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Sylow Prime Group

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Abstract: **Sylow *p* – subgroup is a vital part of the discussion in any algebraic activity dealing with group theory. So, it is natural that, is every group can have Sylow *p* – subgroups or any specific distinction can be put forward that confirms a particular group is either a group having Sylow *p* – subgroup or there is no single Sylow *p* – subgroup. With a view to characterize the groups whether possessing a Sylow *p* – subgroup or not here is an activity that progresses the discussion by one step ahead. This discussion leads to groups of order *p* and order *pq* that has applications in Galvan theory.**

INTRODUCTION:

Suppose *n* is a positive integer. By the fundamental theorem of arithmetic, either *n* is a group of prime order or it is a product of primes expressible in a unique manner as $n = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k}$ where p_i '*s* are prime numbers with the respective multiplicities $\alpha_i, 1 \le i \le k$.

Since each p_i is a prime number, $1 \le i \le k$, there exists a cyclic group $G_i, 1 \le i \le k$ such that $|G_i| = p_i^{\alpha_i}, 1 \le i \le k$

Now, $G = G_1 \times G_2 \times ... \times G_k$

1. *Sylow Prime Group*:

Definition : if *G* is a finite group, *p* is a prime number such that $p^n \mid |G|$ and $p^{n+1} \nmid |G|$, then any subgroup of *G* of order p^n is called a Sylow *p* – subgroup of *G*. (1.1)

Definition 2: a group (*G*, *) is said to be a Sylow prime group if every non trivial subgroup of *G* is a Sylow p_i - subgroup for some prime factor p_i of the order of *G*.
(1.2)

Since $|G| = n = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k}$, and in view of Lagrange's theorem of finite groups, and properties of divisibility, it follows that $p_i^{\alpha_i} \mid |G|$ for each $1 \le i \le k$ and $p_{i+1}^{\alpha_{i+1}} \nmid |G|$ by the unique representation of the integer *n*.

It is not necessary that there is a subgroup of *G* of order $p_i^{\alpha_i}$ for every $1 \le i \le k$. This confirms that every group of finite order is not a Sylow prime group.

To verify these observations, the following instances will show a finite group that admits the definition of Sylow prime group and another instance for not.

2. *Working on Sylow Prime Groups*:

Consider the symmetric group of order 6 or the symmetric group on 3 symbols.

$S_3 = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ where $f_i : A \to A$ is a bijection for each $1 \le i \le 6$ and $A = \{a, b, c\}$

$f_1 = \{(a,a),(b,b),(c,c)\}$

$f_2 = \{(a,b),(b,a),(c,c)\}$

$f_3 = \{(a,a),(b,c),(c,b)\}$

$f_4 = \{(a,c),(b,b),(c,a)\}$

$f_5 = \{(a,b),(b,c),(c,a)\}$

$f_6 = \{(a,c),(b,a),(c,b)\}$

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The composition of mappings '$\circ$' is the operation that makes S_3 a group such that $|S_3| = 6 = 2^1 \times 3^1$, the unique representation by fundamental theorem of arithmetic

It can be easily seen that $H_1 = \{f_1, f_4\}, H_2 = \{f_1, f_5, f_6\}$ are the only non trivial subgroups such that $|H_1| = 2^1, 2^2 \nmid |S_3|$ and so, H_1 is a Sylow 2- subgroup of S_3

Similarly, $|H_2| = 3^1, 3^2 \nmid |S_3|$ and so, H_2 is a Sylow 3 – subgroup of S_3 (2.1)

Take another instance.

$\square \big/ 12 = \{[1],[5],[7],[11]\}$ is a group under multiplication modulo 12 denoted by $\times_{12}$.

$\left| \square \big/ 12 \right| = 4 = 2^2$

$H_1 = \{[1],[11]\}$ is a non trivial subgroup of order 2^1.

Also, $2^2 \mid \left| \square \big/ 12 \right|$ which shows H_1 is not a Sylow 2 – subgroup of $\square \big/ 12$

So, this is an example of a group that is not a Sylow prime group. (2.2)

The working (2.1) and (2.2) will confirm that all finite groups are not Sylow Prime Groups.

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