-\frac{t^3}{3}+\frac{2t^5}{15}-\frac{17t^7}{315}+\frac{11t^9}{2835}-\frac{134t^{11}}{51975}+\frac{4t^{13}}{12285}-\frac{t^{15}}{59535}+.........=1+\tanh(t)$$

This is the exact Taylor series of the analytical solution.

Test Problem 3

Consider the nonlinear initial value problem

$$u'+(1=t^2)u^2=t^4+2t^3+2t+1$$

with u(0)=1, Hosseini and Nasdzadeh (2006). Applying theTDM, We have that

$$u(t)=u(0)+\int(1+2t+2t^3+t^4)dt-\int(1+t^2)u^2dt$$

$$u(t)=1+\int(1+2t+2t^3+t^4)dt-\int(1+t^2)u^2dt$$

take

$$u^0=1+2t+t^2+\frac{2t^3}{3}+\frac{t^4}{2}+\frac{t^5}{5}$$

$$u^1=-\int[(1+t^2)(u^0)^2]dt$$

$$=-\int[(1+t^2)(1+2t+t^2+\frac{2t^3}{3}+\frac{t^4}{2}+\frac{t^5}{5})^2]dt$$

$$=-t-2t^2-\frac{7t^3}{3}-\frac{7t^4}{3}-\frac{32t^5}{15}-\frac{68t^6}{45}-\frac{311t^7}{315}-\frac{3t^8}{5}-\frac{497t^9}{1620}-\frac{19t^{10}}{150}-\frac{167t^{11}}{3300}-\frac{t^{12}}{60}-\frac{t^{13}}{325}$$

$$u^2=-\int[(1+t^2)(u^0+u^1)^2]dt-\int[(1+t^2)(u^0)dt$$

$$u^2=-\int[(1+t^2)(1+t-t^2-\frac{5t^3}{3}-\frac{11t^4}{6}-\frac{29t^5}{15}-\frac{68t^6}{45}-\frac{311t^7}{315}-\frac{3t^8}{5}-\frac{497t^9}{1620}-\frac{19t^{10}}{150}-\frac{167t^{11}}{3300}-\frac{t^{12}}{60}-\frac{t^{13}}{325})^2]dt$$

$$-(-t-2t^2-\frac{7t^3}{3}-\frac{7t^4}{3}-\frac{32t^5}{15}-\frac{68t^6}{45}-\frac{311t^7}{315}-\frac{3t^8}{5}-\frac{497t^9}{1620}-\frac{19t^{10}}{150}-\frac{167t^{11}}{3300}-\frac{t^{12}}{60}-\frac{t^{13}}{325})$$

$$u^2=t^2+\frac{7t^3}{3}+\frac{19t^4}{6}+\frac{53t^5}{15}+\frac{31t^6}{10}+\frac{601t^7}{315}+\frac{211t^8}{420}-\frac{2031t^9}{2835}-\frac{90727t^{10}}{56700}-....$$

$$\therefore u(t)=1+t+\frac{2t^3}{3}+\frac{4t^4}{3}+\frac{8t^5}{5}+\frac{143t^6}{90}+\frac{58t^7}{63}+\frac{41t^8}{420}-\frac{11603t^9}{11340}-\frac{23221t^{10}}{8100}-....$$

Is the analytic solution of the equation.

To showcase the accuracy of the TDM, we have computed the numerical values for some values of t compares to ADM and the results depicted in Table 1 below.

Table1: The Solution of u(t) for different values of t.

T	Exact solution	Adomian Method	Telescoping Method	Adomian Absolute Error	Telescoping Absolute Error
0.0	1.000000000	1.000000000	1.000000000	0.000000000	0.000000000
0.1	1.100000000	1.101273788	1.100817677	0.001273788	0.000817677
0.2	1.200000000	1.209530299	1.208091029	0.009530299	0.008091029
0.3	1.300000000	1.359283717	1.333942014	0.059283717	0.033942014
0.4	1.400000000	1.590982619	1.500438418	0.190982619	0.100438418
0.5	1.500000000	2.019639740	1.741629316	0.519639740	0.241629316
0.6	1.600000000	2.882724292	2.082235597	1.282724292	0.482235597
0.7	1.700000000	4.683061501	2.520426644	2.983061501	0.820426644
0.8	1.800000000	8.479851566	2.031286526	6.679851566	0.231286526
0.9	1.900000000	16.495189280	2.12062087	14.59518928	1.77937913

