

# $\tau_1\tau_2\#RG$ - Homeomorphisms in Bitopological Spaces

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**Abstract** - A bijection  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $\tau_1\tau_2\#$ regular generalized  $\#$ -homeomorphism if  $f$  and  $f^{-1}$  are  $\tau_1\tau_2\#rg$ -continuous. Also we introduce new class of maps, namely  $\tau_1\tau_2\#rgc$ -homeomorphisms which form a subclass of  $\tau_1\tau_2\#rg$ -homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all  $\tau_1\tau_2\#rgc$ homeomorphisms forms a group under the operation composition of maps.

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## 1. INTRODUCTION

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces  $X$  and  $Y$  is a bijective map  $f: X \rightarrow Y$  when both  $f$  and  $f^{-1}$  are continuous. It is well known that as Jänich [[9], p.13] says correctly: homeomorphisms play the same role in topology that linear isomorphism play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphism in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Maki et al. [12] introduced  $g$  -homeomorphisms and  $gc$  -homeomorphisms in topological spaces.

In this paper, we introduce the concept of  $\tau_1\tau_2\#rg$ -homeomorphism and study the relationship between homeomorphisms,  $\tau_1\tau_2\#g$ homeomorphism,  $g\tau_1\tau_2\#$  homeomorphism and  $\tau_1\tau_2\#rgc$ homeomorphism.

Also we introduce new class of maps  $\tau_1\tau_2\#rgc$ -homeomorphism which form a subclass of  $\tau_1\tau_2\#rg$ -homeomorphism. This class of maps is closed under composition of maps. We prove that the set of all  $\tau_1\tau_2\#rgc$ homeomorphisms forms a group under the operation composition of maps.

Let us recall the following definition which we shall require later.

**Definition 1.1.** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called:

- (1)  $\tau_1\tau_2$  preopen set if  $A \subseteq \tau_1\text{int}\tau_2\text{cl}(A)$  and a  $\tau_1\tau_2$  preclosed set if  $\tau_2\text{cl}\tau_1\text{int}(A) \subseteq A$ .
- (2)  $\tau_1\tau_2$  semiopen set[1] if  $A \subseteq \tau_2\text{cl}\tau_1\text{int}(A)$  and a  $\tau_1\tau_2$  semiclosed set if  $\tau_1\text{int}\tau_2\text{cl}(A) \subseteq A$ .
- (3)  $\tau_1\tau_2$  regular open set if  $A = \tau_1\text{int}\tau_2\text{cl}(A)$  and a  $\tau_2$  regular closed set if  $A = \tau_2\text{cl}\tau_1\text{int}(A)$ .
- (4)  $\tau_1\tau_2$   $\pi$ - open set if  $A$  is a finite union of regular open sets.
- (5)  $\tau_1\tau_2$  regular semi open if there is a  $\tau_1$  regular open  $U$  such  $U \subseteq A \subseteq \tau_2\text{cl}(U)$ .

**Definition 1.2.** A subset  $A$  of  $(X, \tau_1, \tau_2)$  is called

- (1)  $\tau_1\tau_2$  generalized closed set (briefly,  $\tau_1\tau_2g$ -closed) if  $\tau_2\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

- (2)  $\tau_1\tau_2$  regular generalized closed set (briefly,  $\tau_1\tau_2$ rg-closed) if  $\tau_2\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -regular open in  $X$ .
- (3)  $\tau_1\tau_2$  generalized preregular closed set (briefly,  $\tau_1\tau_2$ gpr-closed) if  $\tau_2\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -regular open in  $X$ .
- (4)  $\tau_1\tau_2$  regular weakly generalized closed set (briefly,  $\tau_1\tau_2$ wg-closed) if  $\tau_2\text{cl}\tau_1\text{int}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -regular open in  $X$ .
- (5)  $\tau_1\tau_2$  rw-closed if  $\tau_2\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ regular semi open.
- (6)  $\tau_1\tau_2$ #rg-closed if  $\tau_2\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ rw-open.

The complements of the above mentioned closed sets are their respective open sets.

**Definition: 1.3.** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called #rg-continuous if  $f^{-1}(V)$  is  $\tau_1\tau_2$ #rg-closed in  $(X, \tau_1, \tau_2)$  for every closed subset  $V$  of  $(Y, \sigma_1, \sigma_2)$ .

**Definition: 1.4.** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $\tau_1\tau_2$ #rg-irresolute if  $f^{-1}(V)$  is  $\tau_1\tau_2$ #rg-closed in  $X$  for every  $\tau_1\tau_2$ #rg-closed subset  $V$  of  $Y$ .

