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Sum Divisor and Divided Square Sum Cordial Labeling of Fuzzy Graph

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Abstract - In this work, we introduce the concept of sum divisor and divided square sum cordial labeling of fuzzy graph. In this paper we explore the idea of Fuzzy Divisor Cordial Labeling (FDCL) of graphs, which combines divisor cordial graph labeling with fuzzy set theory. A bijective mapping or membership assignment on the vertices that induces an edge membership mapping based on divisibility conditions and guarantees that the membership values differ by no more than one is known as an FDCL. For a split graphs $K_{1,n}$, we examine whether fuzzy sum divisor cordial labeling and divided square sum cordial labeling exists. Additionally, we determine the structural characteristics that must be met for a fuzzy graph to accept our labeling scheme.

Key words: Fuzzy Labeling, cordial Labeling, sum divisor cordial labeling , divided square sum cordial labeling, Split graph.

1.INTRODUCTION

A significant area of mathematics that is frequently used to simulate real-world networks is graph theory. Graphs can be used to illustrate a wide range of real-world issues in biology, computer science, communication, transportation, and social sciences.

In the fascinating field of graph labelling, a graph's vertices and edges are given numbers or symbols based on predetermined guidelines. Because it seeks to maintain a balanced distribution of labels, amicable labeling is one of the most widely used graph labeling techniques. If a graph's vertex and edge labels meet certain balance requirements, it is considered cordial.

Relationships between items are ambiguous or lacking in many real-world scenarios. Such uncertainty cannot be adequately represented by classical graphs. Fuzzy graphs were developed to get around this restriction. Each vertex and edge in a fuzzy graph is given a membership value between 0 and 1, which indicates the strength or degree of the association.

A novel method for researching uncertain networks is to combine the idea of amicable labeling with fuzzy graphs. When fuzzy graphs are cordially labeled, the fuzzy structure is preserved and the cordial labeling requirements are met. This expansion aids in the analysis of systems where balanced labeling and uncertainty are crucial. The notion of sum divisor and divided square sum cordial labeling of fuzzy graphs is presented and its fundamental characteristics are examined in this study. For split graphs, the existence of sum divisor and divided square sum cordial labeling is examined, and the instances are shown. Additionally, this study lays the groundwork for further investigation into sophisticated labeling techniques in fuzzy and neutrosophic graph theory.

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2.PRELIMINARIES

Definition.2. 1

Let C^* = (V, E) be a simple graph. Then C^* = (φ, ψ) is called a fuzzy graph on C^*, if φ: V → [0, 1] and ψ: E → [0, 1] and for all $x,y \in E$.

$$\psi\,(x, y) \leq \min\,[\varphi(x), \varphi\,(y)]$$

A fuzzy graph C= (φ, ψ)) on C^* is called a fuzzy labeling graph if φ and ψ one to one map for all $x, y \in E$.

Definition. 2.2

A cordial labeling is a binary vertex labeling of a graph G = (V, E) where the absolute difference between the count of vertices labeled 0 and 1, and the count of edges labeled 0 and 1, both differ by at most 1. A graph that admits this specific assignment is called a cordial graph.

Definition. 2.3

Let f: V(G)→ { 1,2,3… | $V(G)$|} be a bijective function and let the edges are defined as f*: E(G) →[0,1], be defined as f (e = uv) = 1; if 2| [f(u) + f(v)]

0; otherwise

Then f is called sum divisor cordial labeling of graph G if $|\mathrm{ef}(0) - \mathrm{ef}(1)| \leq 1$.

A graph with a sum divisor cordial labeling is called sum divisor cordial graph.

Definition. 2.4

A bijective mapping σ from the vertex set V to the interval [0, 1]. Each edge uv is assigned an induced label d (where d ϵ(0, 1) if either σ(u) divides σ (v) or σ (v) divides σ (u). If neither condition is met, the edge gets a label of 0 is called fuzzy divisor cordial labeling graphs.

Definition. 2.5

Let G= (V, E) be a simple graph and f: V→{1,2,3,..|V|} be bijection. For each edge uv, assign the label d if $\frac{(\mathrm{f(u)})2+(\mathrm{f(v)})2}{\mathrm{f(u)+f(v)}}$ is even and the label 0 otherwise. f is called divided square sum cordial labeling if

$|ef(0) - ef(\mathrm{d})| \leq 1$, where $ef(\mathrm{d})$ *and* ef (0) denote the number of edges labeled with d and not labeled with d respectively.