Test Problem 4

Consider the nonlinear inhomogeneous advection partial differential equation

$$u_t + uu_x = x + xt^2, u(x,0) = 0$$

Abdul Majid (1999)

Applying TDM (2.3), we have that

$$u(x,t) = u(x,0) + \int xdt + \int xt^2 dt - \int uu_x dt$$

With

$$u^0 = xt$$

$$u^1 = \frac{xt^3}{3} - \int xt^2 dt = \frac{xt^3}{3} - \frac{xt^3}{3} = 0$$

$$\therefore u(x,t) = xt$$

This gives the exact solution with only a step, unlike the ADM which until the noise terms cancels each other in $u^0$and u^2, u^2 and u^3 and so on before arriving at the exact solution.

4. THE TDM FOR SOLVING SECOND ORDER LINEAR AND NONLINEAR DIFFERENTIAL EQUATIONS.

In this section, we have modified equation (2.1) to read as

$u_t=f(t,x,u,u_x,u_{xx})$ or $u_{tt}=f(u,u_x,u_{xx})$ 4.1

If we are solving for u_t then write

$$u^1 = \int_0^t f(\tau, u^0, u_x^0, u_{xx}^0) dt$$

$$u^2 = \int f(\tau, \sum_{k=0}^{1} u^k(x,t), \sum_{o}^{1} u_x^k(x,t), \sum_{0}^{1} u_{xx}^k(x,t))dt - \int f(\tau, u^0, u_x^0, u_{xx}^0)dt$$

$$u^3(x,t) = \int_0^t f(\tau, \sum_{k=0}^{2} u^k(x,\tau), \sum_{k=0}^{2} u_x^k(x,\tau), \sum_{k=0}^{2} u_{xx}^k(x,\tau)d\tau - \int_0^t f(\tau, \sum_{k=0}^{1} u^k(x,\tau), \sum_{k=0}^{1} u_x^k(x,\tau), \sum_{k=0}^{1} u_{xx}^k(x,\tau))d\tau.$$

$$u^4(x,t) = \int_0^t f(\tau, \sum_{k=0}^{3} u^k(x,\tau), \sum_{k=0}^{3} u_x^k(x,\tau), \sum_{k=0}^{3} u_{xx}^k(x,\tau)d\tau - \int_0^t f(\tau, \sum_{k=0}^{2} u^k(x,\tau), \sum_{k=0}^{2} u_x^k(x,\tau), \sum_{k=0}^{2} u_{xx}^k(x,\tau))d\tau.$$

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$$u^n(x,t) = \int_0^t f(\tau, \sum_{k=0}^{n-1} u^k(x,\tau), \sum_{k=0}^{n-1} u_x^k(x,\tau), \sum_{k=0}^{n-1} u_{xx}^k(x,\tau)d\tau - \int_0^t f(\tau, \sum_{k=0}^{n-2} u^k(x,\tau), \sum_{k=0}^{n-2} u_x^k(x,\tau), \sum_{k=0}^{n-2} u_{xx}^k(x,\tau))d\tau.$$

and we are solving for u_{tt} then a two-fold integration is then considered.

Test problem 5

We consider the following nonlinear reaction- diffusion equation.

$$u_t - u_{xx} = u^2 - u_x^2, u(x,t) = u_0 = e^x \quad (4.3)$$

Applying TDM (4.1-4.2), we have

$$u_t = f(u, u_x, u_{xx})$$

$$\therefore u(x,t) = u(x,0) + \int f(u, u_x, u_{xx})dt$$

Take

$$u_0 = u^0 = e^x$$

$$u^1 = \int f(u^0, u_x^0, u_{xx}^0)dt = \int (u_{xx}^0 + (u^0)^2 - (u_x^0)^2)dt = \int (e^x + (e^x)^2 - (e^x)^2)dt = te^x.$$

$$u^2 = \int f(\sum_{k=0}^{1} u^k(x,t), \sum_{o}^{1} u_x^k(x,t), \sum_{0}^{1} u_{xx}^k(x,t))dt - \int f(u^0, u_x^0, u_{xx}^0)dt$$