**Definition 1.5.** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $\tau_1\tau_2$ #rg-closed (resp.  $\tau_1\tau_2$ #rg-open) if for every #rg-closed (resp.  $\tau_1\tau_2$ #rg-open) set  $U$  of  $X$  the set  $f(U)$  is  $\tau_1\tau_2$ #rg-closed (resp.  $\tau_1\tau_2$ #rg-open) in  $Y$ .

**Definition 1.6.** A map  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be

- (i)  $\tau_1\tau_2$  g homeomorphism[12] if both  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  g-continuous,
- (ii)  $\tau_1\tau_2$  gs- homeomorphism [6] if both  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  gs-continuous,
- (iii)  $\tau_1\tau_2$  rwg- homeomorphism[14] if both  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  rwg-continuous,
- (iv)  $\tau_1\tau_2$  gc- homeomorphism[12] if both  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  gc-irresolute.

## 2. $\tau_1\tau_2$ #RG-homeomorphism in Bitopological Spaces

**Definition 2.1.** A bijection  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $\tau_1\tau_2$  #regular generalized homeomorphism (briefly,  $\tau_1\tau_2$  #rg-homeomorphism) if  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-continuous. We denote the family of all  $\tau_1\tau_2$  #rg homeomorphisms of a topological space  $(X, \tau_1, \tau_2)$  onto itself by  $\tau_1\tau_2$ #rg-h $(X, \tau_1, \tau_2)$ .

**Example 2.2.** Consider  $X = Y = \{a, b, c, d\}$  with topologies  $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$  and  $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$ ,  $\sigma_1 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$  and  $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity map. Then  $f$  is bijective,  $\tau_1\tau_2$ #rgcontinuous and  $f^{-1}$  is  $\tau_1\tau_2$ #rg-continuous. Hence  $f$  is  $\tau_1\tau_2$ #rg-homeomorphism.

**Theorem 2.3.** Every homeomorphism is  $\tau_1\tau_2$  #rg homeomorphism, but not conversely.

**Proof.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a homeomorphism. Then  $f$  and  $f^{-1}$  are continuous and  $f$  is bijection. Since every continuous function is  $\tau_1\tau_2$  #rg-continuous,  $f$  and  $f^{-1}$  is  $\tau_1\tau_2$ #rg-continuous. Hence  $f$  is  $\tau_1\tau_2$  #rghomeomorphism. The converse of the above theorem need not be true, as seen from the following example.

**Example 2.4.** Consider  $X = Y = \{a, b, c, d\}$  with topologies  $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$  and  $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$ ,  $\sigma_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$  and  $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity map. Then  $f$  is  $\tau_1\tau_2$ #rg-homeomorphism it is not homeomorphism, since the inverse image of closed set of  $\{a,d\}$  in  $X$  is  $\{a,d\}$  which is not closed in  $Y$ .

**Theorem 2.5.** Every  $\tau_1\tau_2$  #rg-homeomorphism is g-homeomorphism.

**Proof.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a  $\tau_1\tau_2$  #rg-homeomorphism. Then  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-continuous and  $f$  is bijection. Since every  $\tau_1\tau_2$  #rg-continuous function is  $g$ -continuous,  $f$  and  $f^{-1}$  are  $g$ -continuous. Hence  $f$  is  $\tau_1\tau_2$  g-homeomorphism.

**Corollary 2.6.** Every  $\tau_1\tau_2$  #rg-homeomorphism is  $\tau_1\tau_2$  gs-homeomorphism.

**Proof.** By the fact that every  $\tau_1\tau_2$  g homeomorphism is  $\tau_1\tau_2$  gs-homeomorphism and by theorem 2.5.

**Corollary 2.7.** Every  $\tau_1\tau_2$  #rg-homeomorphism is  $\tau_1\tau_2$  gsp-homeomorphism.

**Proof.** By the fact that every gshomeomorphism is  $\tau_1\tau_2$  gsp-homeomorphism and by corollary 2.6.

**Theorem 2.8.** Every  $\tau_1\tau_2$  #rg-homeomorphism is  $\tau_1\tau_2$  rg-homeomorphism.

**Proof.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a  $\tau_1\tau_2$  #rg-homeomorphism. Then  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-continuous and  $f$  is bijection. Since every #rg-continuous function is  $\tau_1\tau_2$  rg-continuous,  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  rg-continuous. Hence  $f$  is  $\tau_1\tau_2$  rg homeomorphism.