A fuzzy graph G is called divided square sum cordial if it admits divided square sum cordial labeling.

Definition. 2.6

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Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.

Definition. 2.7

For a graph G is the split graph is obtained by adding each vertex v a new vertex v′ such that v′ is adjacent to every vertex that is adjacent to v in G noted as *spl*(G).

3.SOME RESULTS ON SUM DIVISOR AND DIVIDED SQUARE SUM CORDIAL LABELLING OF A FUZZY GRAPH

Preposition:3.1

If G is a fuzzy divisor cordial graph, then G + e is also a fuzzy divisor cordial graph for all e∈E(G)**.**

Proof:

Case 1: Let n be the even size of the fuzzy divisor cordial graph G, then it follows that $e_{f(d)}= e_{f(0)}=\frac{n}{2}$, Let e be any edge in G which is labeled either 0 or d.Then we draw G+e ,we labelled the new edge as 0 ,we have $e_{f(0)}=\frac{n+1}{2}$ or label the new edge as d,we have $e_{f(d)}=\frac{n+1}{2}$ edges for d. Hence we increase the edge labelled either 0 or d we have $|\,\text{ef}(0) - \text{ef}(1)|=\left|\frac{n+1}{2}-\frac{n}{2}\right| \leq 1$ or $\left|\frac{n}{2}-\frac{n+1}{2}\right| \leq 1$.

Case 2: Let n be the odd size of the fuzzy divisor cordial graph G, then it follows that either $e_{f(0)}$ is $\frac{n}{2}$ edges; another $e_{f(d)} = \frac{n-1}{2}$ edges. Then we draw G + e, by adding edge labelled either 0 or d. If we add the edge e labelled d from G, we have $e_{f(d)} = \frac{n-1+1}{2}=\frac{n}{2}$ edges.Thus we have $e_{f(d)}= e_{f(0)}=\frac{n}{2}$; $|\,\text{ef}(0) - \text{ef}(1)| \leq 1$.Therefore G+e is fuzzy divisor cordial labeling graph.

Preposition:3.2

Spl($K_{1,n}$) is a sum divisor and divided square sum cordial labeling of fuzzy graph.

Proof: Let ϕ be the apex and $\phi_1, \phi_2,\ldots\ldots \phi_n$ be the pendent vertices of $K_{1,n}$ and $\phi', \phi'_1, \phi'_2, \phi'_3\ldots\ldots \phi'_n$ are the vertices corresponding to $\phi, \phi_1, \phi_2,\ldots\ldots \phi_n$ in Spl($K_{1,n}$). To define f:V(G) → {1,2,3…} ,we consider the following cases.

Case 1(i): Spl($K_{1,2}$) is a sum divisor cordial fuzzy graph.

Let us label the vertices as follows ϕ=0.2 , ,ϕ_1= 0.1, ϕ_2=0.4; $\phi' = 0.5$ $\phi'_1 = 0.6$ $\phi'_2 = 0.3$.The edge values are calculated by using fuzzy sum divisor cordial labeling condition

f(e = uv) = d ; if 2| [f(u) + f(v)]

0; otherwise

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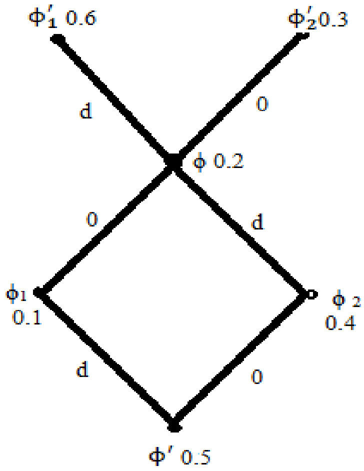

Split graph Spl($K_{1,2}$)

$\phi\ \phi'_1$=(0.2+0.6)/2=0.4 ; $\phi\ \phi'_2$= (0.2+0.3)/2=0.25 ; ϕ,ϕ_1=(0.2+0.1)/2= 0.15; ϕ,ϕ_2=(0.2+0.4)/2= 0.3; $\phi'\phi 1$ =(0.5+0.1)/2= 0.3; $\phi'\phi 2$ =(0.5+0.4)/2=0.45..Bythe definition of fuzzy sum divisor cordial labeling condition we noted that $e_{f(d)}$=3 and $e_{f(0)}$=3 and it also satisfy the cordial labeling condition $|\mathrm{ef}(0)-\mathrm{ef}(1)| \leq 1$.Therefore Spl($K_{1,2}$) is a fuzzy sum divisor cordial graph.