$$= \int f[(e^x + te^x), (e^x + te^x), (e^x + te^x)]dt - te^x$$

$$= \int [(e^x + te^x) + (e^x + te^x)^2 - (e^x + te^x)^2]dt - te^x$$

$$= \int (e^x + te^x)dt - te^x = \frac{t^2 e^x}{2}$$

$$u^3(x,t) = \int_0^t f(\sum_{k=0}^{2} u^k(x,\tau), \sum_{k=0}^{2} u_x^k(x,\tau), \sum_{k=0}^{2} u_{xx}^k(x,\tau)d\tau - \int_0^t f(\sum_{k=0}^{1} u^k(x,\tau), \sum_{k=0}^{1} u_x^k(x,\tau), \sum_{k=0}^{1} u_{xx}^k(x,\tau))d\tau.$$

$$= \int [(e^x + te^x + \frac{t^2 e^x}{2}) + (e^x + te^x + \frac{t^2 e^x}{2})^2 - (e^x + te^x + \frac{t^2 e^x}{2})^2]dt - (te^x + \frac{t^2 e^x}{2})$$

$$= \int [(e^x + te^x + \frac{t^2 e^x}{2})]dt - (te^x + \frac{t^2 e^x}{2}) = \frac{t^3 e^x}{6}$$

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Thus,

$$u(x,t) = \sum_{n=0}^{\infty} u^n(x,t) = e^x + te^x + \frac{t^2 e^x}{2} + \frac{t^3 e^x}{6} + \frac{t^4 e^x}{24} + ...$$

$$ie, u(x,t) = \sum_{n=0}^{\infty} u^n(x,t) = e^x(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + ...)$$

And in closed form, coincide with the exact solution given by $u(x,t)=e^{x+t}$

Problem 6

Consider a nonhomogenous wave equation

$$u_{xx} - uu_{tt} = 2 - 2(t^2 + x^2), Kaya, (1999)$$

With initial conditions $u(x,0)=x^2$, $u(o,t)=t^2$, $u_x(0,t)=0$.

Applying TDM, we have that

$$u_{xx} = f(x, t, u, u_{tt})$$

$$u(x,t) = u(x,0) + xu_x(x,o) + \iint f(x,t,u,u_{tt})dxdx$$
$$= t^2 + \iint [2 - 2(t^2 + x^2) + uu_{tt}]dxdx$$
$$= t^2 + x^2 - x^2t^2 - \frac{x^2}{6} + \iint [uu_{tt}]dxdx$$

Take

$$u^0 = t^2 + x^2$$
$$u^1 = -x^2t^2 - \frac{x^2}{6} + \iint [2(t^2 + x^2)]dxdx$$
$$u^1 = -x^2t^2 - \frac{x^2}{6} + x^2t^2 + \frac{x^2}{6} = 0$$
$$\therefore u(x,t) = \sum_{n=0}^{\infty} u^n(x,t) = t^2 + x^2.$$

Which is the exact solution of the problem, unlike the ADM, which can be easily observed that the self-canceling noise terms appear between various components as follows:

$$u(x,t) = t^2 + x^2 - x^2t^2 - \frac{x^4}{6} + x^2t^2 + \frac{x^4}{6} - \frac{x^4t^2}{3} - \frac{7x^6}{90} + \frac{2x^6t^2}{15} + \frac{x^8}{16} + \frac{x^4t^2}{3} + ...$$

Though the result will arrive to the exact solution u(x,t)=x^2 +t^2 by cancelling the third term in u_0 and the first term in u_1, the fourth term in u_0 and the first term in u_2 and so on, thus, consumed much time when compared to TDM.

CONCLUSION

We have presented and analyzed the Telescoping Decomposition Method (TDM) for solving linear and nonlinear initial value problem. Mohammmed Al-Refai et al (2008). We have applied the TDM tosolve some physical problems and obtained closed form as well as exact solutions for these problems. It has been shown in the forgoing discussions that once the initial value is identified, with proper use of TDM it is possible to obtain analytical solution to a class of linear and nonlinear first order differential equations. In our quest to find and discover if this method can be applicable for solving the second order differential equations, we have redefined the function f(x,u)given by Mohammed Al-Refai et al (2008) and with the few test examples used, we have successfully demonstrated the efficiency, accuracy and convergence of this method for second order linear and nonlinear equations. We have also confirmed that the use of TDM reduces the volume of computation by not requiring the Adomian polynomials for nonlinear terms. Finally, we conclude that, the idea of TDM can be developed to deal with higher order differential equations and various types of functional equations as well.

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