**Corollary 2.9.** Every  $\tau_1\tau_2$  #rg-homeomorphism is  $\tau_1\tau_2$  rwg-homeomorphism and  $\tau_1\tau_2$  gp rhomeomorphism.

**Proof.** By the fact that every  $\tau_1\tau_2$  rg homeomorphism is  $\tau_1\tau_2$  rwg-homeomorphism and  $\tau_1\tau_2$  gpr-homeomorphism, and by theorem 2.8.

**Theorem 2.10.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a bijective  $\tau_1\tau_2$  #rg- continuous map. Then the following are equivalent.

- (i)  $f$  is a  $\tau_1\tau_2$  #rg- open map
- (ii)  $f$  is  $\tau_1\tau_2$  #rg-homeomorphism,
- (iii)  $f$  is a  $\tau_1\tau_2$  #rg -closed map.

**Proof.** Suppose (i) holds. Let  $V$  be open in  $(X, \tau_1, \tau_2)$ . Then by (i),  $f(V)$  is  $\tau_1\tau_2$  #rg-open in  $(Y, \sigma_1, \sigma_2)$ . But  $f(V) = (f^{-1})^{-1}(V)$  and so  $(f^{-1})^{-1}(V)$  is  $\tau_1\tau_2$  #rgopen in  $(Y, \sigma_1, \sigma_2)$ . This shows that  $f^{-1}$  is  $\tau_1\tau_2$  #rg-continuous and it proves (ii).

Suppose (ii) holds. Let  $F$  be a closed set in  $(X, \tau_1, \tau_2)$ . By (ii),  $f^{-1}$  is  $\tau_1\tau_2$  #rg-continuous and so  $(f^{-1})^{-1}(F) = f(F)$  is  $\tau_1\tau_2$  #rg-closed in  $(Y, \sigma_1, \sigma_2)$ . This proves (iii).

Suppose (iii) holds. Let  $V$  be open in  $(X, \tau_1, \tau_2)$ . Then  $V^c$  is closed in  $(X, \tau_1, \tau_2)$ . By (iii),  $f(V^c)$  is  $\tau_1\tau_2$  #rg-closed in  $(Y, \sigma_1, \sigma_2)$ . But  $f(V^c) = (f(V))^c$ . This implies that  $(f(V))^c$  is  $\tau_1\tau_2$  #rg-closed in  $(Y, \sigma_1, \sigma_2)$  and so  $f(V)$  is  $\tau_1\tau_2$  #rg-open in  $(Y, \sigma_1, \sigma_2)$ . This proves (i).

**Definition 2.11.** A bijection  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $\tau_1\tau_2$  #rgc-homeomorphism if both  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-irresolute. We say that spaces  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  are  $\tau_1\tau_2$  #rgchomeomorphic if there exists a  $\tau_1\tau_2$  #rgchomeomorphism from  $(X, \tau_1, \tau_2)$  onto  $(Y, \sigma_1, \sigma_2)$ . We denote the family of all  $\tau_1\tau_2$  #rgc-homeomorphisms of a topological space  $(X, \tau_1, \tau_2)$  onto itself by  $\tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ .

**Theorem 2.12.** Every  $\tau_1\tau_2$  #rgc-homeomorphism is a  $\tau_1\tau_2$  #rg-homeomorphism but not conversely.

**Proof.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be an  $\tau_1\tau_2$  #rgchomeomorphism. Then  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-irresolute and  $f$  is bijection. By Theorem 4.2 in [22],  $f$  and  $f^{-1}$  are  $\tau_1\tau_2$  #rg-continuous. Hence  $f$  is  $\tau_1\tau_2$  #rg-homeomorphism. The converse of the above theorem is not true in general as seen from the following example.

**Example 2.13.** Consider  $X = Y = \{a, b, c, d\}$  with  $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,b,c\}\}$  and  $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}, \{a,b,c\}\}$ ,  $\sigma_1 = \{X, \emptyset, \{c\}, \{a,b\}, \{a,b,c\}\}$  and  $\sigma_2 = \{Y, \emptyset, \{c\}, \{a,b\}, \{a,b,c\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow$

$(Y, \sigma_1, \sigma_2)$  be defined by  $f(a) = b, f(b) = c, f(c) = a$  and  $f(d) = d$ . Then  $f$  is  $\tau_1\tau_2$  #rg-homeomorphism but it is not  $\tau_1\tau_2$  #rgc-homeomorphism, since  $f$  is not  $\tau_1\tau_2$  #rg-irresolute.