Case 1(ii):Spl($K_{1,2}$) is a divided square sum cordial labeling of fuzzy graph.

Let us label the vertices as follows ϕ=0.2, ϕ_1= 0.1, ϕ_2 =0.5; $\phi' = 0.4\ \phi'_1 = 0.6\ \phi'_2 = 0.3$.The edge values are calculated by using fuzzy divided square sum cordial labeling condition. we assign the label d if

if $\frac{(f(u))2+(f(v))2}{f(u)+f(v)}$ is even and assign label 0 otherwise. Let us calculate the edge as follows

$\phi\ \phi'_1 = \frac{(0.2)2+(0.6)2}{0.2+0.6}$ =0.5; $\phi\ \phi'_2 = \frac{(0.2)2+(0.3)2}{0.2+0.3}$ =0.26; $\phi\ \phi_1 = \frac{(0.2)2+(0.1)2}{0.2+0.1}$ =0.17 ; $\phi\ \phi_2 = \frac{(0.2)2+(0.5)2}{0.2+0.5}$ =0.41;

$\phi'\phi_1 = \frac{(0.4)2+(0.1)2}{0.4+0.1}$ =0.34; $\phi'\phi_2 = \frac{(0.4)2+(0.5)2}{0.4+0.1}$ =0.46.Here $e_{f(d)}$=3 and $e_{f(0)}$=3 and it also satisfy the cordial labeling condition $|\mathrm{ef}(0)-\mathrm{ef}(1)| \leq 1$.Therefore Spl($K_{1,2}$) is a fuzzy divided square sum cordial graph.

Case II(i): Spl($K_{1,3}$) is a sum divisor cordial fuzzy graph

Let us label the vertices as follows ϕ=0.2 , ,ϕ_1= 0.1, ϕ_2 =0.8; ϕ_3 =0.4: $\phi' = 0.5\ \phi'_1 = 0.6\ \phi'_2 = 0.7$:.

$\phi'_3 = 0.3$ The edge values are calculated by using fuzzy sum divisor cordial labeling condition

f(e = uv) = d ; if 2| [f(u) + f(v)]

0; otherwise

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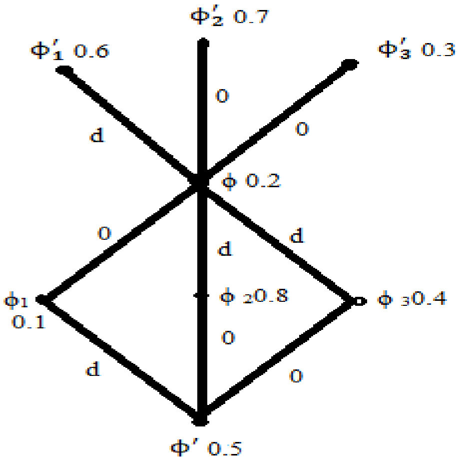

Split graph Spl($K_{1,3}$)

$\phi\,\phi'_1$=(0.2+0.6)/2=0.4 ; $\phi\,\phi'_2$= (0.2+0.7)/2=0.45: $\phi\,\phi'_3$= (0.2+0.3)/2=0.25 ; $\phi\phi_1$=(0.2+0.1)/2= 0.15; $\phi\phi_2$=(0.2+0.8)/2=0.5;$\phi\phi_3$=(0.2+0.4)/2= 0.3: $\phi'\phi 1$=(0.5+0.1)/2=0.3; $\phi'\phi 2$=(0.5+0.8)/2=0.65. $\phi'\phi 3$= (0.5+0.4)/2=0.45. By the definition of fuzzy sum divisor cordial labeling condition, we noted that $e_{f(d)}$=4 and $e_{f(0)}$=5 and it also satisfy the cordial labeling condition $|\mathrm{ef}(0) - \mathrm{ef}(1)| \leq 1$.Therefore Spl($K_{1,3}$) is a fuzzy sum divisor cordial graph.

Case II(ii): Spl($K_{1,3}$) is a divided square sum cordial labeling of fuzzy graph.