**Theorem 2.14.** Every  $\tau_1\tau_2$  #rgc-homeomorphism is  $\tau_1\tau_2$  g-homeomorphism but not conversely.

**Proof.** Proof follows from Theorems 2.5 and Theorem 2.12.

**Remark 2.15**  $\tau_1\tau_2$  #rgc-homeomorphism and  $\tau_1\tau_2$  gc - homeomorphism are independent as seen from the following example.

**Example 2.16** Let  $X = Y = \{a,b,c,d\}$  with  $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$  and  $\tau_2 = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$ ,  $\sigma_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$  and  $\sigma_2 = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity map. Then  $f$  is  $\tau_1\tau_2$  #rgc - homeomorphism but it is not  $\tau_1\tau_2$  gc-homeomorphism, since  $f$  is not  $\tau_1\tau_2$  gc-irresolute.

**Theorem 2.17.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \rho_1, \rho_2)$  are  $\tau_1\tau_2$  #rgc-homeomorphisms, then their composition  $gof: (X, \tau_1, \tau_2) \rightarrow (Z, \rho_1, \rho_2)$  is also  $\tau_1\tau_2$  #rgc-homeomorphism.

**Proof.** Let  $U$  be a  $\tau_1\tau_2$  #rg-closed set in  $(Z, \rho_1, \rho_2)$ . Since  $g$  is  $\tau_1\tau_2$  #rg-homeomorphism,  $g^{-1}(U)$  is  $\tau_1\tau_2$  #rgclosed in  $(Y, \sigma_1, \sigma_2)$ . Since  $f$  is  $\tau_1\tau_2$  #rg-homeomorphism,  $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$  is  $\tau_1\tau_2$  #rg closed in  $(X, \tau_1, \tau_2)$ . Therefore  $gof$  is  $\tau_1\tau_2$  #rgirresolute. Also for a  $\tau_1\tau_2$  #rg-closed set  $G$  in  $(X, \tau_1, \tau_2)$ , We have  $(gof)(G) = g(f(G)) = g(W)$ , where  $W = f(G)$ . By hypothesis,  $f(G)$  is  $\tau_1\tau_2$  #rg-closed in  $(Y, \sigma_1, \sigma_2)$  and so again by hypothesis,  $g(f(G))$  is a  $\tau_1\tau_2$  #rg-closed set in  $(Z, \rho_1, \rho_2)$ . That is  $(gof)(G)$  is a  $\tau_1\tau_2$  #rg-closed set in  $(Z, \rho_1, \rho_2)$  and therefore  $(gof)^{-1}$  is  $\tau_1\tau_2$  #rg-irresolute. Also  $gof$  is a bijection. Hence  $gof$  is  $\tau_1\tau_2$  #rg-homeomorphism.

**Theorem 2.18.** The set  $\tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  is a group under the composition of maps.

**Proof.** Define a binary operation  $*$ :  $\tau_1\tau_2$  #rgch $(X, \tau_1, \tau_2) \times \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2) \rightarrow \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  by  $f * g = gof$  for all  $f, g \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  and  $o$  is the usual operation of composition of maps. Then by theorem 2.17,  $gof \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ . We know that the composition of maps is associative and the identity map  $I: (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2)$  belonging to  $\tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  serves as the identity element. If  $f \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ , then  $f^{-1} \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  such that  $f \circ f^{-1} = f^{-1} \circ f = I$  and so inverse exists for each element of  $\tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ . Therefore  $(\tau_1\tau_2$  #rgch $(X, \tau_1, \tau_2), o)$  is a group under the operation of composition of maps.

**Theorem 2.19.** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a  $\tau_1\tau_2$  #rgc-homeomorphism. Then  $f$  induces an isomorphism from the group  $\tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$  onto the group  $\tau_1\tau_2$  #rgc-h $(Y, \sigma_1, \sigma_2)$ .

**Proof.** Using the map  $f$ , we define a map  $\varphi_f: \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2) \rightarrow \tau_1\tau_2$  #rgc-h $(Y, \sigma_1, \sigma_2)$  by  $\varphi_f(h) = f \circ h \circ f^{-1}$  for every  $h \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ . Then  $\varphi_f$  is a bijection. Further, for all  $h_1, h_2 \in \tau_1\tau_2$  #rgc-h $(X, \tau_1, \tau_2)$ ;  $\varphi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \varphi_f(h_1) \circ \varphi_f(h_2)$ . Hence  $\varphi_f$  is a homomorphism and so it is an isomorphism induced by  $f$ .

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