Let us label the vertices as follows ϕ=0.2 , ϕ_1= 0.1, ϕ_2 =0.8, ϕ_3 =0.5; $\phi' = 0.4$ $\phi'_1 = 0.6$ $\phi'_2 = 0.7$, $\phi'_3 = 0.3$.The edge values are calculated by using fuzzy divided square sum cordial labeling condition we assign the label d if

if $\frac{(f(u))2+(f(v))2}{f(u)+f(v)}$ is even and assign label 0 otherwise. Let us calculate the edge as follows

$\phi\,\phi'_1$= $\frac{(0.2)2+(0.6)2}{0.2+0.6}$ |=0.5; $\phi\,\phi'_2$= $\frac{(0.2)2+(0.7)2}{0.2+0.7} = 0.59$, $\phi\,\phi'_3$= $\frac{(0.2)2+(0.3)2}{0.2+0.3}$ =0.26; $\phi\,\phi_1$= $\frac{(0.2)2+(0.1)2}{0.2+0.1}$ =0.17 $\phi\,\phi_2$= $\frac{(0.2)2+(0.8)2}{0.2+0.8}$ =0.68; $\phi\,\phi_3$= $\frac{(0.2)2+(0.5)2}{0.2+0.5}$ =0.41;

$\phi'\phi_1$= $\frac{(0.4)2+(0.1)2}{0.4+0.1}$ =0.34; $\phi'\phi_2$ = $\frac{(0.4)2+(0.8)2}{0.4+0.8}$ =0.67, $\phi'\phi_3$ = $\frac{(0.4)2+(0.5)2}{0.4+0.5}$ =0.46.Here $e_{f(d)}$=4 and $e_{f(0)}$=5 and it also satisfy the cordial labeling condition $|\mathrm{ef}(0) - \mathrm{ef}(1)| \leq 1$.Therefore Spl($K_{1,3}$) is a fuzzy divided square sum cordial graph.

Case III(i): Spl($K_{1,7}$) is a sum divisor cordial fuzzy graph

Let us label the vertices as follows

ϕ=0.02, $\phi' = 0.03$; ϕ_i =0.15-(n-1)0.01 ;1 $\leq$i$\leq$7; n=1,2,3,4,5,6,7.

ϕ'_i= 0.08-(n-1)0.01; n=1,2,3,4,5,8

ϕ'_i= 0.08+ (n-1)0.01; n=9

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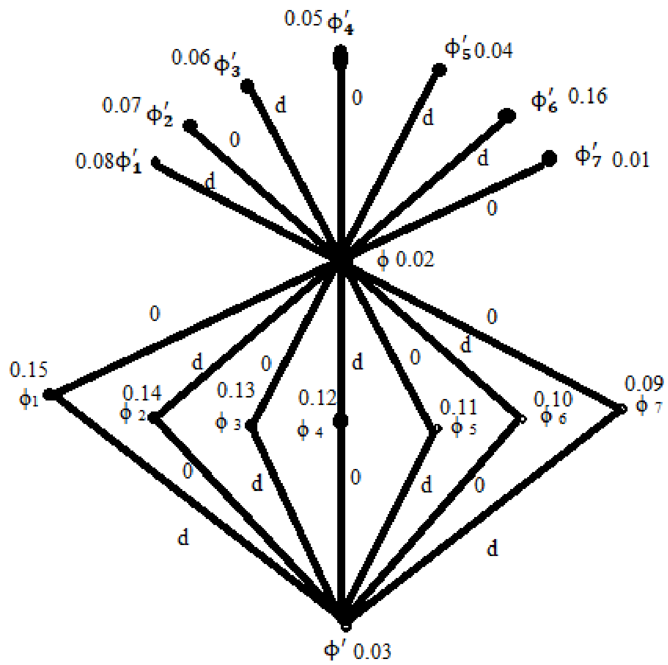
Vol. 15 Issue 08 , August - 2026

$\phi\ \phi'_1$=(0.02+0.08)/2=0.05 ; $\phi\ \phi'_2$= (0.02+0.07)/2=0.045: $\phi\ \phi'_3$= (0.02+0.06)/2=0.04 ; $\phi\ \phi'_4$= (0.02+0.05)/2 =0.035;$\phi\ \phi'_5$= (0.02+0.04)/2 =0.03; $\phi\ \phi'_6$= (0.02+0.16)/2=0.09;
$\phi\ \phi'_7$=(0.02+0.01)/2=0.015;$\phi\phi_1$=(0.02+0.15)/2=0.085;$\phi\phi_2$=(0.02+0.14)/2=0.08;$\phi\phi_3$=(0.02+0.13)/2=0.075;$\phi\phi_4$=(0.02+0.12)/2=0.07;$\phi\phi_5$=(0.02+0.11)/2=0.065;$\phi\phi_6$=(0.02+0.10)/2=0.06;$\phi\phi_7$=(0.02+0.09)/2=0.055; $\phi'\phi1$=(0.03+0.15)/2=0.09;
$\phi'\phi2$=(0.03+0.14)/2=0.085;
$\phi'\phi3$=(0.03+0.13)/2=0.08; $\phi'\phi4$=(0.03+0.12)/2=0.075; $\phi'\phi5$=(0.03+0.11)/2=0.07; $\phi'\phi6$=(0.03+0.10)/2=0.065;
$\phi'\phi7$=(0.03+0.09)/2=0.06

In the labeling pattern we have $e_{f(d)}$=11 and $e_{f(0)}$=10.Thus we have $|\,\mathrm{ef}(0)\ -\mathrm{ef}(1)| \leq 1$.



Split graph Spl($K_{1,7}$)

Case III(ii): Spl($K_{1,7}$) is a divided square sum cordial fuzzy graph

Let us label the vertices as follows

ϕ=0.02, $\phi' = 0.03$; ϕ_i=0.15-(n-1)0.01 ;1 ≤i≤7; n=1,2,3,4,5,6,8.

ϕ'_i= 0.08-(n-1)0.01; n=1,2,3,4,5,8

ϕ'_i= 0.08+ (n-1)0.01; n=2,9

$\phi\ \phi'_1= \frac{(0.02)2+(0.09)2}{0.02+0.09}$ |=0.08; $\phi\ \phi'_2= \frac{(0.02)2+(0.07)2}{0.02+0.07} = 0.06$; $\phi\ \phi'_3= \frac{(0.02)2+(0.06)2}{0.02+0.06} = 0.05$; $\phi\ \phi'_4= \frac{(0.02)2+(0.05)2}{0.02+0.05} =$ 0.04; $\phi\ \phi'_5= \frac{(0.02)2+(0.04)2}{0.02+0.04} = 0.03$; $\phi\ \phi'_6= \frac{(0.02)2+(0.16)2}{0.02+0.16} = 0.14$; $\phi\ \phi'_7= \frac{(0.02)2+(0.01)2}{0.02+0.01} = 0.02$; $\phi\ \phi_1= \frac{(0.02)2+(0.15)2}{0.02+0.15}$ =0.14; $\phi\ \phi_2= \frac{(0.02)2+(0.14)2}{0.02+0.14}$ =0.13; $\phi\ \phi_3= \frac{(0.02)2+(0.13)2}{0.02+0.13}$ =0.12; $\phi\ \phi_4= \frac{(0.02)2+(0.12)2}{0.02+0.12}$ =0.11; $\phi\ \phi_5= \frac{(0.02)2+(0.11)2}{0.02+0.11}$ =0.10; $\phi\ \phi_6= \frac{(0.02)2+(0.10)2}{0.02+0.10}$ =0.09; $\phi\ \phi_7= \frac{(0.02)2+(0.08)2}{0.02+0.08}$ =0.07; $\phi'\phi_1= \frac{(0.03)2+(0.15)2}{0.03+0.15}$ =0.13; $\phi'\phi_2 = \frac{(0.03)2+(0.14)2}{0.03+0.14}$ =0.12; $\phi'\phi_3 = \frac{(0.03)2+(0.13)2}{0.03+0.13}$ =0.11; $\phi'\phi_4 = \frac{(0.03)2+(0.12)2}{0.03+0.12}$ =0.10; $\phi'\phi_5 = \frac{(0.03)2+(0.11)2}{0.03+0.11}$ =0.09; $\phi'\phi_6 = \frac{(0.03)2+(0.10)2}{0.03+0.10}$ =0.08; $\phi'\phi_7 = \frac{(0.03)2+(0.08)2}{0.03+0.08}$ =0.08;

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In the labeling pattern we have $e_{f(d)}$=11 and $e_{f(0)}$=10.Thus we have $|\ \mathrm{ef}(0)\ -\mathrm{ef}(1)| \leq 1$. From the above cases we conclude that the split graph Spl($K_{1,n}$) is a sum divisor cordial graph and divided square sum cordial labeling of fuzzy graph.

4.Conclusion

The definitions of sum divisor cordial labelling and divided square sum cordial labeling of fuzzy graphs for the split graph Spl($K_{1,n}$) have been investigated in this paper.

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